CONFRONTI ASINTOTICI E SERIE

criterio del confronto asintotico

$\sum_{n=0}^{+\infty} a_n$ e $\sum_{n=0}^{+\infty} b_n$ 1. serie a termini positivi

2. (con $b_n > 0$ almeno definitivamente)

3. esiste il $\lim_n \frac{a_n}{b_n}$

$\Rightarrow$ 1. se $\lim_n \frac{a_n}{b_n} = L \neq 0 \Rightarrow \sum_n a_n < +\infty \Leftrightarrow \sum_n b_n < +\infty$

2. se $\lim_n \frac{a_n}{b_n} = 0 \Rightarrow \sum_n b_n < +\infty \Rightarrow \sum_n a_n < +\infty$

equivale

$\sum_n a_n = +\infty \Leftarrow \sum_n b_n = +\infty$

3. se $\lim_n \frac{a_n}{b_n} = +\infty \Rightarrow \sum_n a_n < +\infty \Rightarrow \sum_n b_n < +\infty$

equivale

$\sum_n b_n = +\infty \Leftarrow \sum_n a_n = +\infty$

$$\sum_{n=1}^{+\infty} \left(1 - \cos \frac{1}{n}\right)$$

$$-1 < \cos \frac{1}{n} < 1$$

$$-1 < -\cos \frac{1}{n} < 1 \qquad \text{cambio verso/segno } (-1)$$

$$1 - 1 < 1 - \cos \frac{1}{n} < 1 + 1 \qquad \text{aggiungo } 1$$

$$0 < 1 - \cos \frac{1}{n} < 1 \qquad \text{Serie T.p.}$$

$$\boxed{n \to \infty \qquad \frac{1}{n} \to 0}$$

sviluppo $\cos x = 1 - \frac{x^2}{2} + o(x^2)$

sostituendo $\frac{1}{n} = x \quad \Rightarrow \quad \cos \frac{1}{n} = 1 - \frac{\left(\frac{1}{n}\right)^2}{2} + o\left(\left(\frac{1}{n}\right)^2\right)$

$$\Rightarrow \quad a_n = 1 - \cos \frac{1}{n} = \cancel{1} - \cancel{1} + \frac{1}{2} \frac{1}{n^2} + o\left(\left(\frac{1}{n}\right)^2\right) = \frac{1}{2} \frac{1}{n^2} + o\left(\frac{1}{n^2}\right)$$

$$\lim_{n \to +\infty} \frac{1 - \cos \frac{1}{n}}{\frac{1}{n^2}} =$$

successione confronto

$b_n = \frac{1}{n^2}$

serie armonica generalizzata

$$= \lim_{n \to +\infty} \frac{\frac{1}{2} \frac{1}{n^2} + o\left(\frac{1}{n^2}\right)}{1/n^2} = \frac{1}{2}$$

ma attenzione

equivale

$$= \lim_{x \to 0} \frac{\frac{1}{2} x^2 + o(x^2)}{x^2} = \frac{1}{2}$$

$\Rightarrow \sum \frac{1}{n^2}$ converge

$$\lim_{n \to +\infty} \frac{1 - \cos \frac{1}{n}}{\frac{1}{n^2}} = \frac{1}{2} < +\infty$$

$\Rightarrow \sum \left(1 - \cos \frac{1}{n}\right)$ conv.

attenzione

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

Ricavato come limitato del notevole

posto $x = \frac{1}{n}$ $\quad a_n = 1 - \cos\frac{1}{n}$

$b_n = \frac{1}{n^2}$

• $\sum_{n=1}^{+\infty} \left(\frac{1}{n} - \sin \frac{1}{n} \right)$

$a_n = \frac{1}{n} - \sin \frac{1}{n}$

$\lim_{n \to +\infty} \frac{a_n}{b_n} = ?$ ricercare b_n come serie confronto

si considera lo sviluppo di $\sin x = x - \frac{1}{6} x^3 + o(x^3)$ $\underline{\underline{x \to 0}}$

$\frac{1}{n} = x \quad \to a_n = \frac{1}{n} - \sin \frac{1}{n} \to x - \sin x =$

$= x - \left(x - \frac{1}{6} x^3 + o(x^3) \right) =$

$= x - x + \frac{1}{6} x^3 + o(x^3) \sim \frac{1}{6} x^3 + o(x^3)$

$\sin x = \frac{1}{6}x^3 + o(x^3) \underset{\text{sostituendo}}{\Rightarrow} \frac{1}{6}\frac{1}{n^3} + o\left(\frac{1}{n^3}\right)$

la serie confronto $\sum \frac{1}{n^3}$ serie armonica generalizzata

$\downarrow$

convergente

$$\lim_{n \to +\infty} \frac{\frac{1}{6}\frac{1}{n^3} + o\left(\frac{1}{n^3}\right)}{1/n^3} \Leftrightarrow \lim_{x \to 0} \frac{\frac{1}{6}x^3 + o(x^3)}{x^3} = \frac{1}{6}$$

$\Rightarrow \sum \left(\frac{1}{n} - \sin\frac{1}{n}\right)$ convergente

- $\sum_{n=1}^{+\infty} (-1)^n \operatorname{Tg}\left(\frac{1}{n}\right)$ serie a segni alterni

1. convergenza assoluta $\Rightarrow \sum_{n=1}^{+\infty} \operatorname{Tg}\left(\frac{1}{n}\right)$ Termini positivi

considerando lo sviluppo:

$$\operatorname{tg} x = x + o(x) \qquad x \to 0$$

$$\Downarrow$$

$x = \frac{1}{n}$ $$\operatorname{tg}\left(\frac{1}{n}\right) = \frac{1}{n} + o\left(\frac{1}{n}\right) \sim \frac{1}{n} \qquad n \to +\infty$$

$\sum \frac{1}{n}$ serie armonica, non convergente (diverge)

$\Rightarrow \sum \operatorname{Tg} \frac{1}{n}$ non è convergente (diverge) $\Rightarrow \sum (-1)^n \operatorname{tg} \frac{1}{n}$ non conv. ass.

