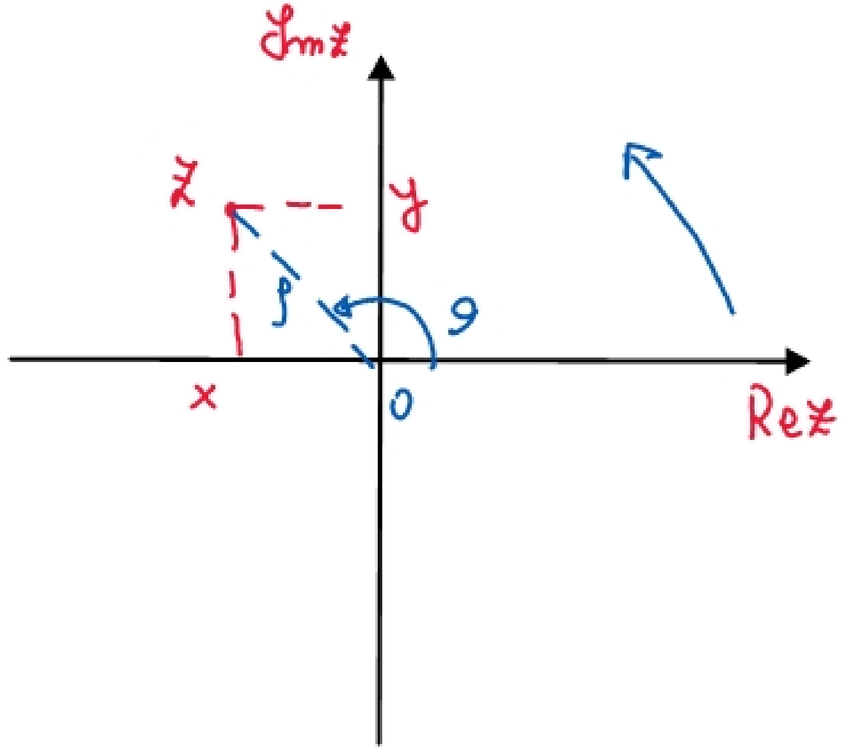
I NUMERI COMPLESSI

$z \in \mathbb{C}$:

forma algebrica $= \underbrace{x}_{\mathrm{Re}\, z} + i \underbrace{y}_{\mathrm{Im}\, z}$ $\quad x, y \in \mathbb{R}$

forma trigonometrica $= \rho(\cos\theta + i \,\mathrm{sen}\,\theta)$

forma esponenziale $= \rho\, e^{i\theta}$



1) $\rho = \sqrt{x^2+y^2}$

$$\theta : \begin{cases} \cos\theta = \dfrac{x}{\sqrt{x^2+y^2}} \\ \mathrm{sen}\,\theta = \dfrac{y}{\sqrt{x^2+y^2}} \end{cases}$$

$\theta = \mathrm{arctg}\, \dfrac{b}{a}$ (*)

2) $$\begin{cases} x = \rho \cos\theta \\ y = \rho\, \mathrm{sen}\,\theta \end{cases}$$

L'unità immaginaria

$$\boxed{i = \sqrt{-1}}$$

$$\begin{cases} i^0 = 1 \\ i^1 = i \\ i^2 = -1 \\ i^3 = -i \end{cases}$$

R $\begin{cases} i^4 = 1 \\ i^5 = i \\ i^6 = -1 \\ i^7 = -i \end{cases}$

DIVISIONE MOD 4

Calcolare

$i^{23} = i^{20} \cdot i^3 = i^3 = -i$

$i^{20} \to i^0 = 1$

20 è multiplo di 4
è divisibile per 4

$i^{231} = i^{228}\, i^3 = i^0 \cdot i^3 = i^3 = -i$

$231 : 4 = 57$ Resto 3

$i^{738} = i^{736} \cdot i^2 = i^2 = -1$

$738 : 4 = 184$ resto 2

$i^{736} = (i^4)^{184}$

es 1

Calcolare il valore della seguente espressione

$$\left[\frac{(2i^{25}-3i)^2+(-i^{10}+2i^2)^3}{(2i+5i^7)\,(5i^{14}+2i^{32})}\right]^2=\left[\frac{(2i-3i)^2+(-(-1)+2(-1))^3}{(2i+5(-i))\,(5(-1)+2)}\right]^2=$$

