Lez. 27

STUDIO COMPLETO DI FUNZIONE

- $y = \sqrt[3]{(2+x)^2} \cdot e^x$

dom $\mathbb{R}$

simmetrie

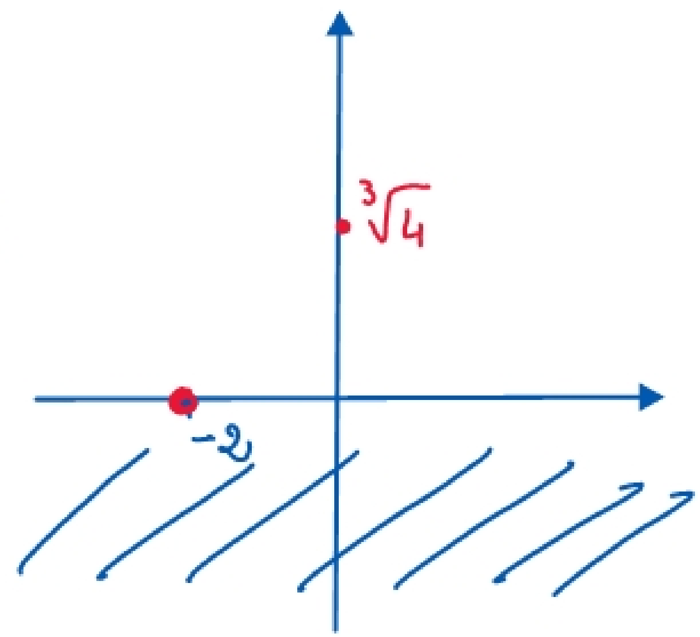
$f(x) \geq 0$ $\qquad \underbrace{\sqrt[3]{(2+x)^2}}_{+} \cdot \underbrace{e^x}_{+} \geq 0$

$f(x) = 0 \iff x = -2$

$\forall x \in \mathbb{R} \setminus \{-2\} \quad f(x) > 0$

intersezione asse ordinate

$$x=0 \rightarrow f(0) = \sqrt[3]{2^2} \cdot e^0 = \sqrt[3]{4} > 1$$



$$\lim_{x \to +\infty} \Big(\underbrace{\sqrt[3]{(2+x)^2}}_{+\infty} \; \underbrace{e^x}_{+\infty} \Big) = +\infty$$

$$\Big\downarrow \text{CN}$$

$$\boxed{\begin{array}{l} \text{ASINTOTO OBLIQUO} \quad y = mx+q \\[1em] m = \displaystyle\lim_{x \to +\infty} \frac{f(x)}{x} < \infty \\[1em] q = \displaystyle\lim_{x \to +\infty} \big[f(x) - mx \big] < \infty \end{array}}$$

N.B. non ammette asintoto obliquo

$$f(x) = \underset{\downarrow\,0}{h(x)} + \underset{\downarrow\,\infty}{g(x)} \qquad f(x) \sim g(x)$$

comportamento asintotico

$$y = \frac{x^2 - 2x + 1}{x} = \lim_{x \to +\infty} \frac{x^2 - 2x + 1}{x} = +\infty$$

$$= \underbrace{x - 2}_{g(x)} + \underbrace{\frac{1}{x}}_{h(x)}$$

$$m = \lim_{x \to +\infty} \frac{x^2 - 2x + 1}{x^2} = 1$$

$$q = \lim_{x \to +\infty} \left[\frac{x^2 - 2x + 1}{x} - x \right] =$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^2} - 2x + 1 - \cancel{x^2}}{x} = -2$$

ASINTOTO OBLIQUO

$$\boxed{y = x - 2}$$

$$\lim_{x \to -\infty} \underbrace{\sqrt[3]{(2+x)^2}}_{\substack{\downarrow \\ (2+x)^{2/3} \\ \downarrow \\ +\infty}} \; \underbrace{e^{x}}_{\downarrow \, 0} =$$

$$= \lim_{x \to -\infty} \frac{\sqrt[3]{(2+x)^2}}{e^{-x}} \sim \lim_{x \to -\infty} \frac{x^{2/3} \to \infty}{e^{-x} \to \infty} = 0$$

