Funzioni

domini

Radicando ≥ 0

$\sqrt{\ }$ radice pari

log $\rightarrow$ Argomento > 0

- $y = \sqrt{\ln(2x+3) - \ln(x-1)} = \sqrt{\ln\left(\frac{2x+3}{x-1}\right)}$

$$\begin{array}{l} \text{Arg.} \\ \text{Arg.} \\ \sqrt{\ } \end{array} \begin{cases} 2x+3 > 0 \\ x-1 > 0 \\ \ln(2x+3) - \ln(x-1) \geq 0 \end{cases}$$

DIVERSE

$$\begin{cases} x > -3/2 \\ x > 1 \\ \ln(2x+3) \geq \ln(x-1) \end{cases}$$

confronto logaritmi

entrambi crescenti $(e > 1)$

confronto degli argomenti mantenendo il verso

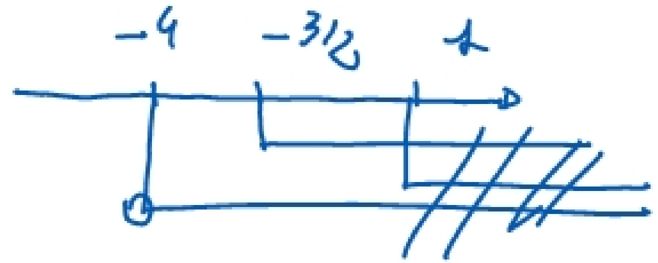
$2x+3 \geq x-1 \quad \rightarrow \quad x \geq -4$

ATTENZIONE (NO)

$$\begin{cases} \ln\left(\frac{2x+3}{x-1}\right) \geq 0 \rightarrow \frac{2x+3}{x-1} \geq 1 \\ \frac{2x+3}{x-1} > 0 \end{cases}$$

