Sia $\alpha \in \mathbb{R}$, la serie $\sum_{n=1}^{+\infty} n^\alpha \left(1-\sqrt{\cos\frac{1}{n}}\right)$ converge $\iff$:

$(+)$ $(+)$

la serie è a termini positivi perché:

$$0 < \frac{1}{n} \le 1 \Rightarrow 0 < \cos\frac{1}{n} \le 1 \Rightarrow 1 - \sqrt{\cos\frac{1}{n}} > 0$$

$$0 < \sqrt{\cos\frac{1}{n}} \le \sqrt{1}$$

CRITERIO DEL CONFRONTO ASINTOTICO

$$a_n = n^\alpha \left(1-\sqrt{\cos\frac{1}{n}}\right) \simeq n^\alpha \left(1 - \underbrace{\left(1-\frac{1}{2}\frac{1}{n^2}\right)^{1/2}}\right) \simeq$$

$$\cos x = \underbrace{1-\frac{1}{2}x^2}_{(1+x)^\alpha} + \frac{1}{4!}x^4 \dots + o(x)^2$$

$$(1-x)^\alpha = 1 - \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 \dots$$

$$\alpha = \frac{1}{2}$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 \dots$$

$$\simeq n^\alpha \left(1 - \left(1 - \underset{\alpha}{\frac{1}{2}}\ \underset{x}{\frac{1}{2n^2}}\right) + o\left(\frac{1}{n}\right)^2\right) = n^\alpha \left(\cancel{1} - \cancel{1} + \frac{1}{2}\frac{1}{2n^2} + o\left(\frac{1}{n}\right)^2\right) = \frac{n^\alpha}{4n^2} = \frac{1}{4}\left(\frac{1}{n}\right)^{2-\alpha}$$

$\Rightarrow$ procedo a confronto con la serie armonica generalizzata $\sum \left(\frac{1}{n}\right)^{2-\alpha}$

che converge se $2-\alpha > 1$ $\quad -\alpha > -1 \quad \alpha < 1$

$$\lim_{n\to\infty} \frac{n^{\alpha}\left(1-\sqrt{\cos\frac{1}{n}}\right)}{\left(\frac{1}{n}\right)^{2-\alpha}} =$$

con lo sviluppo

$$= \lim_{n\to\infty} \frac{\frac{1}{4}\left(\frac{1}{n}\right)^{2-\alpha}}{\left(\frac{1}{n}\right)^{2-\alpha}} = \frac{1}{4}$$

valore finito

$\downarrow$

convergenze

purché $\alpha < 1$

- determinare il carattere della serie

$$\sum_{n=0}^{+\infty} \frac{n!}{\left(2+(n+1)!\right)^{\alpha}} \qquad \text{con } \alpha \in \mathbb{R}$$

$$a_n = \frac{n!}{\left(2+(n+1)!\right)^{\alpha}} = \frac{n!}{\left[(n+1)!\right]^{\alpha} \underbrace{\left[\underbrace{\frac{2}{(n+1)!}}_{\to 0} + 1\right]^{\alpha}}_{1}} \sim$$

asintoticamente $n \to +\infty$

$$\sim \frac{n!}{\left[(n+1)!\right]^{\alpha}} = \frac{n!}{(n+1)^{\alpha} \cdot \left[n!\right]^{\alpha}} =$$

$$= \frac{1}{(n+1)^{\alpha} \left[n!\right]^{\alpha-1}}$$

$$= \frac{1}{(n+1)^{\alpha} (n!)^{\alpha-1}}$$

se $\alpha = 1$

$$= \frac{1}{(n+1)^{1} (n!)^{0}} = \frac{1}{n+1} \qquad \frac{1}{n}$$

serie confronto è la serie armonica

$\Downarrow$ diverge

se $\alpha > 1$

$$= \frac{1}{(n+1)^{\alpha}} \cdot \frac{1}{(n!)^{\alpha-1}} < \frac{1}{(n+1)^{\alpha}} \qquad \frac{1}{n^{\alpha}}$$

serie armonica generalizzata

$\Downarrow$

convergenza

Calcola il valore della serie:

