Lez 7

COMPLESSI

- $\operatorname{Im}\left[3(z-\bar{z}) - 2i\left(|z|^2 + z^2\right) + 7\right] = 0$

gli insieme/e dei valori per cui sia vera l'uguaglianza

$z = x + iy$

calcolo dell'espressione del numero complesso:

$$3\left(x+iy-(x-iy)\right) - 2i\left(x^2+y^2+(x+iy)^2\right)+7\,\big] =$$

$$= 3\Big(\cancel{x}+\overset{2yi}{iy-\cancel{x}+iy}\Big) - 2i\Big(\overset{2x^2}{x^2+\cancel{y^2}+x^2-\cancel{y^2}}+2xyi\Big)+7\,\big] =$$

$$= (4xy+7) + i\,(6y-4x^2) =$$

$Im(P) = 0$

$$6y - 4x^2 = 0 \qquad y = \frac{2}{3}x^2 \quad \text{parabola}$$

oppure altre richieste

1) $Re > 0$ $\qquad 4xy + 1 > 0$ $\qquad$ iperbole equilatera

2) $\begin{cases} Re = 0 \\ Im = 0 \end{cases}$ $\qquad z_1(0,0)$ $\qquad$ numero nullo

DISEQUAZIONI IN 2 VARIABILI

$2x+1>0$ $\quad x>-\frac{1}{2}$

$]-\frac{1}{2}, +\infty[$

$2x-y>0$ $\quad$ 1° prendo 2 variabili $\Rightarrow (x,y)$

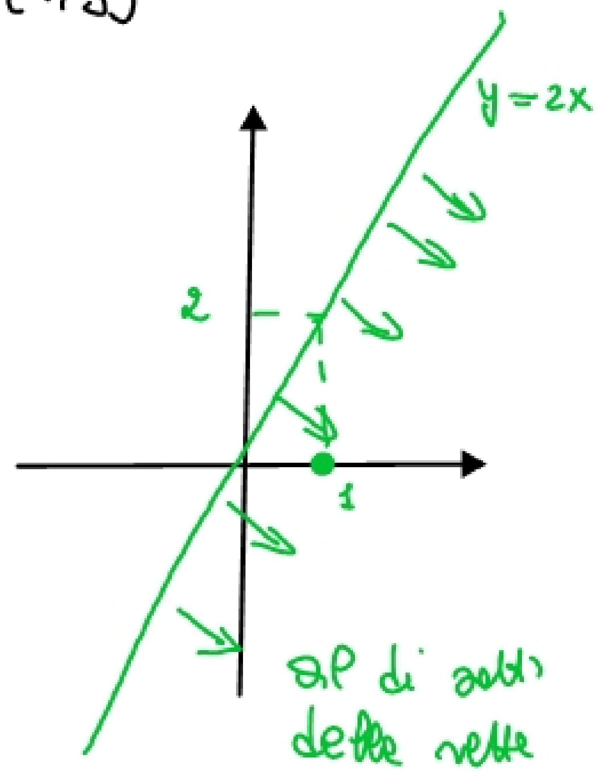
esplicitando

$y<2x$

come uguaglianza

$y=2x$ retta

rappresenta i punti, le coppie (x,y) che verificano l'uguaglianza



$$\begin{cases} 2x - y > 0 \\ y - x + 1 < 0 \end{cases}$$

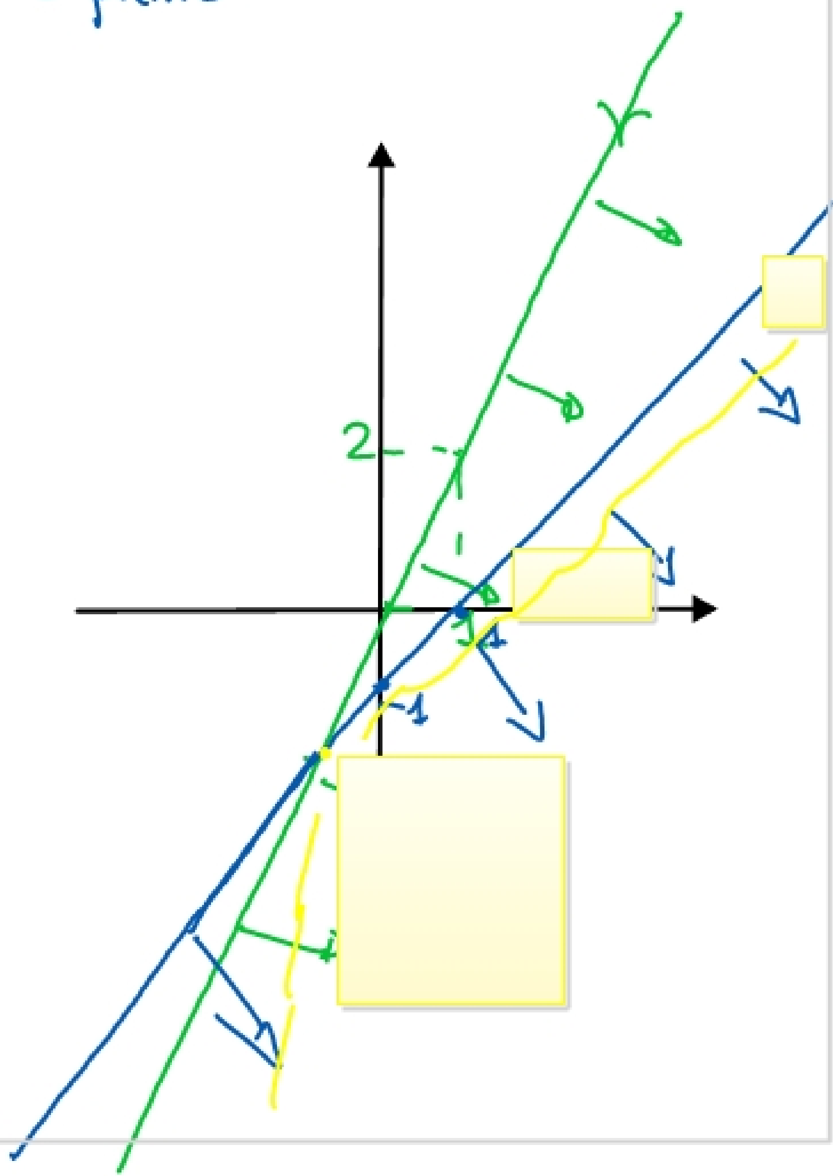
ciascuna retta determina una zona di piano

il sistema è risolto nell' intersezione delle 2 zone di piano

r: $y = 2x$ $\qquad [\, y < 2x \,]$

s: $y = x - 1$ $\qquad [\, y < x - 1 \,]$

zona di piano comune ad entrambi



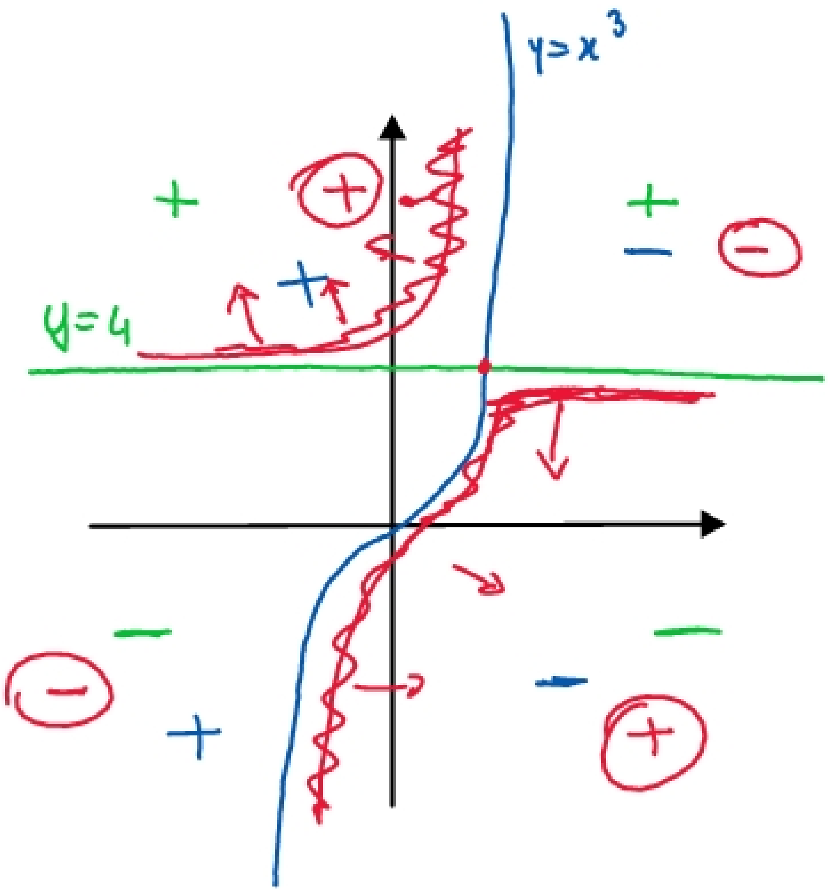
$$(y - x^3)(y - 4) > 0$$

STUDIO DEL SEGNO:

