Limiti di successioni con limiti notevoli e parametrici

$$\lim_n \frac{\frac{1}{n!} - \frac{1}{(n+1)!}}{\log(n!+5) - \log(n!)} =$$

$$(n+1)\,n!$$

$$= \lim_n \frac{n+\cancel{1} - \cancel{1}}{(n+1)\,n!} \cdot \frac{1}{\log\left(\frac{n!+5}{n!}\right)} =$$

$$= \lim_n \frac{n}{(n+1)!} \cdot \frac{1}{\dfrac{\log\left(1+\frac{5}{n!}\right)}{\frac{5}{n!}} \cdot \frac{n!}{5}} = \frac{1}{5}$$

$(n+1)!=(n+1)\,\cancel{n!}$

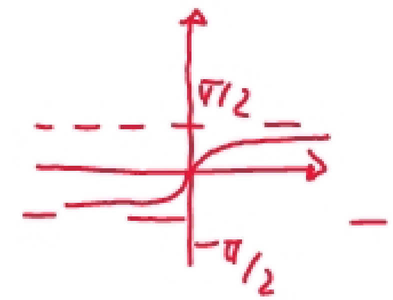
$\frac{5}{n!} \to 0$

$\frac{n}{n+1} \to 1$

$\to 1$

$$\lim_n \frac{\log\left(1+\frac{1}{n}\right)}{\frac{1}{n}} = 1 \qquad \frac{1}{n} \to 0$$

$$\bullet \lim_{n} \frac{(n+7)^n + n\log n + 14\, n^n}{n^n + 8\, n! + 7^n} \cdot \underbrace{\operatorname{arctg} n}_{\to \frac{\pi}{2}} =$$



$$= \frac{\pi}{2} \lim_{n} \frac{\cancel{n^n}\left[\overbrace{\frac{(n+7)^n}{n^n}}^{\to e^7} + \overbrace{\frac{n\cdot \log n}{n^n}}^{\to 0} + 14\right]}{\cancel{n^n}\left[1 + 8\,\overbrace{\frac{n!}{n^n}}^{\to 0} + \overbrace{\frac{7^n}{n^n}}^{\to 0}\right]} = \frac{\pi}{2}\left(e^7+14\right)$$

$$\frac{(n+7)^n}{n^n} = \left(\frac{n+7}{n}\right)^n = \left[\left(1+\frac{7}{n}\right)^{\frac{n}{7}}\right]^7 \to e^7$$

$$\frac{n\cdot \log n}{n^n} = \underbrace{\frac{n^2}{n^n}}_{\to 0}\cdot \underbrace{\frac{\log n}{n}}_{\to 0} \to 0$$

$$\frac{7^n}{n^n} = \left(\frac{7}{n}\right)^n$$

$n > 7$ $\quad \frac{7}{n} < 1$

CONFRONTO DI INFINITI

$n! < n^n$

$7^n < n^n$

$$\cdot \lim_{n} n^3 \left(e^{\frac{1}{n}}-1\right)\left(\sqrt{n^2+\log\left(1+\frac{3}{n}\right)}-\sqrt{n^2+1}\right) =$$

$$= \lim_{n} n^3 \left(e^{\frac{1}{n}}-1\right) \frac{\left(\sqrt{n^2+\log\left(1+\frac{3}{n}\right)}-\sqrt{n^2+1}\right)\left(\sqrt{n^2+\log\left(1+\frac{3}{n}\right)}+\sqrt{n^2+1}\right)}{\sqrt{n^2+\log\left(1+\frac{3}{n}\right)}+\sqrt{n^2+1}} =$$

$\sqrt{n^2} \to n$ $\qquad \sqrt{n^2} = n$ no $|\ |$

$$= \lim_{n} n^3 \left(e^{\frac{1}{n}}-1\right) \frac{\cancel{n^2}+\log\left(1+\frac{3}{n}\right)-\cancel{n^2}-1}{n\left[\sqrt{1+\frac{\log\left(1+\frac{3}{n}\right)}{n^2}}+\sqrt{1+\frac{1}{n^2}}\right]} =$$

$$= \lim_{n} n^{\cancel{3}} \underbrace{\frac{e^{\frac{1}{n}}-1}{\frac{1}{n}}}_{\to 1} \cdot \cancel{n} \; \frac{\log\left(1+\frac{3}{n}\right)-1}{\cancel{n}} \cdot \frac{1}{\underbrace{\left[\sqrt{1+\frac{\log\left(1+\frac{3}{n}\right)}{n^2}}+\sqrt{1+\frac{1}{n^2}}\right]}_{2}}$$

$\to 0 \qquad \to 0$

LIMITE RICAVATO DAL LIMITE NOT.