$$\sum_{n=1}^{+\infty} (-1)^n \frac{2^{n+1}}{3^{n+2} n!}$$

$$= \sum_{n=1}^{+\infty} (-1)^n \frac{2^{n+1}}{3^{n+1} \cdot 3 \cdot n!} = \frac{1}{3} \sum_{n=1}^{+\infty} (-1)^n \left(\frac{2}{3}\right)^{n+1} \frac{1}{n!} =$$

$$= \frac{1}{3} \sum_{n=1}^{+\infty} \frac{(-1)^n \left(\frac{2}{3}\right)^{n+1}}{n!} = \frac{1}{3} \sum_{n=1}^{+\infty} \frac{(-1)^n \left(\frac{2}{3}\right)^n}{n!} \cdot \frac{2}{3} =$$

$$= \frac{2}{9} \left(\sum_{n=0}^{+\infty} \frac{\left(-\frac{2}{3}\right)^n}{n!} - \underbrace{\frac{(-1)^0 \left(\frac{2}{3}\right)^0}{0!}}_{=1} \right) =$$

$0! = 1$

$$= \frac{2}{9} \left(e^{-2/3} \underbrace{- 1}_{=1} \right)$$

Vi ricorda:

$$\sum_{n=0}^{+\infty} \frac{x^n}{n!} = e^x$$

con $x = -2/3$

Calcolare il valore della serie:

$$\sum_{h=1}^{+\infty} \frac{2h+1}{(2h)!} =$$

$$= \sum_{h=1}^{+\infty} \frac{2h \cdot 1^{2h}}{(2h)!} + \sum_{h=1}^{+\infty} \frac{1}{(2h)!} =$$

$$\sum_{h=1}^{+\infty} \frac{1}{(2h)!} = \cosh 1 - 1 \quad (*)$$

N.B.: $\sum_{h=0}^{+\infty} \frac{x^{2h}}{(2h)!} = \cosh x$

$x = 1$

$a_0 = \cosh 0 = \frac{e^0 + e^{-0}}{2} \quad \frac{1+1}{2} = 1$

$\sum_{h=0}^{+\infty} \frac{x^{2h+1}}{(2h+1)!} = \operatorname{senh} x$

$x = 1$

$b_0 = \operatorname{senh} 0 = \frac{e^0 - e^{-0}}{2} \quad \frac{1-1}{2} = 0$

$$\sum_{h=1}^{+\infty} \frac{2h}{(2h)!} = \sum_{h=1}^{+\infty} \frac{\cancel{2h}}{\cancel{(2h)}(2h-1)!} = \sum_{h=1}^{\infty} \frac{1}{(2h-1)!}$$

dispari

$n \in \mathbb{N}$; $m \in \mathbb{N}$

dispari: $2n-1 = 2m+1$

$2m+2 = 2n$

$n = m+1$

cambio indice

$n \to m$

$n=1 \to m+1=1$

$\sum$ $m=0$

nuovo indice

cambio di indice:

$$\sum_{n=1}^{+\infty} \frac{1}{(2n-1)!} = \sum_{m=0}^{+\infty} \frac{1}{(2(m+1)-1)!} = \sum_{m=0}^{+\infty} \frac{1}{(2m+2-1)!} =$$

$$= \sum_{m=0}^{+\infty} \frac{1}{(2m+1)!} = \sinh 1 \qquad x=1$$

$$\Sigma := \sinh 1 + \overbrace{\cosh 1 - 1}^{(*)} = \frac{e^1 - e^{-1}}{2} + \frac{e^1 + e^{-1}}{2} - 1 =$$

$$= \frac{e^1 - \cancel{e^{-1}} + e^1 + \cancel{e^{-1}} - 2}{2} = e - 1$$

CONTINUITA'