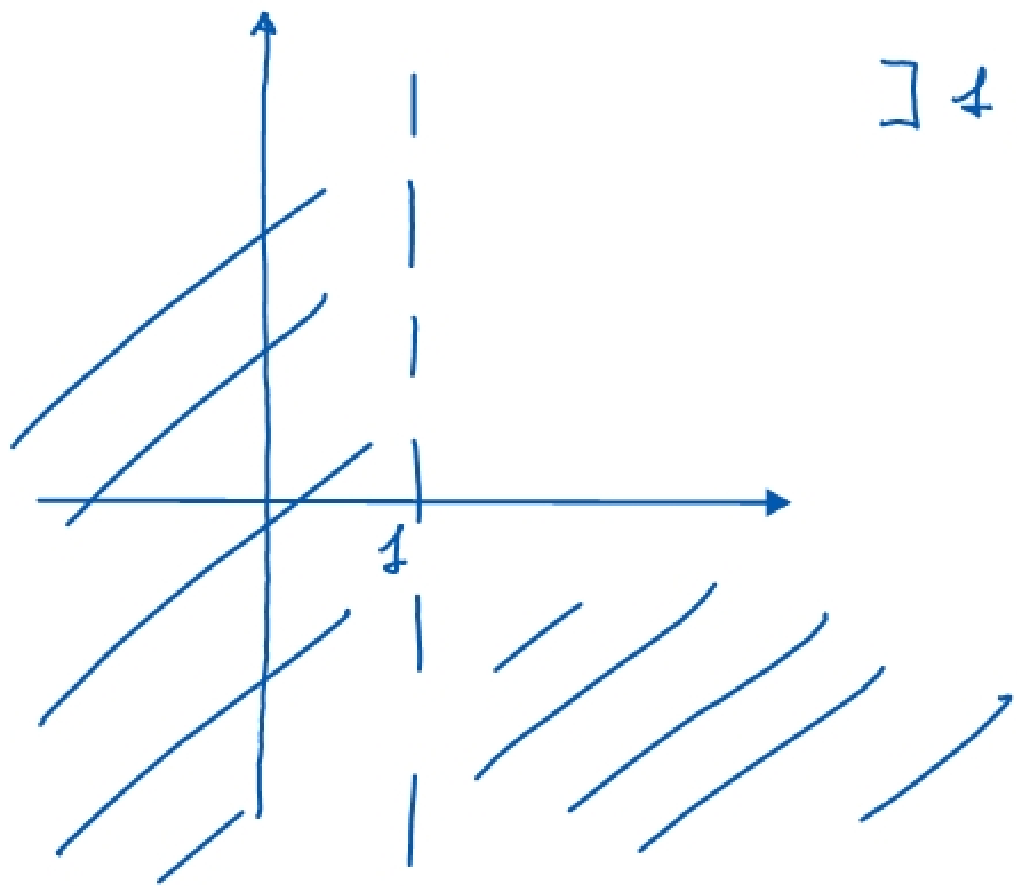
$-3/2 \qquad 1$

$$\begin{cases} x > -3/2 \\ x > 1 \\ x \geq -4 \end{cases} \Rightarrow$$



DOMINIO $X > 1$

$]1, +\infty[$



$y = \sqrt{f(x)}$

$y \geq 0$

- $y = (x^2 - x)^{x+1}$ esponenziale

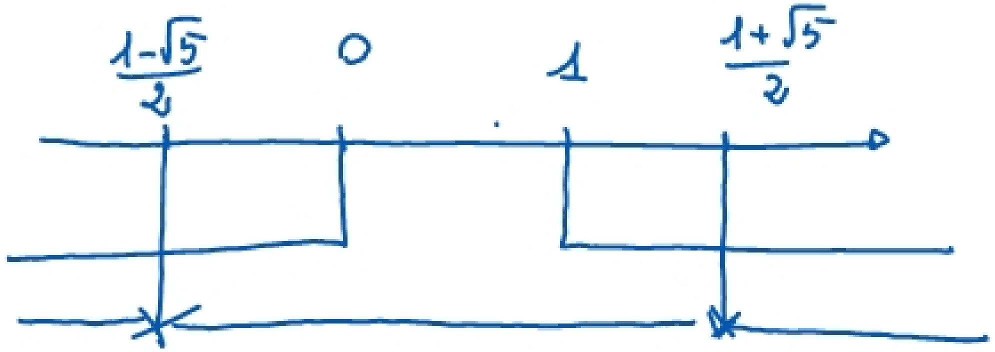
base variabile !

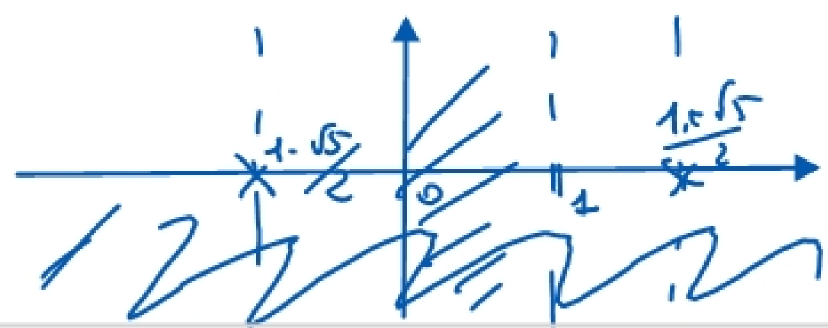
base > 0

base $\neq 1$

$$\begin{cases} x^2 - x > 0 \\ x^2 - x \neq 1 \end{cases} \qquad \begin{cases} x < 0 \cup x > 1 \\ x^2 - x - 1 \neq 0 \end{cases}$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2}$$





• $y = \operatorname{arcsen}\left(\dfrac{x^3+2x^2+4}{x^3} \right)$

dominio

$$-1 \le \frac{x^3+2x^2+4}{x^3} \le 1$$

$$\Rightarrow \begin{cases} \dfrac{x^3+2x^2+4}{x^3} \ge -1 \\[2ex] \dfrac{x^3+2x^2+4}{x^3} \le 1 \end{cases} \qquad \begin{cases} \dfrac{x^3+2x^2+4+x^3}{x^3} \ge 0 \\[2ex] \dfrac{\cancel{x^3}+2x^2+4-\cancel{x^3}}{x^3} \le 0 \end{cases}$$

$$\begin{cases} 2\,\dfrac{x^3+x^2+2}{x^3} \ge 0 \\[2ex] \dfrac{2x^2+4}{x^3}^{+} \le 0 \end{cases} \qquad \mathrm{I} \begin{cases} \\[2ex] x < 0 \end{cases}$$

$$\frac{x^3+x^2+2}{x^3} \boxed{\geq 0} \quad \Rightarrow \quad x \leq \alpha \cup x > 0$$