$$\frac{e^{\frac{1}{n}}-1}{\frac{1}{n}} \to 1$$

$$= \lim_{n} \frac{n\left[\overbrace{\log\left(1+\overbrace{\frac{3}{n}}^{\to 0}\right)}^{\to 0} - 1\right]}{2} = \frac{(+\infty)(-1)}{2} = -\infty$$

$$\cdot \lim_n \underbrace{\left(\sqrt{n^{\alpha-6}+8n+1}-\sqrt{n^2+7n+1}\right)}_{\text{Limite 2}} + \underbrace{\frac{2\log n+\sin^2 n-\cos^2 n}{\log[(n+2)!]-\log(n!)}}_{\text{Limite 1}}$$

LIMITE 1

$$\lim_n \frac{2\log n+\overset{[0,1]}{\sin^2 n}-\overset{[0,1]}{\cos^2 n}}{\log[(n+2)!]-\log(n!)} = \lim_n \frac{\log n\left[2+\overset{\to 0}{\dfrac{\sin^2 n}{\log n}}-\overset{\to 0}{\dfrac{\cos^2 n}{\log n}}\right]}{\log\left[\dfrac{(n+2)!}{n!}\right]} =$$

$$\sim \lim_n \frac{2\log n}{\log\left[\dfrac{(n+2)(n+1)\cancel{n!}}{\cancel{n!}}\right]} =$$

$$\lim_n \sin n = \nexists$$

$$\lim_n \sin^2 n = \nexists$$

$$= \lim_{n} \frac{2\log n}{\log\left[n^2\left(1+\frac{2}{n}\right)\left(1+\frac{1}{n}\right)\right]} =$$

$$= \lim_{n} \frac{2\log n}{\log n^2 + \underbrace{\log\left(1+\frac{2}{n}\right)}_{\downarrow 0} + \underbrace{\log\left(1+\frac{1}{n}\right)}_{\downarrow 0}}$$

$$\simeq \lim_{n} \frac{2\log n}{2\log n} = 1$$

LIMITE 2

$$\lim_{n} \left(\sqrt{n^{\alpha-6}+8n+1} - \sqrt{n^2+7n+1} \right) =$$

RAZIONALIZZANDO

$$= \lim_{n} \frac{n^{\alpha-6}+8n+\cancel{1} - n^2-7n-\cancel{1}}{\sqrt{n^{\alpha-6}+8n+1} + \sqrt{n^2+7n+1}} =$$

$$= \lim_{n} \frac{n^{\alpha-6} + n - n^2}{\underbrace{n^{\frac{\alpha-6}{2}}}\sqrt{1+\frac{8}{n^{\alpha-5}}+\frac{1}{n^{\alpha-6}}} + \underbrace{n}\sqrt{1+\frac{7}{n}+\frac{1}{n^2}}}$$

$$= \lim_n \frac{n^{\alpha-6} - n^2 + n}{n^{\frac{\alpha-6}{2}}\left(\sqrt{1+\frac{8}{n^{\alpha-5}}+\frac{1}{n^{\alpha-6}}} + n^{1-\frac{\alpha-6}{2}}\sqrt{1+\frac{7}{n}+\frac{1}{n^2}}\right)}$$

$1-\frac{\alpha-6}{2}<0$ $\qquad 2-\alpha+6<0$ $\qquad \alpha>8$

$n^{1-\frac{\alpha-6}{2}} \to 0$

$(\alpha-6) > \left(\frac{\alpha-6}{2}\right)$

grado del N > grado del D

LIM N°2 $\to +\infty$

LIM N°2 + LIM N°1 $\to +\infty$

$1 - \frac{\alpha - 6}{2} = 0$ $\qquad \alpha = 8$

$$\text{LIM } 2 = \lim_{n} \frac{\overset{n^2}{\cancel{n^{8-6}}} - \cancel{n^2} + \cancel{n}}{\cancel{n^{8-6}}\left(\sqrt{\boxed{1} + \underset{\to 0}{\frac{8}{n^{8-5}}} + \underset{\to 0}{\frac{1}{n^{8-6}}}} + \underset{\to 1}{n^{0}}\sqrt{\boxed{1} + \underset{\to 0}{\frac{7}{n}} + \underset{\to 0}{\frac{1}{n^2}}}\right)} = \frac{1}{2}$$

$$\text{LIM } 2 + \text{LIM } 1 = \frac{1}{2} + 1 = 3/2$$

$1 - \frac{\alpha - 6}{2} > 0$ $\qquad \alpha < 8$

prevale $(-n^2)$ $\qquad$ LIM 2 $\to -\infty$

$$\bullet\ \lim_{n} \sin\left(\frac{1}{\sqrt{n}}\right)(n+1)^{\alpha}\underbrace{\left(\sqrt{n+e^{\frac{1}{n}}}-\sqrt{n+1}\right)}_{\text{RAZIONALIZZARE}} =$$