$y - x^3 \geq 0$ $\qquad$ $y = x^3$ $\qquad$ sono (x, y) zero per il prodotto

$y - 4 \geq 0$ $\qquad$ $y = 4$

(x, y)



$\mathrm{Re}\ z_1 > 0$

$4xy + 7 > 0$

$4xy + 7 = 0$

$xy = -\frac{7}{4}$ $\quad x \neq 0$

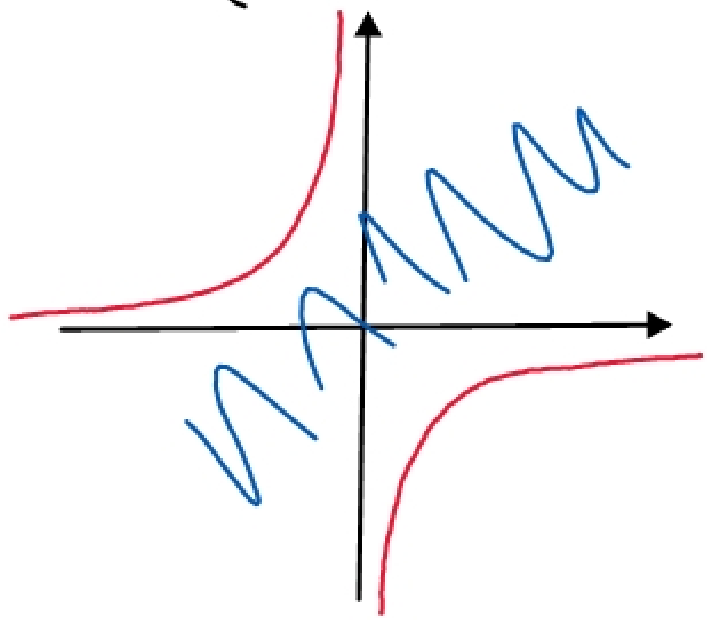
$y = -\frac{7}{4x} = -\frac{7}{4} \cdot \frac{1}{x}$

iperbole equilatera $2° - 4°$ quad.
$(K = -7/4)$

$(0,0)$ $\quad 4 \cdot 0 \cdot 0 + 7 > 0$

$7 > 0$

Zona di piano illimitato
che ha come frontiera
i 2 rami dell' iperbole



$$\underbrace{\left[\,|z-3i|^2 + \mathrm{Re}\,(z+6\bar{z})\cdot \mathrm{Im}\,(z-\bar{z})\,i-(7i+1)\,z\bar{z}\,\right]}_{\mathrm{Im}\,[\quad]\,=\,0} \in \mathbb{R}$$

$$z = x+iy$$

$$\Big|\,x+\underbrace{iy-3i}_{i(y-3)}\,\Big|^2 + \mathrm{Re}\,\big(\overbrace{x+iy+6(x-iy)}^{7x\,-\,6iy}\big)\cdot \mathrm{Im}\,\big(\overbrace{\cancel{x}+iy-(\cancel{x}-iy)}^{2iy}\big)\,i\ -$$

$$+\,(7i+1)\,\underbrace{(x+iy)(x-iy)}_{(x^2+y^2)} =$$

$$= x^2+(y-3)^2+\underbrace{7x\cdot 2y\cdot i}_{\mathrm{Im}} - (x^2+y^2)\ -\underbrace{7(x^2+y^2)}_{\mathrm{Im}}\,i$$

per essere un vettore reale

$$\mathrm{Im}(\quad) \Rightarrow$$

$$\cancel{14}^{\,2}\, xy - \cancel{4}(x^2+y^2) = 0$$

$$2xy - x^2 - y^2 = 0$$

$$x^2 + y^2 - 2xy = 0$$

$$(x-y)^2 = 0 \qquad \Rightarrow \qquad x = y$$

bisettrice 1°-3° quad.

