Lez 8

Limiti di successioni

- $$\lim_{n} \frac{n + \sin n}{\operatorname{arctg} n - n} = \frac{+\infty}{\frac{\pi}{2} - \infty} = \frac{\infty}{\infty} \quad F.I.$$

$$-1 \leq \sin n \leq 1$$

$$\lim_{n} \sin n = \nexists$$

$$\lim_{n} \cos n = \nexists$$

$$\lim_{n} \operatorname{tg} n = \nexists$$

$$\lim_{n} \operatorname{ctg} n = \nexists$$

$$\lim_{n} (n + \sin n) = \underbrace{+\infty + [-1, 1]}_{+\infty}$$

$$= \lim_{n} \frac{\cancel{n}\left(1 + \overset{\to 0}{\frac{\sin n}{n}}\right)}{\cancel{n}\left(\underset{\to 0}{\frac{\operatorname{arctg} n}{n}} - 1\right)} = -1$$

$$0 + \infty$$

$$\lim_{n} \frac{3^n + \sin n}{\operatorname{arctg} n - 5 \cdot 2^n} =$$

$$= \lim_{n} \frac{3^n \left(1 + \overbrace{\frac{\sin n}{3^n}}^{\to 0}\right)}{2^n \left(\underbrace{\frac{\operatorname{arctg} n}{2^n}}_{\to 0} - 5\right)} = \lim_{n} \left(-\frac{1}{5}\right) \underbrace{\left(\frac{3}{2}\right)^n}_{\to +\infty} = -\infty$$

$$\lim_{n} \frac{n^{n-3} + (n-3)^n}{6 n^n + 7 n!} =$$

$$= \lim_{n} \frac{\cancel{n^n} \left[\overbrace{n^{-3}}^{\to 0} + \frac{(n-3)^n}{n^n} \right]}{\cancel{n^n} \left[6 + 7 \underbrace{\frac{n!}{n^n}}_{\to 0} \right]} = \frac{e^{-3}}{6}$$

$$\left(\frac{n-3}{n}\right)^n = \left(1 - \frac{3}{n}\right)^n \to e^{-3}$$

CONFRONTO

$n! < n^n$

LIMITE NO TE VOLE

$$\lim_{n} \left(1+\frac{1}{n}\right)^{n} = e$$

$$\lim_{n} \left(1-\frac{3}{n}\right)^{n} = \lim_{n} \left[\left(1+\frac{1}{-\frac{n}{3}}\right)^{\frac{n}{-3}}\right]^{-3} = e^{-3}$$

$$+\frac{1}{-\frac{n}{3}} = t \qquad \left(1+\frac{1}{t}\right)^{t} \to e$$

$$\lim_{n} \left(1+\frac{1}{n}\right)^{n+5} =$$

$$= \lim_{n} \left[\underbrace{\left(1+\frac{1}{n}\right)^{n}}_{\to e} \cdot \left(1+\underbrace{\frac{1}{n}}_{\to 0}\right)^{5}\right] = e \qquad 1^5 = 1$$

$$\bullet \lim_{n} \left\{ \log_a \left[\underset{+\infty - \infty}{n - \sqrt{n^2-1}} \right] + \log_a n \right\} \qquad \begin{array}{l} a > 0 \\ a \neq 1 \end{array}$$

$$= \lim_{n} \left\{ \log_a \left(\left[n - \sqrt{n^2-1} \right] \cdot n \right) \right\} =$$

$$= \log_a \left\{ \lim_{n} \left(n \cdot \frac{(n - \sqrt{n^2-1})(n + \sqrt{n^2-1})}{n + \sqrt{n^2-1}} \right) \right\} =$$

$$= \log_a \left\{ \lim_{n} \left(\frac{n\,(\cancel{n^2} - \cancel{n^2} + 1)}{n + \sqrt{n^2-1}} \right) \right\} =$$

$$= \log_a \left\{ \lim_{n} \frac{n}{n + \sqrt{n^2-1}} \right\} = \log_a \left\{ \lim_{n} \frac{\cancel{n}}{\cancel{n}\left(1 + \sqrt{1 - \underset{\to 0}{\frac{1}{n^2}}}\right)} \right\} = \log_a \frac{1}{2}$$