$$= \lim_{n} \frac{\sin\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}\cdot\sqrt{n}}\, n^{\alpha}\left(1+\frac{1}{n}\right)^{\alpha} \frac{\cancel{n}+e^{\frac{1}{n}}-\cancel{n}-1}{\sqrt{n+e^{\frac{1}{n}}}+\sqrt{n+1}} =$$

$$= \lim_{n} \underbrace{\frac{\sin\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}}_{\to 1}\cdot\frac{n^{\alpha}}{\sqrt{n}}\cdot\underbrace{\left(1+\frac{1}{n}\right)^{\alpha}}_{\to 1}\underbrace{\frac{e^{\frac{1}{n}}-1}{\frac{1}{n}}}_{\to 1}\cdot n\cdot\frac{1}{\sqrt{n}\left(\sqrt{1+\frac{e^{\frac{1}{n}}}{n}}+\sqrt{1+\frac{1}{n}}\right)}$$

$$\frac{e^{\frac{1}{n}}}{n} \to 0$$

LIMITE NOTEVOLE

$$\frac{\sin\frac{1}{n}}{\frac{1}{n}} \to 1 \qquad \frac{1}{n} \to 0$$

$$= \frac{\ell}{n} \quad \frac{n^{\alpha}}{\sqrt{n} \quad n \quad \sqrt{n}} \cdot \frac{1}{2} =$$

$$= \frac{1}{2} \quad \frac{\ell}{n} \quad \frac{n^{\alpha}}{n^{2}} = \frac{1}{2} \quad \frac{\ell}{n} \quad n^{\alpha - 2}$$

se	$\alpha - 2 > 0$	$\alpha > 2$	LIM $\to +\infty$	divergenza
	$\alpha - 2 = 0$	$\alpha = 2$	LIM $\to \frac{1}{2}$	convergenza a $\frac{1}{2}$
	$\alpha - 2 < 0$	$\alpha < 2$	LIM $\to 0$	convergenza a zero

$$\bullet \ \lim_{n} n^{\alpha} \Big[\underbrace{\sqrt{n^2+3n+1} - \sqrt{n^2+n+1}}_{\text{RAZIONALIZZARE}} \Big] \frac{n^{\frac{3}{n^2}} - 1}{4 \log n} =$$

$$= \lim_{n} n^{\alpha} \frac{\cancel{n^2} + 3n \cancel{+1} - \cancel{n^2} - n \cancel{-1}}{\sqrt{n^2+3n+1} + \sqrt{n^2+n+1}} \cdot \frac{e^{\log n^{\frac{3}{n^2}}} - 1}{4 \log n} =$$

$$= \lim_{n} n^{\alpha} \cdot \overbrace{\left[\frac{2n}{n \Big(\underbrace{\sqrt{1+\frac{3}{n}+\frac{1}{n^2}} + \sqrt{1+\frac{1}{n}+\frac{1}{n^2}}}_{\to 2} \Big)} \right]}^{\to 1} \cdot \frac{e^{\boxed{\frac{3}{n^2} \cdot \log n}_{\to 0}} - 1}{4 \boxed{\frac{3}{n^2} \log n} \cdot \frac{n^2}{3}}$$

$$= \lim_{n} n^{\alpha} \cdot \underbrace{\frac{e^{\frac{3}{n^2}\log n} - 1}{\frac{3}{n^2}\cdot \log n}}_{\downarrow \; 1} \cdot \frac{3}{4n^2} =$$

$$= \frac{3}{4} \lim_{n} n^{\alpha - 2}$$

se $\alpha - 2 > 0$ $\qquad \alpha > 2$ $\qquad$ LIM $= +\infty$

se $\alpha - 2 = 0$ $\qquad \alpha = 2$ $\qquad$ LIM $= 3/4$

se $\alpha - 2 < 0$ $\qquad \alpha < 2$ $\qquad$ LIM $= 0$

$$\cdot \lim_{n} \left(1-\cos\left(\frac{7}{n}\right)\right) \cdot \sqrt{1+n^{4\alpha}} \cdot \log\left(1+\frac{2}{n^2}\right)$$

$$\frac{\left(1-\cos\frac{7}{n}\right)\left(1+\cos\frac{7}{n}\right)}{1+\cos 7/n} = \frac{1-\cos^2\frac{7}{n}}{1+\cos\frac{7}{n}} = \frac{\operatorname{sen}^2\frac{7}{n}}{1+\cos\frac{7}{n}}$$