prevale l'esponenziale

$\Downarrow$

$y = 0$ è asintoto orizzontale

$$y = \sqrt[3]{(2+x)^2} \cdot e^x = (2+x)^{2/3} \cdot e^x$$

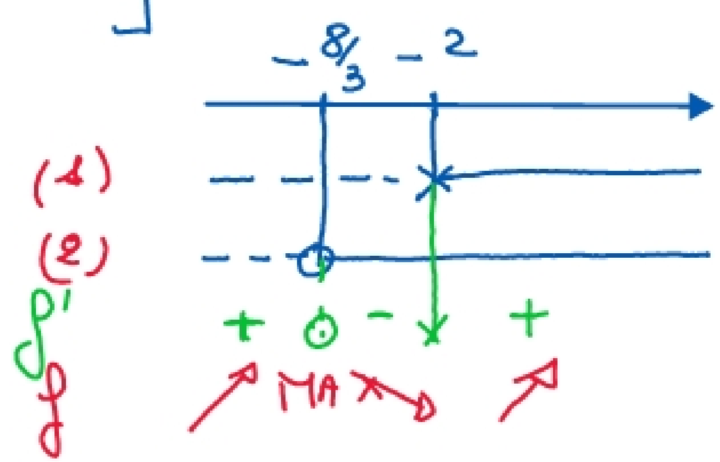
$$y' = \frac{2}{3}(2+x)^{\overset{-1/3}{\frac{2}{3}-1}} \cdot \underbrace{(1)}_{D(2+x)} \cdot e^x + (2+x)^{2/3}\, e^x =$$

$$= e^x \left[\frac{2}{3} \frac{1}{\sqrt[3]{(2+x)}} + \sqrt[3]{(2+x)^2} \right] =$$

$$= \frac{1}{3} \frac{1}{\sqrt[3]{2+x}}\, e^x \left[2 + 3(2+x) \right] =$$

$$= \frac{1}{3} \underbrace{\frac{1}{\sqrt[3]{2+x}}}_{(1)}\, \underbrace{e^x}_{(+)} \underbrace{\left(8 + 3x \right)}_{(2)}$$

STUDIO DEL SEGNO



MAX

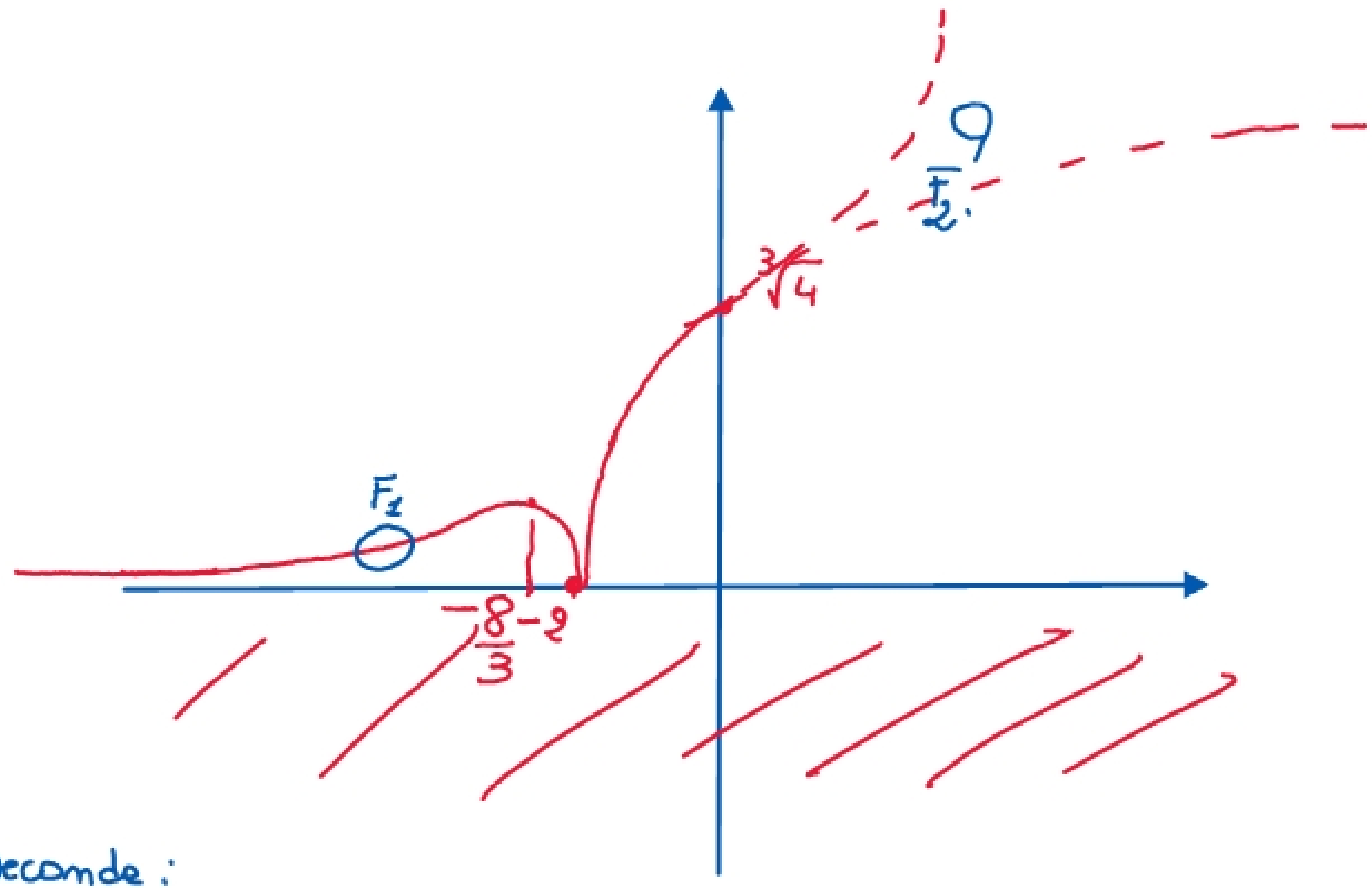
$$f(-8/3) = \sqrt[3]{\left(2-\frac{8}{3}\right)^2}\ e^{-8/3} = \sqrt[3]{\left(-\frac{2}{3}\right)^2}\ e^{-8/3} =$$