STUDIO DEL SEGNO

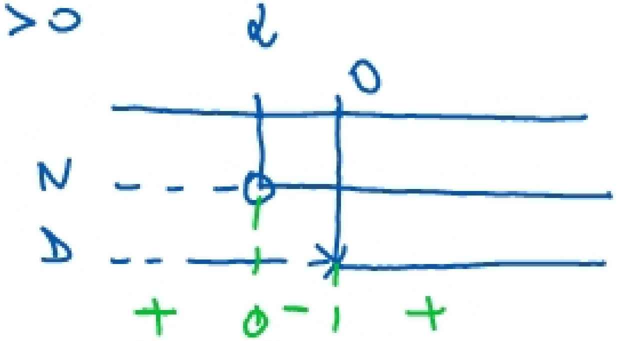
N. $\quad x^3 + x^2 + 2 \geq 0$

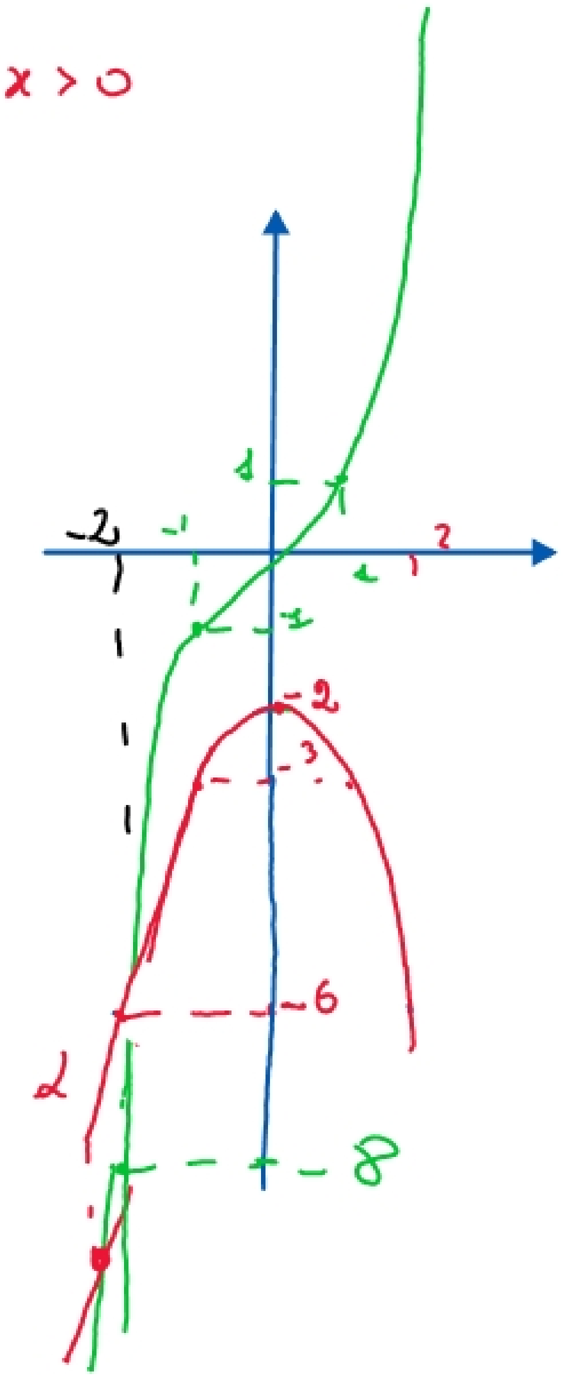
$$x^3 \geq -x^2 - 2$$

$f(-2)=(-8)$ $\qquad f(-2) = -6$

$-2 < \alpha < -1$

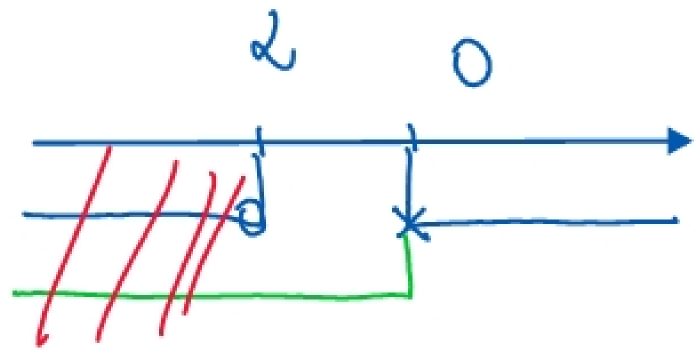
D. $\quad x > 0$





quindi il sistema

$$\begin{cases} x \leq \alpha \cup x > 0 \\ x < 0 \end{cases} \qquad (-2 < \alpha < -1)$$