PARI GRADO

$$\lim_{n \to \infty} \left\{ \underset{+\infty}{\sqrt[3]{n+1}} - \underset{-\infty}{\sqrt[3]{n-3}} \right\} =$$

DIFFERENZA DI CUBI

$$a^3 - b^3 = (a-b)(a^2+b^2+ab)$$

$$a^3 + b^3 = (a+b)(a^2+b^2-ab)$$

$$= \lim_{n} \left\{ \left(\sqrt[3]{n+1} - \sqrt[3]{n-3} \right) \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{(n-3)^2} + \sqrt[3]{(n+1)(n-3)}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{(n-3)^2} + \sqrt[3]{(n+1)(n-3)}} \right\} =$$

$$= \lim_{n} \frac{\cancel{n}+1 - \cancel{n} + 3}{\underbrace{\sqrt[3]{(n+1)^2}}_{+\infty} + \underbrace{\sqrt[3]{(n-3)^2}}_{+\infty} + \underbrace{\sqrt[3]{(n+1)(n-3)}}_{+\infty}} = \frac{4}{+\infty} = 0$$

$$\bullet \lim_{n} \frac{5\,(n+1)^n + e^n \log n}{2\,(n-1)^n - n!\,\cos\left(n\frac{\pi}{2}\right)} =$$

$$= \lim_{n} \frac{\dfrac{5\,(n+1)^n + e^n \log n}{n^n}}{\dfrac{2\,(n-1)^n - n!\,\cos\left(n\frac{\pi}{2}\right)}{n^n}} =$$

$$= \lim_{n} \frac{5\,\dfrac{(n+1)^n}{n^n} + \underbrace{\dfrac{e^n \log n}{n^n}}_{(*)\ 0}}{2\,\dfrac{(n-1)^n}{n^n} - \underbrace{\dfrac{n!}{n^n}}_{\to 0}\cos(n\pi/2)} = \lim_{n} \frac{5\,\overbrace{\left(1+\frac{1}{n}\right)^n}^{\to e}}{2\,\underbrace{\left(1-\frac{1}{n}\right)^n}_{\to e^{-1}}} = \frac{5}{2}\,\frac{e}{e^{-1}} = 5/2\; e^2$$

[assume i valori $(-1; 0; 1)$]

(*) $\dfrac{e}{n}\,\dfrac{e^{n}\log n}{n^{n}} =$

$$= \frac{e}{n}\,\frac{e^{n-1}}{n^{n-1}} \cdot e \cdot \underset{\downarrow\ 0}{\frac{\log n}{n}}$$

esponenziale con base variabile (<1) $\left(\dfrac{e}{n}\right)^{n-1}$

$\log n < n$ confronto

per $n \geq 3$ $\quad 0 < \left(\dfrac{e}{n}\right) < 1$ decrescente

$$\frac{e}{n}\left(\frac{e}{n}\right)^{n-1} = 0$$

$$\bullet \lim_{n} \frac{n! \cdot (n+1)!}{\log(n!+5) - \log(n!)} =$$

- $$\lim_{n} \frac{e^{3\sqrt{\log^2 n + \log n + 1}}}{n^3} = \frac{\infty}{\infty}$$

$$= \lim_{n} \frac{e^{3\sqrt{\log^2 n + \log n + 1}}}{e^{\log n^3}} =$$

$$= \lim_{n} e^{\left\{3\sqrt{\log^2 n + \log n + 1} - \log n^3\right\}} = e$$

passiamo a calcolare il limite dell'esponente

$$= \lim_{n} \left\{ 3\sqrt{\log^2 n + \log n + 1} - 3\log n \right\}$$

$$\cdot \lim_{n} 3 \left\{ \frac{\cancel{\log^2 n} + \log n + 1 - \cancel{\log^2 n}}{\sqrt{\log^2 n + \log n + 1} + \log n} \right\} =$$

