Lez 16

$$\bullet \lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} = \frac{0}{0} = F.I.$$

$$= \lim_{x\to 0} \frac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)} =$$

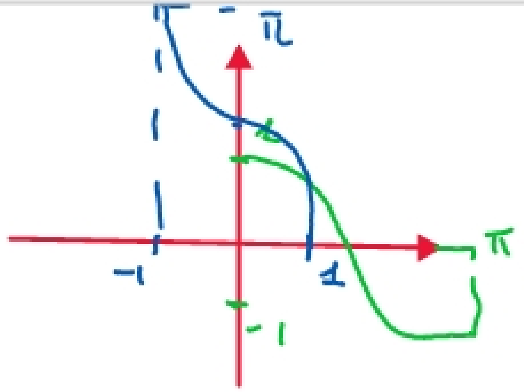
RAZIONALIZZAZIONE INVERSA

$$= \lim_{x\to 0} \frac{\cancel{1}+\cancel{x}-\cancel{1}}{\cancel{x}(\underbrace{\sqrt{1+x}}_{\sqrt{1}}+1)} = \frac{1}{2}$$

$$\bullet \lim_{x\to 0} \frac{\sqrt{1+x^2}-1}{x} = \lim_{x\to 0} \frac{\cancel{1}+x^{\cancel{2}}-\cancel{1}}{\cancel{x}\,(\underbrace{\sqrt{1+x^2}+1}_{\underbrace{\sqrt{1}+1}_{2}})} = \lim_{x\to 0} \frac{x}{2} = 0$$

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{3x^2} = \lim_{x\to 0} \frac{\cancel{1}+\cancel{x}-\cancel{1}}{3x^{\cancel{2}}(\underbrace{\sqrt{1+x}+1}_{2})} = \lim_{x\to 0} \frac{1}{3x} = \infty$$

$$\cdot \lim_{x\to 1} \frac{(\arccos x)^4}{3\,(1-x)^3} = \frac{0}{0} = F.I.$$



cambio di variabile (cambio indice)

$$y = \arccos x \quad\quad x \to 1$$

$$\downarrow \quad\quad y = \arccos x \longrightarrow \underbrace{\arccos 1}_{=0}$$

$$x = \cos y$$

$$= \lim_{y\to 0} \frac{y^4}{3(1-\cos y)^3} = \lim_{y\to 0} \frac{y^2}{1-\cos y}^{\to 2} \cdot \frac{y^2}{1-\cos y}^{\to 2} \frac{1}{\underbrace{1-\cos y}_{\to 0}} \cdot \frac{1}{3} = +\infty$$

Ricavando del limite notevole

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} = \left(\frac{\sin x}{x}\right)^2$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} = \lim_{x\to 0} \underbrace{\frac{\sin^2 x}{x^2}}_{\to 1} \cdot \underbrace{\frac{1}{1+\cos x}}_{\to 2} = \frac{1}{2}$$

$$\lim_{x\to 0^+} \frac{(1+2x)^{\log x}-1}{\operatorname{sen} x \ \log x^5} =$$

DOMINIO

$$\begin{cases} \operatorname{sen} x \neq 0 \\ x > 0 \\ 1+2x > 0 \\ 1+2x \neq 1 \\ \log x^5 \neq 0 \end{cases} \qquad \begin{cases} x \neq k\pi \quad k \in \mathbb{Z} \\ x > 0 \\ x > -\frac{1}{2} \\ x \neq 0 \\ x^5 \neq 1 \Rightarrow x \neq 1 \end{cases}$$

$$\{x>0,\ x\neq 1,\ x\neq k\pi \ \ k\in N\}$$

passeggio all' esponenziale

$$= \lim_{x\to 0^+} \frac{e^{\log (1+2x)^{\log x}}-1}{5 \ \operatorname{sen} x \ \log x} =$$

$$\lim_{x\to 0^+} \log x \cdot \log(1+2x) = 0$$

$$= \lim_{x\to 0^+} \frac{e^{\log x \cdot \log(1+2x)}-1}{5 \ \operatorname{sen} x \ \log x} =$$

$$\lim_{t\to 0} \frac{e^t-1}{t} = 1$$

$$= \lim_{x\to 0^+} \frac{1}{5} \underbrace{\frac{e^{\log x \cdot \log(1+2x)}-1}{\log x \cdot \log(1+2x)}}_{\to 1} \cdot \frac{\cancel{\log x} \cdot \log(1+2x)}{\operatorname{sen} x \cdot \cancel{\log x}} =$$

$$= \frac{1}{5} \lim_{x \to 0^+} \frac{\log(1+2x)}{\operatorname{sen} x} =$$

$$= \frac{1}{5} \lim_{x \to 0^+} \underbrace{\frac{\log(1+2x)}{2x}}_{\downarrow 1} \cdot 2\underbrace{\frac{x}{\operatorname{sen} x}}_{\downarrow 1} = \frac{2}{5}$$

$$\cdot \lim_{x\to+\infty} \frac{\ln(2x + e^x + \overset{[-1,1]}{\cos x})}{x} = \frac{\infty}{\infty}$$

$$= \lim_{x\to+\infty} \frac{\ln\left[e^x\left(\frac{2x}{e^x} + 1 + \frac{\cos x}{e^x}\right)\right]}{x} =$$

$$= \lim_{x\to+\infty} \frac{\ln e^x + \overbrace{\ln\left(\frac{2x}{e^x} + 1 + \frac{\cos x}{e^x}\right)}^{\to 0}}{x} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}\cdot \overset{=1}{\ln e}}{\cancel{x}} = 1$$