Reale positivo

$$\begin{cases} Im(\quad) = 0 \\ Re(\quad) > 0 \end{cases} \qquad \begin{cases} x = y \\ 9 - 6y > 0 \end{cases} \qquad \begin{cases} y = x \\ y < \frac{9}{6} = \frac{3}{2} \end{cases}$$

$$Re(\quad) = x^2 + (y-3)^2 - (x^2+y^2) =$$

$$= \cancel{x^2} + \cancel{y^2} + 9 - 6y - \cancel{x^2} - \cancel{y^2} =$$

$$= 9 - 6y$$

SUCCESSIONI

PREMESSE

$$a_n = \cos n\frac{\pi}{2}$$

$$n! = n \cdot (n-1)\,(n-2) \cdots \underbrace{(n-n+1)}_{1} \qquad n \in \mathbb{N}$$

prodotto del numero per i suoi precedenti

$$\frac{n!}{(n-3)!} = \frac{n \cdot (n-1)\,(n-2)\,\cancel{(n-3)!}}{\cancel{(n-3)!}} = n\,(n-1)\,(n-2)$$

$$a_n = \log (n+1)! - \log(n!) = \log \frac{(n+1)!}{n!} = \log \frac{(n+1)\,\cancel{(n!)}}{\cancel{n!}} =$$

$$= \log (n+1)$$

LIMITI

$$\lim_{n} n^{\to +\infty} \sin\left(\frac{n(n+1)}{2}\pi\right) = 0$$

N.B si tratta di successione identicamente nulla.

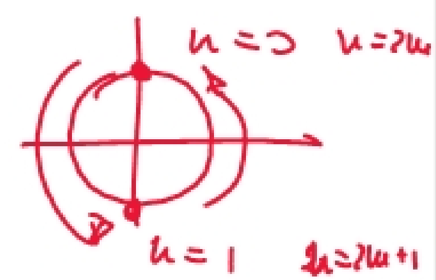
$n(n+1)$ pari $\qquad \frac{n(n+1)}{2}\pi$ multiplo di π

$$\Downarrow$$

$$\sin\left(\frac{n(n+1)}{2}\pi\right) = 0 \qquad \forall n$$

$$\lim_{n \to +\infty} 3^{n \to +\infty} \cos\left(\frac{2n+1}{2}\pi\right) = 0$$

$$\frac{2n+1}{2}\pi = n\pi + \frac{\pi}{2} \qquad \begin{cases} \frac{\pi}{2} \\ \frac{3}{2}\pi \end{cases}$$



$$\cos\left(\frac{2n+1}{2}\pi\right) = 0$$

$$\lim_{n} \left\{ \underbrace{3^{n}}_{+\infty} + \underbrace{\cos\left(\frac{2n+1}{2}\pi\right)}_{0} \right\} = +\infty$$

$$\lim_{n} 3^{-n} \cos n =$$

$$-1 \leq \cos n \leq +1 \qquad \text{quantità limitata}$$

$$3^{-n}\cdot(-1) \leq 3^{-n}\cos n \leq (+1)\, 3^{-n} \qquad 3^{-n} > 0$$

$$-3^{-n} \leq 3^{-n} \cos n \leq 3^{-n}$$

$$\underbrace{-\left(\frac{1}{3}\right)^{n}}_{0} \leq \underbrace{\left(\frac{1}{3}\right)^{n} \cos n}_{0} \leq \underbrace{\left(\frac{1}{3}\right)^{n}}_{0} \qquad \text{Teo Carabinieri}$$

$$\bullet \lim_{n} \Big(\underbrace{\sqrt{n^2+5n+3}}_{+\infty} - \underbrace{n}_{+\infty} \Big) = \qquad \text{F.I. } +\infty - \infty$$

$$(a+b)(a-b) = a^2 - b^2$$

$$= \lim_{n} \left(\sqrt{n^2+5n+3} - n \right) \cdot \frac{\sqrt{n^2+5n+3} + n}{\sqrt{n^2+5n+3} + n} =$$

$$= \lim_{n} \frac{\cancel{n^2} + 5n + 3 - \cancel{n^2}}{\sqrt{n^2+5n+3} + n} = \qquad \text{F.I. } \frac{+\infty}{+\infty}$$

$$= \lim_{n} \frac{\cancel{n}\left(5 + \underbrace{\frac{3}{n}}_{\to 0}\right)}{\cancel{n}\left(\sqrt{1 + \underbrace{\frac{5}{n}}_{0} + \underbrace{\frac{3}{n^2}}_{0}} + 1\right)} = \frac{5}{2}$$