$$= 3 \lim_{n} \frac{\cancel{\log n}\left(1 + \frac{1}{\log n}\right)^{\to 0}}{\cancel{\log n}\left(\sqrt{1 + \underbrace{\frac{1}{\log n}}_{\to 0} + \underbrace{\frac{1}{\log^2 n}}_{\to 0}} + 1\right)} =$$

$$= 3 \cdot \frac{1}{2} = 3/2$$

$$= \text{limite Totale} = e^{3/2}$$

$$\bullet \ \lim_{n} \frac{n^n + 3^n}{4^{n \log n}} =$$

$$= \lim_{n} \left[\frac{n^n}{4^{n \log n}} + \frac{3^n}{4^{n \log n}} \right] =$$

$$= \lim_{n} \left[\frac{e^{\log n^n}}{4^{\log n^n}} + \left(\frac{3}{4^{\log n}} \right)^n \right] =$$

$$= \lim_{n} \left[\underbrace{\left(\frac{e}{4} \right)^{\log n^n}}_{\substack{\to 0 \\ \text{base } < 1}} + \underbrace{\left(\frac{3}{4^{\log n}} \right)^n}_{\substack{\to 0 \\ \text{base } < 1}} \right] = 0$$

$$\frac{3}{4^{\log n}} < \frac{3}{4} < 1$$

$$n \geq 3$$

$$\cdot \lim_{n} \frac{(n-1)!\; n^{n+1} - (n+1)!\; n^{n-1}}{n^{n}\left((n-1)! + \log n \right)} =$$

$$= \lim_{n} \frac{(n-1)!\; n^{n+1} - (n+1)\, n\, (n-1)!\; n^{n-1}}{n^{n}\, (n-1)! \left(1 + \frac{\log n}{(n-1)!} \right)} =$$

$$= \lim_{n} \frac{\cancel{(n-1)!}\; \cancel{n^{n}} \overset{\cancel{n} - \cancel{n} - 1}{\left[n - (n+1)\cancel{n} \cdot \frac{1}{\cancel{n}} \right]}}{\cancel{n^{n}}\; \cancel{(n-1)!} \left(1 + \underbrace{\frac{\log n}{(n-1)!}}_{\to 0} \right)} = -1$$

$$\bullet \quad \underset{n}{\ell} \quad \frac{\log\left((n+2)!\right) - \log(n!)}{\log\left(2n^6\right)} =$$

$$= \underset{n}{\ell} \quad \frac{\log\left\{\dfrac{(n+2)!}{n!}\right\}}{\log 2 + \log n^6} =$$

$$= \underset{n}{\ell} \quad \frac{\log\left\{\dfrac{(n+2)(n+1)\cancel{n!}}{\cancel{n!}}\right\}}{\log 2 + 6\log n} =$$

$$= \underset{n}{\ell} \quad \frac{\log\left\{n^2\left(1+\frac{2}{n}\right)\left(1+\frac{1}{n}\right)\right\}}{\log n\left(\dfrac{\log 2}{\log n} + 6\right)} =$$

$$= \lim_{n} \frac{\log n^2 + \log\left(1+\frac{2}{n}\right) + \log\left(1+\frac{1}{n}\right)}{\log n \left(\frac{\log 2}{\log n} + 6\right)} =$$

$$= \lim_{n} \frac{2\,\cancel{\log n} + \overbrace{\log\left(1+\frac{2}{n}\right)}^{\to 0} + \overbrace{\log\left(1+\frac{1}{n}\right)}^{\to 0}}{\cancel{\log n} \left(\underbrace{\frac{\log 2}{\log n}}_{\to 0} + 6\right)} =$$

$$= \frac{2}{6} = \frac{1}{3}$$