- $\int \frac{dx}{\sin x} = \int \frac{\frac{2}{1+t^2}}{\frac{2t}{1+t^2}}\,dt = \int \frac{1}{t}\,dt = \ln|t| + c =$

$\uparrow$ SOSTITUENDO

$= \ln\left|\tg\frac{x}{2}\right| + c$

FORMULE PARAMETRICHE

$t = \tg\frac{x}{2}$

$\sin x = \frac{2t}{1+t^2} = \frac{2\tg\frac{x}{2}}{1+\tg^2\frac{x}{2}}$

$\cos x = \frac{1-t^2}{1+t^2} = \frac{1-\tg^2\frac{x}{2}}{1+\tg^2\frac{x}{2}}$

se $t = \tg\frac{x}{2}$ $\quad dx = \frac{2}{1+t^2}\,dt$

$$\int \frac{\sin x}{1-\sin x}\,dx =$$

$$= \int \frac{\operatorname{sen} x - 1 + 1}{1-\operatorname{sen} x}\,dx = \int \frac{\overbrace{\operatorname{sen} x - 1}^{-(1-\operatorname{sen} x)}}{1-\operatorname{sen} x}\,dx + \int \frac{1}{1-\operatorname{sen} x}\,dx =$$

$$= \int (-1)\,dx + \underbrace{\int \frac{1}{1-\operatorname{sen} x}\,dx}_{J(x)} = -x - \frac{2}{\operatorname{tg}\frac{x}{2} - 1} + c$$

$$J(x) = \int \frac{1}{1-\operatorname{sen} x}\,dx = \int \frac{1}{1-\frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt = \int \frac{2}{1+t^2-2t}\,dt =$$

$$= \int \frac{2}{(t-1)^2}\,dt = -\frac{2}{t-1} + c_1$$

$t = \operatorname{tg}\frac{x}{2}$

Formule parametriche

$$\cdot \int \frac{dx}{8-4\sin x+7\cos x} = \qquad \text{posto } t = \operatorname{tg}\frac{x}{2}$$

$$dx = \frac{2}{1+t^2}\,dt$$

$$= \int \frac{\frac{2}{1+t^2}}{8-4\frac{2t}{1+t^2}+7\frac{1-t^2}{1+t^2}}\,dt = 2\int \frac{1}{\frac{8+8t^2-8t+7-7t^2}{1+t^2}}\cdot\frac{1}{1+t^2}\,dt$$

$$= 2\int \frac{1}{t^2-8t+15}\,dt = 2\int \frac{1}{(t-5)(t-3)}\,dt$$

come frazioni

$$\frac{1}{(t-5)(t-3)} = \frac{A}{t-5}+\frac{B}{t-3} = \frac{A(t-3)+B(t-5)}{(t-3)(t-5)}$$

$$At - 3A + Bt - 5B = 1$$

$$t(A+B) - 3A - 5B = 1$$

$$3\cdot\begin{cases} A+B = 0 \\ -3A-5B = 1 \end{cases} \quad \begin{cases} 3B - 5B = 1 \\ A = -B \end{cases} \rightarrow \begin{cases} -2B = 1 \\ A = -B \end{cases} \quad \begin{cases} B = -\frac{1}{2} \\ A = \frac{1}{2} \end{cases}$$

$$= \cancel{2}\int \frac{1}{\cancel{2}(t-5)}\,dt + \cancel{2}\int -\frac{1}{\cancel{2}(t-3)}\,dt =$$

$$= \ln|t-5| - \ln|t-3| + c = \ln\left|\frac{t-5}{t-3}\right| + c =$$

$$= \ln\left|\frac{\operatorname{tg}\frac{x}{2} - 5}{\operatorname{tg}\frac{x}{2} - 3}\right| + c$$

$$\int \frac{\overbrace{\cos x \quad\quad\quad\quad\quad dx}^{dy}}{\sin^2 x - 6 \operatorname{sen} x + 5} =$$

den. è un polinomio in sen x

Num contiene la derivata di sen x

sostituzione $y = \operatorname{sen} x$

$$dy = \cos x \, dx$$

$$= \int \frac{1}{y^2 - 6y + 5} \, dy = \int \frac{1}{(y-5)(y-1)} \, dy$$

$$\frac{1}{(y-5)(y-1)} = \frac{A}{y-5} + \frac{B}{y-1} = \frac{A(y-1) + B(y-5)}{(y-1)(y-5)} =$$

$$= \frac{(A+B)y - A - 5B}{(y-1)(y-5)}$$

$$\begin{cases} A + B = 0 \\ -A - 5B = 1 \end{cases} \qquad \begin{cases} B - 5B = 1 \\ A = -B \end{cases} \qquad \begin{cases} B = -\dfrac{1}{4} \\ A = -B = \dfrac{1}{4} \end{cases}$$

$$= \int \frac{1}{4} \cdot \frac{1}{y-5}\, dy - \frac{1}{4} \int \frac{1}{y-1}\, dy =$$

$$= \frac{1}{4} \ln|y-5| - \frac{1}{4} \ln|y-1| + C = \frac{1}{4} \ln\left|\frac{y-5}{y-1}\right| + C =$$

$$= \frac{1}{4} \ln\left|\frac{\operatorname{sen} x - 5}{\operatorname{sen} x - 1}\right| + C$$