$x \leq \alpha$

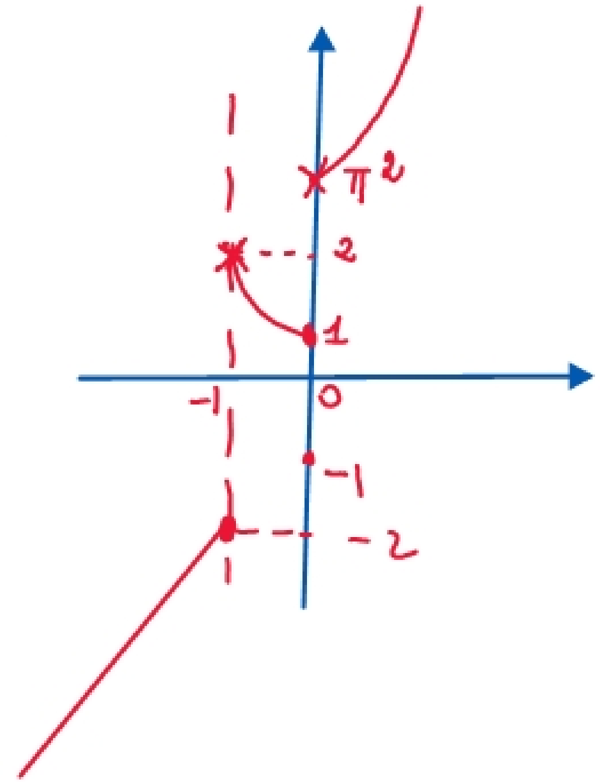
Limiti

1. definizioni

2. calcolo

    - teoremi dell'algebra dei limiti
    - forme di indeterminazione
    - teoremi
        - confronto
        - dei 2 carabinieri
        - esistenza e unicità

• $$y = f(x) = \begin{cases} x-1 & x \le -1 \quad (1) \\ x^2+1 & -1 < x \le 0 \quad (2) \\ (x+\pi)^2 & x > 0 \quad (3) \end{cases}$$



$$\lim_{x \to -1^-} f(x) = \underset{\text{retta}(1)}{f(-1)} = -2$$

$$\lim_{x \to 0^+} f(x) = \underset{\text{parabola }(3)}{\pi^2}$$

$$\lim_{x \to -1^+} f(x) = \underset{\text{parabola }(2)}{2}$$

$$\lim_{x \to 0^-} f(x) = \underset{\text{parabola }(2)}{1} = f(0)$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

(Dominio $\mathbb{R}$
presenta 2 discontinuità salto)

$$\bullet \quad \lim_{x \to 4} f(x) = 2 \qquad \lim_{x \to 4} g(x) = -3$$

calcolare :

$$\lim_{x \to 4} \left(g(x) + 3 \right) = \lim_{x \to 4} g(x) + 3 = -3+3 = 0$$

$$\lim_{x \to 4} x \cdot f(x) = \left(\lim_{x \to 4} x \right) \cdot \lim_{x \to 4} f(x) = 4 \cdot 2 = 8$$

$$\lim_{x \to 4} \left(g(x) \right)^2 = \left(\lim_{x \to 4} g(x) \right)^2 = (-3)^2 = 9$$

$$\lim_{x \to 4} \frac{g(x)}{f(x) - 1} = \frac{\lim_{x \to 4} g(x)}{\left(\lim_{x \to 4} f(x) \right) - 1} = \frac{-3}{2 - 1} = -3$$

— CALCOLO —

$$\lim_{\substack{x\to x_0 \\ (x\to\infty)}} f(x) = \begin{cases} \infty & (+\infty \,/\, -\infty) \\ \ell & \\ \text{F.I.} & \frac{0}{0}\,;\ \frac{\infty}{\infty}\,;\ \infty^{\infty};\ 0^{\infty};\ \infty^{0};\ 1^{\infty};\ +\infty-\infty\,;\ 0\cdot\infty \\ \nexists & \end{cases}$$

1. raccoglimenti totali (→ raccoglimento forzato)
2. scomposizioni [razionalizzazione inversa]
3. comportamento asintotico (catene di infiniti, ordine)
4. cambio di variabile
5. passaggio all'esponenziale $\left(f(x) = e^{\log f(x)} \right)$
6. limiti notevoli
7. De Hospital
8. sviluppi di Taylor $(x\to 0)$, Mc Laurin $(x\to x_0)$
9. teoremi di calcolo

non esistenza del limite

$$\lim_{x\to+\infty} \cos x = \nexists \qquad -1 \le \cos x \le 1$$

$$\lim_{x\to+\infty} (\cos x+5) = \nexists \qquad -1 \le \cos x \le 1$$

$$\underline{4} = -1+5 \le \cos x+5 \le 1+5 = \underline{6}$$

$$\lim_{x\to+\infty} x \underbrace{(\cos x+5)}_{\substack{[4,6]\\ +}} = +\infty$$

$$\lim_{x\to+\infty} x \underbrace{\cos x}_{[-1,+1]} = \nexists$$

$$\lim_{x\to+\infty} \frac{\overbrace{\cos x}^{[-1,1]}}{x} = 0 \qquad \lim_{x\to+\infty} \frac{\operatorname{sen} x}{x} = 0$$