Trasformazione delle potenze di i

$$i^{25}=i^{24}\cdot i=i$$

$$i^{10}=i^{8}\cdot i^{2}=i^{2}=-1$$

$$i^{0}=1$$

$$i^{2}=-1$$

$$i^{7}=i^{4}\cdot i^{3}=i^{3}=-i$$

$$i^{14}=i^{12}\cdot i^{2}=i^{2}=-1$$

$$i^{32}=i^{0}=1$$

$$=\left[\frac{\overbrace{(-i)^2}^{i^2}+(-1)^3}{(-3i)\,(-3)}\right]^2=$$

$$=\left[\frac{-1-1}{+9i}\right]^2=$$

$$=\left[\frac{-2}{9i}\right]^2=\frac{4}{81\underbrace{i^2}_{(-1)}}=-\frac{4}{81}$$

es 2 Dati i numeri complessi

$$z_1 = \underline{2(h-4)} + \underline{3}\,i$$

$$z_2 = \underline{(1-k)} + \underline{(k^2-1)}\,i$$

— Re — Im

determinare i valori dei parametri reali h e k in corrispondenza dei quali

1 - z_1 sia immaginario

$$\mathrm{Re}\, z_1 = 0 \Rightarrow 2(h-4) = 0 \qquad h = 4$$

2 - z_2 sia reale, non nullo

$$\begin{cases} k^2-1 = 0 \\ 1-k \neq 0 \end{cases} \qquad \begin{cases} k = \pm 1 \\ k \neq 1 \end{cases} \Rightarrow k = -1$$

3 - z_1 e z_2 siano uguali :

$$\begin{cases} \operatorname{Re} z_1 = \operatorname{Re} z_2 \\ \operatorname{Im} z_1 = \operatorname{Im} z_2 \end{cases}$$

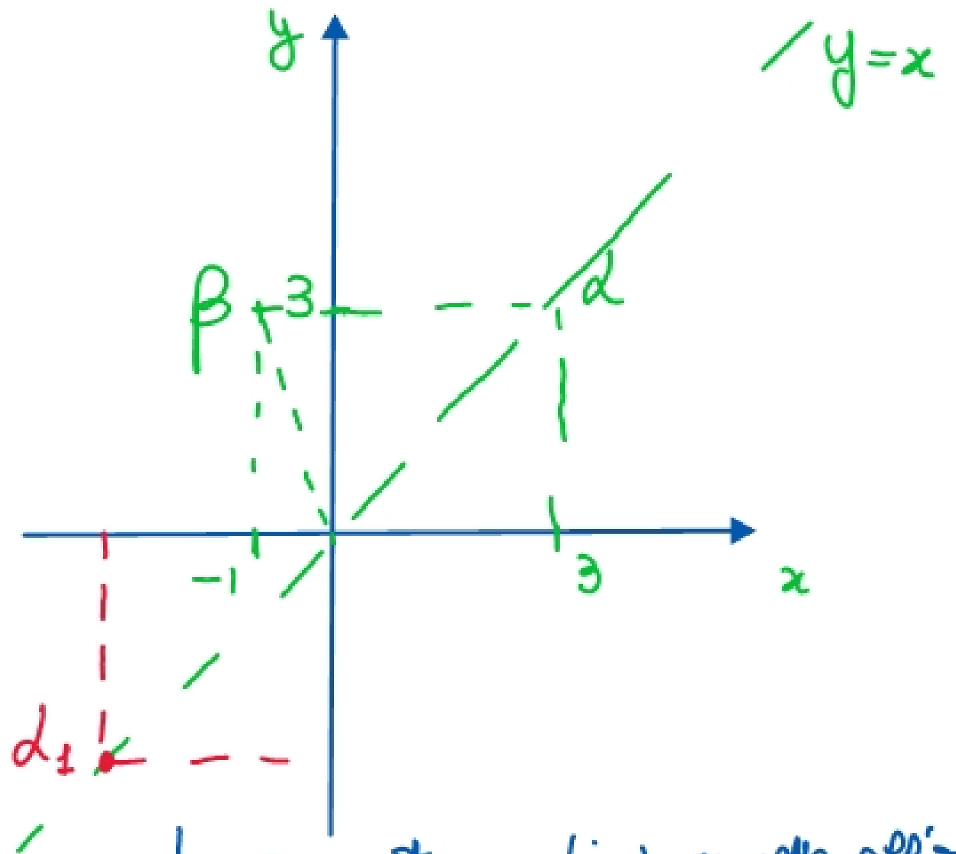
$$\begin{cases} 2(h-4) = 1-k \\ 3 = k^2-1 \end{cases} \qquad \begin{cases} h = \dfrac{9-k}{2} \\ k = \pm 2 \end{cases}$$