$\cdot \int \cosh^2 x \, dx$

$$\boxed{\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)}$$

$$= \int \frac{1}{2}(\cosh 2x + 1)\, dx =$$

$$= \frac{1}{2} \cdot \frac{1}{2} \int 2\cosh 2x \, dx + \frac{1}{2} \int dx =$$

$$= \frac{1}{4} \operatorname{senh} 2x + \frac{1}{2} x + C$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$D(\cosh x) = \frac{e^x - e^{-x}}{2} =$$

$$= \operatorname{senh} x$$

$$\operatorname{senh} x = \frac{e^x - e^{-x}}{2}$$

$$D(\operatorname{senh} x) = \frac{e^x + e^{-x}}{2} =$$

$$= \cosh x$$

$$\int \cosh^3 x \, dx = \int \cosh^2 x \cdot \underbrace{\cosh x \, dx}_{} =$$

$$d(\sinh x) = \cosh x \, dx$$

$$= \int (1 + \sinh^2 x) \, d(\sinh x) =$$

$$\boxed{\cosh^2 x - \sinh^2 x = 1}$$

$$\cosh^2 x = 1 + \sinh^2 x$$

$$= \int d(\sinh x) + \int \sinh^2 x \, d(\sinh x) =$$

$$= \sinh x + \frac{1}{3} \sinh^3 x + C$$

$$\int \frac{dx}{\sinh x \cosh^2 x} = \qquad \frac{1}{\cosh^2 x} = D(\tanh x)$$

$$= \int \frac{1}{\sinh x} \cdot d(\tanh x) \underset{pp.}{=} \frac{1}{\sinh x} \cdot \tanh x - \int \tanh x \cdot \frac{\cosh x}{\sinh^2 x}\, dx =$$

$$D((\sinh x)^{-1}) = -\frac{1}{\sinh^2 x} \cdot \cosh x$$

$$= \frac{1}{\cancel{\sinh x}} \cdot \frac{\cancel{\sinh x}}{\cosh x} + \int \frac{\cancel{\sinh x}}{\cancel{\cosh x}} \cdot \frac{\cancel{\cosh x}}{\sinh^{\cancel{2}} x}\, dx =$$

$$= \frac{1}{\cosh x} + \int \frac{1}{\sinh x}\, dx = \frac{1}{\cosh x} + \ln\left|\tanh \frac{x}{2}\right| + C$$

$\to \ln\left|\tanh \frac{x}{2}\right|$ parametriche

$$\int \sqrt{3-2x-x^2}\,dx =$$

$$3-2x-x^2 = 4-1-2x-x^2 = 4-(1+2x+x^2) = 4-(1+x)^2$$

$$\left.\begin{aligned} x+1 &= 2\,\mathrm{sen}\,t \\ dx &= 2\cos t\,dt \end{aligned}\right\}$$ sostituzione

$$= \int \sqrt{4-(1+x)^2}\,dx = \int \sqrt{4-4\,\mathrm{sen}^2 t}\cdot 2\cos t\,dt =$$

$$= 4\int \underbrace{\sqrt{1-\mathrm{sen}^2 t}}_{\cos t}\cdot \cos t\,dt = 4\int \cos^2 t\,dt = 4\int \frac{\cos 2t+1}{2}\,dt =$$

$$= \frac{4}{2}\left(\frac{\mathrm{sen}\,2t}{2} + t\right) + c = \mathrm{sen}\,2t + 2t + c$$

$$\operatorname{sen} 2t = 2 \operatorname{sen} t \cos t$$

$$\cos t = \sqrt{1 - \operatorname{sen}^2 t} \qquad \operatorname{sen} t = \frac{x+1}{2}$$

$$t = \operatorname{arcsen} \frac{x+1}{2}$$

$$= \operatorname{sen} 2t + t + C = \underline{2 \operatorname{sen} t} \cos t + t + C =$$

$$= (x+1) \cdot \sqrt{1 - \left(\frac{x+1}{2}\right)^2} + \operatorname{arcsen} \frac{x+1}{2} + C =$$

$$\sqrt{4} = 2$$

$$= \frac{x+1}{2} \sqrt{4 - x^2 - 1 - 2x} + \operatorname{arcsen} \frac{x+1}{2} + C = \frac{x+1}{2} \sqrt{3 - 2x - x^2} + \operatorname{arcsen} \frac{x+1}{2} + C$$