2. convergenza semplice

a. successione infinitesima

$$\lim_{n\to+\infty} \operatorname{tg}\underbrace{\frac{1}{n}}_{\to 0} = 0 \qquad \text{infinitesima}$$

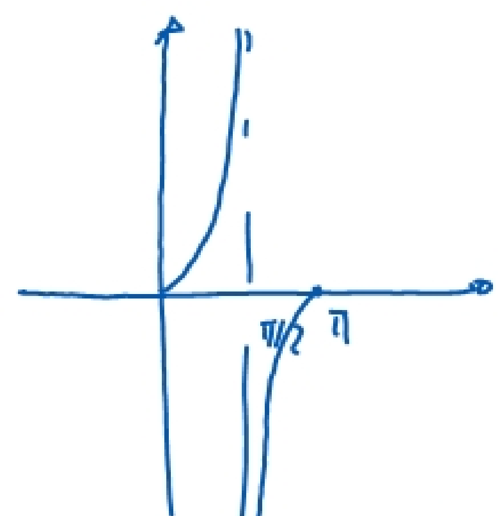
b. successione decrescente

sono soddisfatti $\Rightarrow$ converge semplicemente

$$0 < 1 \le n < +\infty$$

$$0 < \frac{1}{n} \le 1 < \frac{\pi}{2}$$

$$\operatorname{tg}\frac{1}{n}$$



$$0 < \frac{1}{n} < \frac{\pi}{2}$$

$n < n+1$ $\qquad \operatorname{tg}\frac{1}{n+1} < \operatorname{tg}\frac{1}{n} \Rightarrow$ a_n è decresc.

$\frac{1}{n} > \frac{1}{n+1}$ $\quad$ tg mantiene le disuguaglianze

$$\cdot \sum_{n=1}^{+\infty} \left[2\,\mathrm{arctg}(n+1) - \pi\right] \cdot \underbrace{\left[\cos\left((n+1)\pi\right)\right]}_{(-1)^{n+1}} =$$

$$= \sum_{n=1}^{+\infty} (-1)^{n+1} \left[2\,\mathrm{arctg}\,(n+1) - \pi\right] \text{ = segno alterno}$$

$$= \sum_{n=1}^{+\infty} (-1)^{n} \left[\pi - 2\,\mathrm{arctg}\,(n+1)\right]$$

1. convergenza assoluta

$$\boxed{\mathrm{arctg}\,(n+1) = \frac{\pi}{2} - \mathrm{arctg}\,\frac{1}{n+1}}$$

$\operatorname{arctg} x = x + o(x) \qquad x \to 0$

$$\pi - 2\operatorname{arctg}(n+1) = \pi - 2\left(\frac{\pi}{2} - \operatorname{arctg}\frac{1}{n+1}\right) =$$

$$= \cancel{\pi} - \cancel{\pi} + 2\operatorname{arctg}\frac{1}{n+1} = 2\operatorname{arctg}\frac{1}{n+1} \sim$$

considerando lo sviluppo

$$\sim 2\left(\frac{1}{n+1} + \underbrace{o\left(\frac{1}{n+1}\right)}\right) = \qquad n \to +\infty$$

$$= \frac{2}{n+1} + o\left(\frac{1}{n+1}\right) \sim \frac{2}{n+1}$$

scelgo come serie confronto $\sum \frac{1}{n+1}$ serie armonica divergente

NO CONV. ASS.

2. convergenza semplice:

$$\sum_{n=1}^{\infty} \Big[\underbrace{\pi - 2 \arctg (n+1)}_{b_n} \Big]$$

1. infinitesimo
2. decrescente

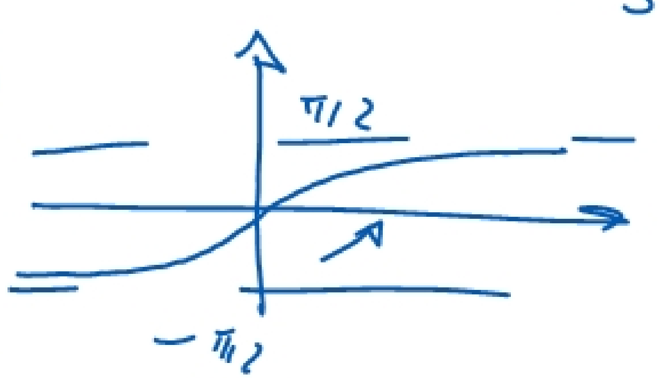
$\Downarrow$

conv. semplice.

$$b_n = \pi - 2 \arctg (n+1) = 2 \arctg \frac{1}{n+1}$$

1) $\lim_{n \to +\infty} 2 \arctg \underbrace{\frac{1}{n+1}}_{0} = 0$ infinitesimo

2)



$\arctg x \nearrow$ monotono

$n < n+1$

$\frac{1}{n} > \frac{1}{n+1}$

$\arctg \frac{1}{n} > \arctg \frac{1}{n+1} \Rightarrow \arctg \frac{1}{n+1} \searrow$

Decrescente