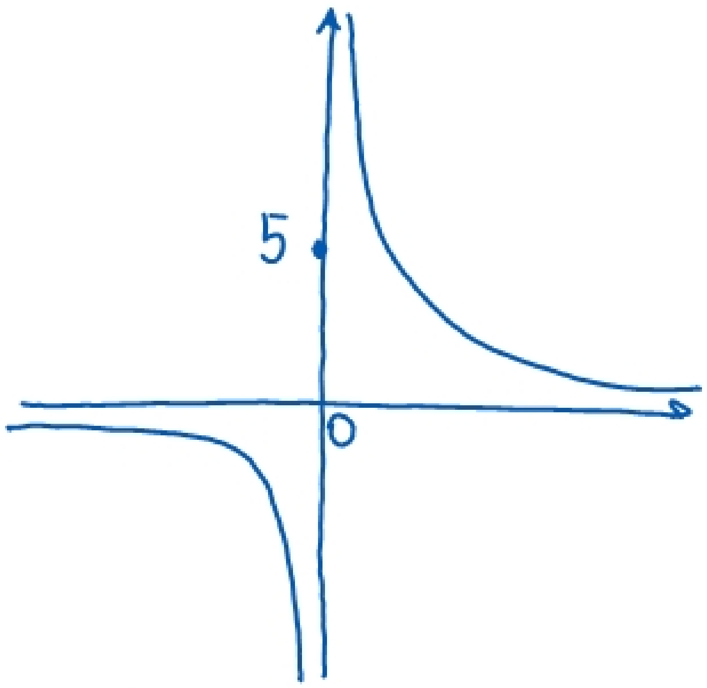
- DOMINIO
- punti di accumulazione

$$y = \begin{cases} \frac{1}{x} & x \neq 0 \\ 5 & x = 0 \end{cases}$$

dominio è $\mathbb{R}$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x \to 0^+} \frac{1}{x} = +\infty$$