$$= \sqrt[3]{\frac{4}{9}} \cdot e^{-8/3} < 1$$

$x = -2$ punto di non derivabilità, non appartiene al dominio della derivata prima, ma appartiene al dominio di $f(x)$ e ivi è continua

$$\lim_{x \to -2^{\pm}} \frac{e^x}{3\sqrt[3]{2+x}} \cdot (3x+8) = \frac{\overset{(+)}{e^{-2}}}{\underset{(\pm)}{3\sqrt[3]{0}}} \overset{(+)}{(-6+8)} = \pm\infty$$

$$\boxed{x = -2 \text{ è un punto di cuspide}}$$



Derivata seconda:

$$y' = \frac{1}{3}\underbrace{\frac{3x+8}{\sqrt[3]{2+x}}}_{D} \qquad e^x$$

$$D\left[(3x+8)\,(2+x)^{-\frac{1}{3}}\right] =$$

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$$= 3\,(2+x)^{-\frac{1}{3}} + (3x+8)\left(-\frac{1}{3}\right)(2+x)^{-4/3} =$$

$$(2+x)^{-\frac{4}{3}} = \frac{1}{\sqrt[3]{(2+x)^4}} =$$

$$= \frac{3}{\sqrt[3]{2+x}} - \frac{3x+8}{3\,(2+x)\sqrt[3]{2+x}}$$

$$= \frac{9\,(2+x) - 3x - 8}{3\,(2+x)\sqrt[3]{2+x}} = \frac{10+6x}{3\,(2+x)\sqrt[3]{2+x}}$$

$$y'' = \frac{1}{3}\left(\frac{10+6x}{3(2+x)\sqrt[3]{2+x}}\cdot e^x + e^x\cdot\frac{3x+8}{\sqrt[3]{2+x}}\right) =$$

$$= \frac{e^x}{3}\;\frac{10+6x+3\,(2+x)(3x+8)}{3\,(2+x)\sqrt[3]{2+x}} = \frac{e^x}{9}\;\frac{10+6x+18x+9x^2+48+24x}{(2+x)\sqrt[3]{2+x}} =$$

$$y'' = \frac{e^{x}}{9} \cdot \frac{\overset{(I)}{9x^2 + 48x + 58}}{\underset{(II)}{(2+x)} \, \underset{(III)}{\sqrt[3]{2+x}}}$$

STUDIO DEL SEGNO

$$9x^2 + 48x + 58 \geq 0$$

$$x_{1,2} = \frac{-24 \pm \sqrt{576 - 522}}{9} = \frac{-24 \pm \sqrt{54}}{9} = \frac{-24 \pm 3\sqrt{6}}{9} = \frac{-8 \pm \sqrt{6}}{3}$$