$$\begin{cases} k_1 = -2 \\ h_1 = \dfrac{11}{2} \end{cases} \qquad \begin{cases} k_2 = 2 \\ h_2 = 7/2 \end{cases}$$

$$\alpha \begin{cases} z_1 = 2\left(\frac{11}{2}-4\right)+3i = 3+3i \\ z_2 = 3+3i \end{cases} \qquad \beta \begin{cases} z_1 = 2\left(\frac{7}{2}-4\right)+3i = -1+3i \\ z_2 = -1+3i \end{cases}$$

$\alpha\ (3,\ 3)$

$\beta\ (-1,\ 3)$



α in coordinate polari

$\rho_\alpha = \sqrt{9+9} = 3\sqrt{2}$

$\vartheta_\alpha = \text{arctg}\ \frac{3}{3} = \text{arctg}\ 1 = \pi/4$

simmetrico di α rispetto all'origine

$\rho = 3\sqrt{2}$

$\vartheta = \frac{5}{4}\pi$

$x_{\alpha_1} = \frac{\cos \frac{5}{4}\pi}{\rho} =$

$y_{\alpha_2} = \frac{\text{sen}\ 7/4\ \pi}{\rho} =$

Es 3 CALCOLI ALGEBRICI

$z_1 = 2 - 3i \qquad z_2 = -4 + i$

$$z_1 + z_2 = (2-3i) + (-4+i) = 2 - 3i - 4 + i = -2 - 2i$$

$$3z_1 - 2z_2 = 3(2-3i) - 2(-4+i) = 6 - 9i + 8 - 2i = 14 - 11i$$

$$z_1 \cdot z_2 = (2-3i)(-4+i) = -8 + 12i + 2i - 3\overset{-1}{i^2} =$$

$$= -8 + 3 + 14i = -5 + 14i$$

$$(z_2)^2 = (-4+i)^2 = 16 + \overset{-1}{i^2} - 8i = 15 - 8i$$

$$\frac{1 - z_1}{5 + z_2} = \frac{1-(2-3i)}{5+(-4+i)} = \frac{-1+3i}{1+i} \cdot \frac{1-i}{1-i} = \frac{-1+3i+i-3\overset{+3}{i^2}}{1-\underset{+1}{i^2}} =$$

$$= \frac{2+4i}{2} = 1 + 2i$$

• <u>es 4</u> determinare il coniugato del numero complesso:

$$z = \frac{1+i}{(3+2i)(2-i)} =$$

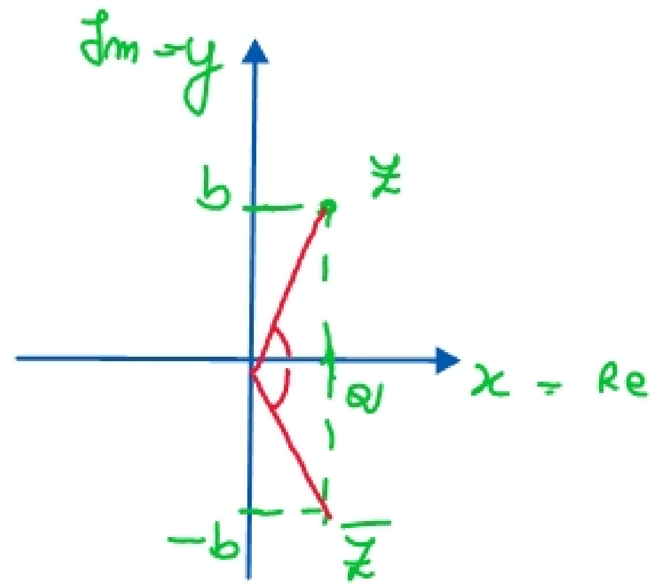
$$= \frac{1+i}{(3+2i)(2-i)} \cdot \frac{(3-2i)(2+i)}{(3-2i)(2+i)} =$$

$$= \frac{(3+3i-2i+2)\,(2+i)}{(9+4)\,(4+1)} =$$

$$= \frac{(5+i)\,(2+i)}{13 \cdot 5} = \frac{10+2i+5i-1}{65} =$$

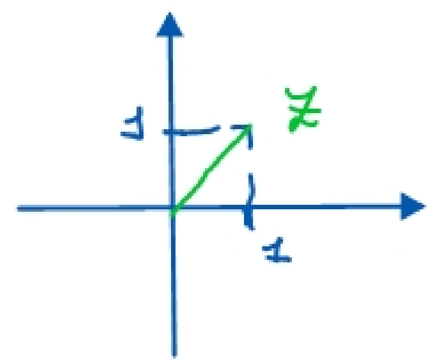
$$= \frac{9}{65} + \frac{7}{65}i \qquad \bar{z} = \frac{9}{65} - \frac{7}{65}i$$

$$\boxed{\begin{array}{l} z = a+ib \\ \bar{z} = a-ib \end{array}}$$



• TRASFORMAZIONE – POTENZA – RADICE

$z = 1+i$



$$\rho = \sqrt{1+1} = \sqrt{2}$$

$$\theta = \operatorname{arctg} \frac{1}{1} = \operatorname{arctg} 1 = \frac{\pi}{4}$$

$$z = 1 + i = \sqrt{2}\, e^{\frac{\pi}{4} i} = \sqrt{2} \left(\cos \frac{\pi}{4} + i \operatorname{sen} \frac{\pi}{4} \right)$$

POTENZA

$$z^5 = (1+i)^5 = (1+i)^2 \, (1+i)^3 =$$ sviluppo binomiale Triangolo di Tartaglia

$$= \left(\sqrt{2}\, e^{\frac{\pi}{4} i} \right)^5 = (\sqrt{2})^5 \, e^{\frac{5}{4}\pi i} =$$

$$= (\sqrt{2})^5 \left(\cos \frac{5}{4}\pi + i \operatorname{sen} \frac{5}{4}\pi \right)$$

in generale

$$z^n = (a+ib)^n = (\rho,\vartheta) \xrightarrow{\text{polari}} = \rho^n e^{n\vartheta i} = \rho^n(\cos n\vartheta + i \operatorname{sen} n\vartheta)$$

RADICE n-esima

$z = 1+i$ $\qquad \sqrt[3]{z} = \sqrt[3]{1+i}$

FORMULA:

$$\sqrt[n]{z} = \sqrt[n]{\rho} \cdot \left(\cos\frac{\vartheta+2k\pi}{n} + i \operatorname{sen}\frac{\vartheta+2k\pi}{n}\right) =$$