$f(0) = 5$

$$y = \frac{x^2 + x - 6}{x^2 - x - 2}$$

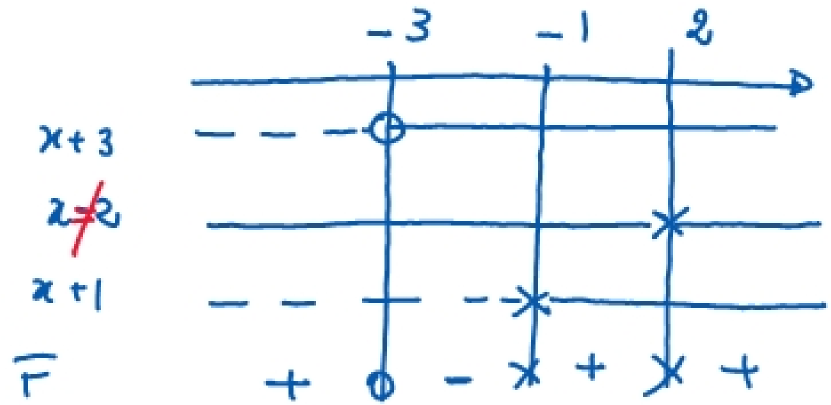
1. dom f: $\quad x^2 - x - 2 \neq 0$

$$(x-2)(x+1) \neq 0 \qquad x \neq 2$$
$$x \neq -1$$

2. segno

$$\frac{x^2 + x - 6}{x^2 - x - 2} \geq 0$$

$$\frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)} \geq 0$$



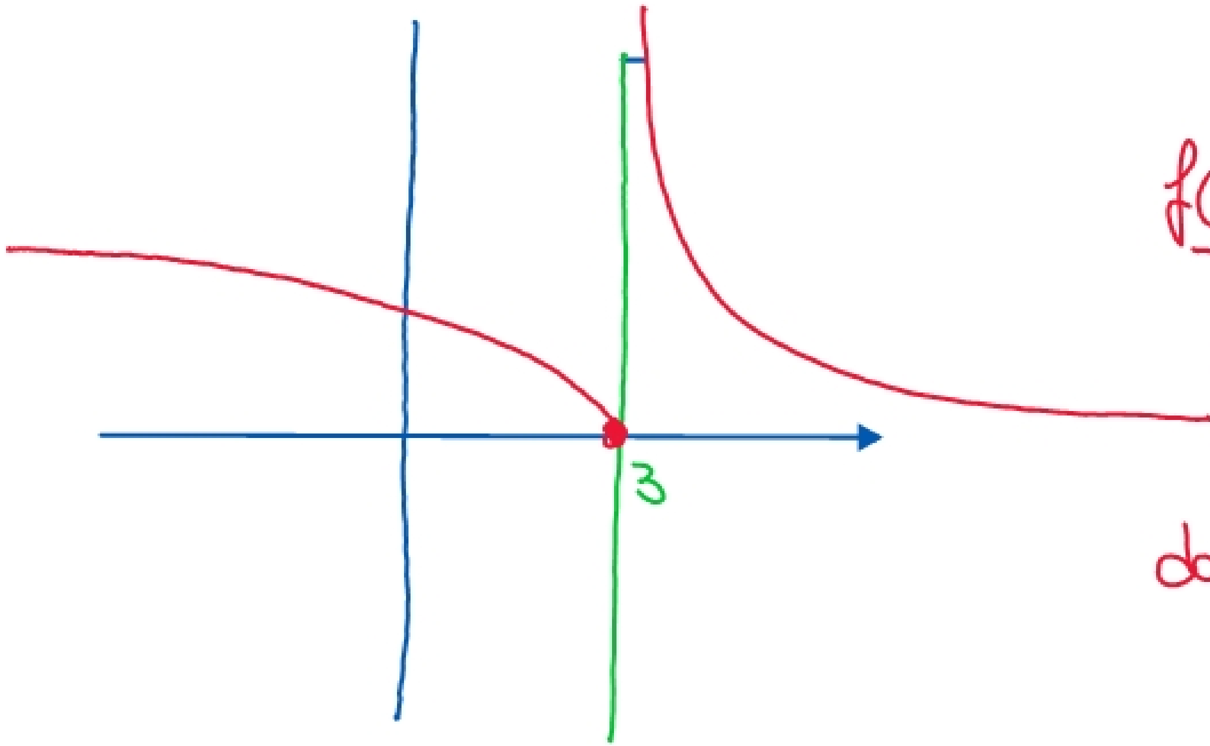
3. limiti

$$\lim_{x \to \pm\infty} \frac{x^2+x-6}{x^2-x-2} = 1$$

$$\boxed{\text{se } \lim_{x\to\infty} f(x) = \ell < \infty \quad y = \ell \text{ asintoto ORIZZONTALE}}$$

$$\lim_{x \to -1} \frac{x^2+x-6}{x^2-x-2} =$$

$$= \lim_{x \to -1^\pm} \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)} = \frac{+2}{0} = \pm\infty$$



$f(3) = 0$

$\lim_{x \to 3^+} f(x) = +\infty$

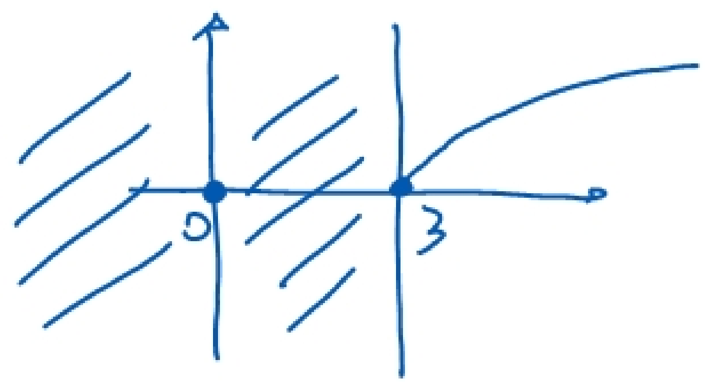
$\operatorname{dom} f = \mathbb{R}$

$f(x) = \sqrt{x^2 (x-3)}$

$x^2 (x-3) \geq 0 \qquad x \geq 3 \ \vee \ x = 0$

$x = 0$ pt. isolato

$y = f(x)$ è continua in $x = 0$



$\times$

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - x - 2} = \frac{0}{0}$$

$$= \lim_{x \to 2} \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)} =$$

$$= \lim_{x \to 2} \frac{x+3}{x+1} = \frac{5}{3}$$

$$y = f(x) \begin{cases} \dfrac{x^2 + x - 6}{x^2 - x - 2} & \text{per } x \neq -1,\ x \neq 2 \\ 5/3 & \text{per } x = 2 \end{cases}$$

Reso continua in $x = 2$
estendendo il dominio