LIMITE NOTEVOLE

$$\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1$$

$$\lim_{n\to+\infty} \frac{\operatorname{sen} 1/n}{1/n} = 1$$

$$\lim_{x\to+\infty} \{x + \underbrace{2\cos x}_{[-2,2]}\} = +\infty$$

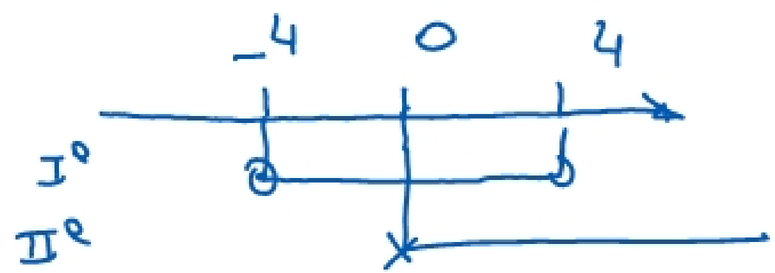
ATTENZIONE: il calcolo del limite avviene per valori appartenenti al dominio o di accumulazione per esso.
Se la $f(x)$ non è definita nell'intorno non ne calcolo il limite (non esiste)

$$\lim_{x\to+\infty}\left(\sqrt{16-x^2}+\log 2x\right) = \nexists$$

DOMINIO

$16-x^2\geq 0 \qquad 2x>0$

$-4\leq x\leq +4 \qquad x>0$



dominio è $0<x\leq 4$

$$\lim_{x\to 0^+}\left(\sqrt{16-x^2}+\log 2x\right) = 4-\infty = -\infty$$

$$\lim_{x\to 4^-}\left(\sqrt{16-x^2}+\log 2x\right) = \log 8 = \log 2^3 = 3\log 2$$

$$\bullet \lim_{x\to 64} \frac{x^{1/3}-4}{x^{1/2}-8} = \frac{\sqrt[3]{64}-4}{\sqrt{64}-8} = \frac{4-4}{8-8} = \frac{0}{0}$$

$$= \lim_{x\to 64} \frac{\sqrt[3]{x}-4}{\sqrt{x}-8} =$$

1. $\sqrt{\ }$ al denominatore

2. $x=64$ zero per il numeratore e per il denominatore

$$= \lim_{x\to 64} \frac{(\sqrt[3]{x}-4)(\sqrt{x}+8)}{(\sqrt{x}-8)(\sqrt{x}+8)} \cdot \frac{(\sqrt[3]{x})^2+4^2+4\sqrt[3]{x}}{(\sqrt[3]{x})^2+16+4\sqrt[3]{x}} =$$

$$= \lim_{x\to 64} \frac{\cancel{x-64}}{\cancel{x-64}} \cdot \frac{\sqrt{x}\overset{\to 8}{}+8}{(\sqrt[3]{x})^2+16+4\sqrt[3]{x}} =$$

$$(a-b)(a^2+b^2+ab) = a^3-b^3$$

$$= \frac{16}{4^2+16+4\cdot 4} = \frac{16}{3\cdot 16} = \frac{1}{3}$$

$$\lim_{x\to 2} \frac{x^3-2x^2-x+2}{x-2} = \frac{8-8-2+2}{2-2} = \frac{0}{0} \quad \text{F.I}$$

$\Rightarrow$ $(x-2)$ è divisore del Numeratore

$$N(x) = \underbrace{x^3-2x^2}-x+2 = x^2(x-2)-(x-2) = (x-2)(x^2-1)$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}\,(x^2-1)}{\cancel{x-2}} = 4-1 = 3$$

Ruffini

$(x-2)$

$$\begin{array}{c|ccc|c}
 & x^3 & x^2 & x^1 & x^0 \\
 & 1 & -2 & -1 & +2 \\
2 & & 2 & 0 & -2 \\
\hline
 & 1 & 0 & -1 & 0 \\
 & x^2 & x^1 & x^0 &
\end{array}$$

per i divisori

(2) = T.N.

divisori $+1$; -1

$+2$; -2

$P(1)$; $P(-1)$; $P(2)$; $P(-2)$