- $$\lim_{x\to 0} \frac{e^{x}-1+\operatorname{sen}x}{(1+x)^{2}-1+\operatorname{tg}x} = \frac{1-1+0}{1-1+0} = \text{F.I.}$$

$$= \lim_{x\to 0} \frac{\cancel{x}\left(\overbrace{\frac{e^{x}-1}{x}}^{\to 1}+\overbrace{\frac{\operatorname{sen}x}{x}}^{\to 1}\right)}{\cancel{x}\left(\underbrace{\frac{(1+x)^{2}-1}{x}}_{\to 2}+\underbrace{\frac{\operatorname{tg}x}{x}}_{\to 1}\right)} = \frac{2}{3}$$

$$\lim_{x\to 0} \frac{(1+x)^{2}-1}{x} = \lim_{x\to 0} \frac{\cancel{1}+x^{2}+2x\cancel{-1}}{x} = \lim_{x\to 0} \frac{x^{2}+2x}{x} = \lim_{x\to 0}(x+2)=2$$

$$\lim_{x\to 0} \frac{\operatorname{tg}x}{x} = \lim_{x\to 0} \frac{\operatorname{sen}x}{x\cdot\cos x} = \lim_{x\to 0} \frac{1}{\cos x}\cdot\underbrace{\frac{\operatorname{sen}x}{x}}_{\to 1} = 1$$

• $$\lim_{x \to \frac{\pi}{2}} \left[1 + \operatorname{sen}\left(\frac{\pi}{2} - x\right)\right]^{\frac{2}{\operatorname{sen}\left(\frac{\pi}{2}-x\right)}} \cdot \log\left[5 \, \frac{1-\cos\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)^2}\right]$$

pongo $\frac{\pi}{2} - x = p$ $\quad x \to \frac{\pi}{2} \quad p \to 0$

$$= \lim_{p \to 0} \left[1 + \operatorname{sen} p\right]^{\frac{2}{\operatorname{sen} p}} \cdot \log\left[5 \, \frac{1-\cos p}{p^2}\right] =$$

Risolvere con cambio di variabile e scissione:

$t = \frac{1}{\operatorname{sen} p} \quad p \to 0 \quad t \to \infty \quad \left(\operatorname{sen} p = \frac{1}{t}\right)$

$$= \lim_{t \to +\infty} \underbrace{\left(1 + \frac{1}{t}\right)^{2 \cdot t}}_{\to e^2} \cdot \lim_{p \to 0} \log\left[5 \, \underbrace{\frac{1-\cos p}{p^2}}_{\to \frac{1}{2}}\right] = e^2 \cdot \log\frac{5}{2}$$

$$\cdot \lim_{x \to 0^+} \frac{(1+3x)^{\log x} - 1}{\operatorname{sen} x \cdot \log x^2} =$$

$$= \lim_{x \to 0^+} \frac{e^{\log\left[(1+3x)^{\log x}\right]} - 1}{\operatorname{sen} x \, \log x^2} =$$

$$= \lim_{x \to 0^+} \frac{e^{\log x \, \log(1+3x)} - 1}{2 \log x \, \operatorname{sen} x} =$$

$$= \lim_{x \to 0^+} \underbrace{\frac{e^{\log x \, \log(1+3x)} - 1}{\log x \cdot \log(1+3x)}}_{1} \cdot \underbrace{\frac{\log(1+3x) \cdot 3x}{2 \operatorname{sen} x \cdot 3x}}_{1} = 3/2$$

$$\lim_{x\to 0^+} \frac{1-\operatorname{sen}x-\cos^2 x}{e^{x^2}-1} = \frac{1-0-1}{1-1} = \frac{0}{0}$$

$1-\cos^2 x = \operatorname{sen}^2 x$

$$= \lim_{x\to 0^+} \frac{\operatorname{sen}^2 x - \operatorname{sen} x}{x^2} \cdot \frac{x^2}{e^{x^2}-1} =$$

$$= \lim_{x\to 0^+} \left(\underbrace{\frac{\operatorname{sen}^2 x}{x^2}}_{\left(\frac{\operatorname{sen} x}{x}\right)^2 \to 1} - \underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1} \cdot \underbrace{\frac{1}{x}}_{\frac{1}{0^+} \to +\infty} \right) \cdot \underbrace{\left(\frac{e^{x^2}-1}{x^2} \right)^{-1}}_{\to 1} = -\infty$$

$$\underbrace{1 - 1 \cdot \frac{1}{0^+}}_{-\infty} \cdot 1$$

$$\cdot \lim_{x\to 0^+} \frac{\log(1-\cos x)}{\log x} = \frac{-\infty}{-\infty} \quad \text{F.I.}$$

$$= \lim_{x\to 0^+} \frac{\log\left[\dfrac{1-\cos x}{x^2}\cdot x^2\right]}{\log x} =$$

$$= \lim_{x\to 0^+} \left\{ \frac{\log\Big(\overset{\to 1/2}{\dfrac{1-\cos x}{x^2}}\Big)}{\underbrace{\log x}_{\to -\infty}} + \frac{\overset{2\,\cancel{\log x}}{\log x^2}}{\cancel{\log x}} \right\} =$$

$$= 0 + 2 = 2$$