$$\sqrt{1+n^{4\alpha}} = \sqrt{n^{4\alpha}\left(\frac{1}{n^{4\alpha}}+1\right)} = n^{2\alpha}\sqrt{\left(\frac{1}{n^{4\alpha}}+1\right)}$$

$$= \frac{e}{n}\,\frac{\sin^2\frac{7}{n}}{\left(1+\cos\frac{7}{n}\right)}\; n^{2\alpha}\sqrt{\frac{1}{n^{4\alpha}}+1}\;\cdot\;\underbrace{\frac{\log\left(1+\frac{2}{n^2}\right)}{\frac{2}{n^2}}}_{\to 1}\cdot\frac{n^2}{2}$$

$$\sin^2\frac{7}{n} = \sin\frac{7}{n}\cdot\sin\frac{7}{n}$$

$$\left(\frac{\sin\frac{7}{n}}{7/n}\cdot\frac{\sin\frac{7}{n}}{7/n} \sim \frac{49}{n^2}\right)$$

$$= \frac{e}{n}\,\underbrace{\frac{\sin^2\frac{7}{n}}{\frac{49}{n^2}}}_{\to 1}\cdot\frac{n^2}{49}\cdot\underbrace{\frac{1}{1+\cos\frac{7}{n}}}_{\downarrow\ \cancel{2}}\; n^{2\alpha}\underbrace{\sqrt{1+\frac{1}{n^{4\alpha}}}}_{\downarrow\ 1}\cdot\frac{\cancel{2}}{n^2}$$

$$= \frac{l}{n} \quad \frac{1}{\frac{n^2}{4g}} \cdot n^{2\alpha} \cdot \frac{1}{n^2} =$$

$$= \frac{l}{n} \quad 4g \quad \frac{n^{2\alpha}}{n^4} = 4g \, \frac{l}{n} \, n^{2\alpha - 4}$$

$2\alpha - 4 > 0 \qquad \alpha > 2 \qquad$ LIM $\to +\infty$

$2\alpha - 4 = 0 \qquad \alpha = 2 \qquad$ LIM $\to 4g$

$2\alpha - 4 < 0 \qquad \alpha < 2 \qquad$ LIM $\to 0$

$$\cdot \lim_{n} \left(\cosh \frac{7}{n}\right)^{\frac{3n}{7 \sinh 7/n}} =$$

passaggio all' esponenziale

$$= \lim_{n} e^{\log\left(\cosh \frac{7}{n}\right)^{\frac{3n}{7 \sinh 7/n}}} =$$

$$\cosh x = \frac{e^{x}+e^{-x}}{2}$$

$$\sinh x = \frac{e^{x}-e^{-x}}{2}$$

$$= \lim_{n} e^{\frac{3n}{7 \sinh \frac{7}{n}} \cdot \log\left(\cosh \frac{7}{n}\right)} =$$

$$= e^{\lim_{n} \left[\frac{3n}{7 \cdot \frac{e^{7/n}-e^{-7/n}}{2}} \cdot \log \frac{e^{\frac{7}{n}}+e^{-\frac{7}{n}}}{2}\right]}$$

$$\frac{e^{\frac{7}{n}} - e^{-\frac{7}{n}}}{2} = \frac{e^{\frac{14}{n}} - 1}{2\, e^{7/n}}$$

$$\frac{e^{\frac{7}{n}} + e^{-\frac{7}{n}}}{2} = \frac{e^{\frac{14}{n}} + 1}{2\, e^{7/n}}$$

considero la parte dell' esponente

$$= \frac{\ell}{n} \left\{ \frac{3n}{7 \cdot \frac{e^{14/n} - 1}{2\, e^{7/n}}} \cdot \log\left(\frac{e^{\frac{14}{n}} + 1}{2\, e^{7/n}} \right) \right\}$$

$$= \frac{\ell}{n} \left\{ \frac{3n}{\frac{7}{2}\, \underset{\downarrow 1}{\frac{1}{e^{7/n}}} \cdot \underbrace{\frac{e^{14/n} - 1}{\frac{14}{n}}}_{\to 1} \cdot \frac{n}{14}} \cdot \log\left(\frac{e^{\frac{14}{n}} + 1}{2\, e^{7/n}} \right) \right\} =$$

$$\frac{3n}{\frac{7}{8}\,\underbrace{\frac{1}{e^{7/n}}}_{\to 1}\cdot\frac{14}{n}}$$ AND $$3n\cdot\frac{8}{7}\cdot\frac{n}{14}\,7 =$$

$$= \frac{e}{n}\;\frac{3}{49}\cdot\underbrace{n^2}_{\to +\infty}\cdot\underbrace{\log\left(\frac{e^{\frac{14}{n}}+1}{2\,e^{7/n}}\right)}_{0}$$

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