$$= \sqrt[n]{\rho} \cdot e^{i\frac{\vartheta+2k\pi}{n}}$$

$k \in N_0 \qquad k = 0, 1, 2, 3 \cdots, n-1$

$z = 1 + i$ $\qquad \rho = \sqrt{2}$

$\vartheta = \frac{\pi}{4}$

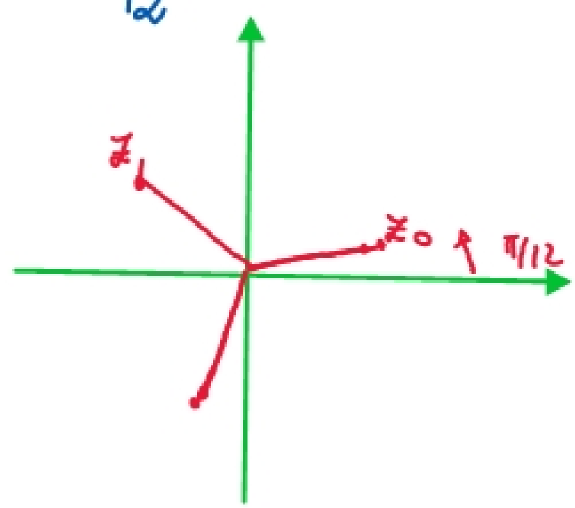
$\sqrt[3]{z} \qquad \sqrt[3]{\rho} = \left(\sqrt{2}\right)^{1/3} = 2^{1/6} = \sqrt[6]{2}$

$$\vartheta_0 = \frac{\frac{\pi}{4} + 2 \cdot 0\pi}{3} = \pi/12 \qquad z_0 = \sqrt[6]{2}\, e^{\frac{\pi}{12} i}$$

$$\vartheta_1 = \frac{\frac{\pi}{4} + 2 \cdot 1\pi}{3} = \frac{\pi/4 + 2\pi}{3} = \frac{9}{12}\pi \qquad z_1 = \sqrt[6]{2}\, e^{\frac{9}{12}\pi i}$$

$$\vartheta_2 = \frac{\frac{\pi}{4} + 2 \cdot 2\pi}{3} = \frac{\frac{\pi}{4} + 4\pi}{3} = \frac{17}{12}\pi \qquad z_2 = \sqrt[6]{2}\, e^{\frac{17}{12}\pi i}$$

$n = 3$



RADICI N-ESIME

genero un poligono regolare
n-vertici n lati

uguaglianze de verificare (de dimostrare)

$z = a + ib$

$\bar{z} = a - ib$

$z + \bar{z} = 2a$

$z - \bar{z} = 2bi$

$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$

$\overline{\left(\frac{1}{z}\right)} = \frac{1}{z}$

$|z| = \sqrt{a^2 + b^2}$

$|z| \geq 0$

$|\mathrm{Re}\, z| \leq |z|$

$|\mathrm{Im}\, z| \leq |z|$

$|z| \leq |\mathrm{Re}\, z| + |\mathrm{Im}\, z|$

$|z_1 + z_2| \leq |z_1| + |z_2|$

$|z_1 + z_2| \geq \big|\, |z_1| + |z_2| \,\big|$

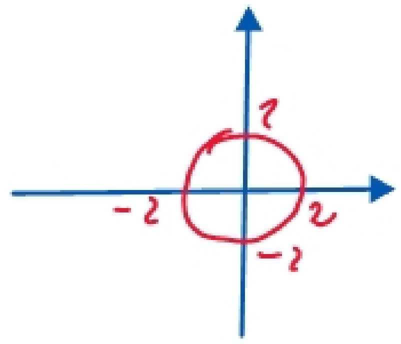
es 5

Trovare il luogo dei punti del piano t.c. $|z| = 2$

$z = x + iy$

$|z| = \sqrt{x^2 + y^2}$

$|z| = 2 \implies \sqrt{x^2+y^2} = 2$



$x^2 + y^2 = 4$ $\quad C(0,0) \quad R = 2$

sono i punti della circonferenza

$|z| \leq 2$ $\quad x^2 + y^2 \leq 4$

sono i punti del cerchio, compresa la circonferenza

$|z| > 2$ $\quad x^2+y^2 > 4$

tutti i punti del piano esterni al cerchio

$1 < |z| < 2$

$1 < x^2+y^2 < 4$

$x^2+y^2 = 1 \quad C(0,0) \quad R = 1$

$x^2+y^2 = 4 \quad C(0,0) \quad R = 2$

zona di piano compresa tra le 2 circonferenze

$\rightarrow$ corona circolare (bordi esclusi)