CONTINUITA'

1) determinare $\alpha, \beta \in \mathbb{R}$:

$$f(x) := \begin{cases} \log(x+\beta^2) & \text{se } x>0 \\ \dfrac{1-\cos(\alpha x)}{\operatorname{arctg}(x^2)} & \text{se } x<0 \\ 1 & \text{se } x=0 \end{cases}$$

DOM: $x+\beta^2>0$ $\quad x>\underbrace{-\beta^2}_{\leq 0}$ $\quad \beta^2 \geq 0$

$\operatorname{arctg} x^2 \neq 0 \Rightarrow x \neq 0$

definita su $\mathbb{R}$

da destra $\quad f(x) = \log(x+\beta^2)$

$$\lim_{x\to 0^+} \log(x+\beta^2) = \log \beta^2$$

$\Rightarrow \boxed{\log \beta^2 = 1}$ $\quad \beta^2 = e$ $\quad \boxed{\beta = \pm\sqrt{e}}$

da sinistra $\quad f(x) = \dfrac{1-\cos \alpha x}{\operatorname{arctg} x^2}$

$$\lim_{x\to 0^-} \frac{\overbrace{1-\cos(\alpha x)}^{1-1}}{\underbrace{\operatorname{arctg} x^2}_{0}} = \frac{0}{0} \quad f.i.$$

$$= \lim_{x\to 0^-} \frac{1-\left(1-\frac{(\alpha x)^2}{2}+o(x^2)\right)}{x^2+o(x^2)} =$$

$$= \lim_{x\to 0^-} \frac{\cancel{1}-\cancel{1}+\frac{\alpha^2 x^2}{2}+o(x^2)}{x^2+o(x^2)} =$$

$$= \lim_{x\to 0^-} \frac{\frac{\alpha^2 x^2}{2}+o(x^2)}{x^2+o(x^2)} = \frac{\alpha^2}{2}$$

$$f(0)=1 \quad \Rightarrow \quad \frac{\alpha^2}{2}=1 \qquad \alpha^2=2$$

$$\boxed{\alpha=\pm\sqrt{2}}$$

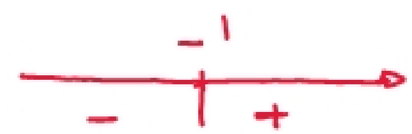
soluzi $\left\{\begin{array}{l}\beta=\pm\sqrt{e}\\ \alpha=\pm\sqrt{2}\end{array}\right\}$

a) $f: \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} \dfrac{e^{x-3}-1}{(x+1)(x-3)} & \text{se } x \neq -1 \text{ e } x \neq 3 \\ 0 & \text{se } x = -1 \text{ o } x = 3 \end{cases}$$

nel punto $x = -1$

$$\lim_{x \to -1^{\pm}} \frac{e^{x-3}-1}{(x+1)(x-3)} = \frac{\overbrace{e^{-4}-1}^{(-)}}{0 \cdot \underbrace{(-4)}_{(-)\,\to\,+}} = \pm\infty$$

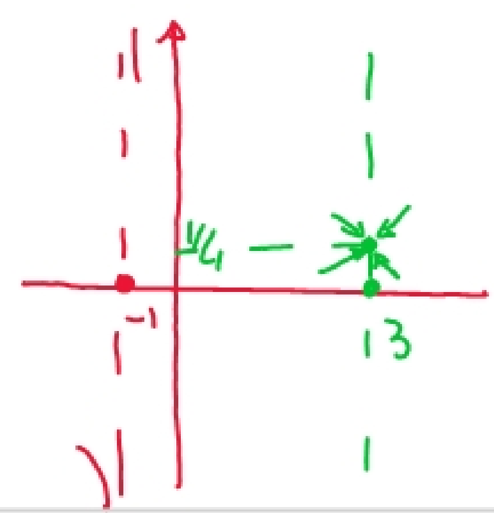


punto di infinito

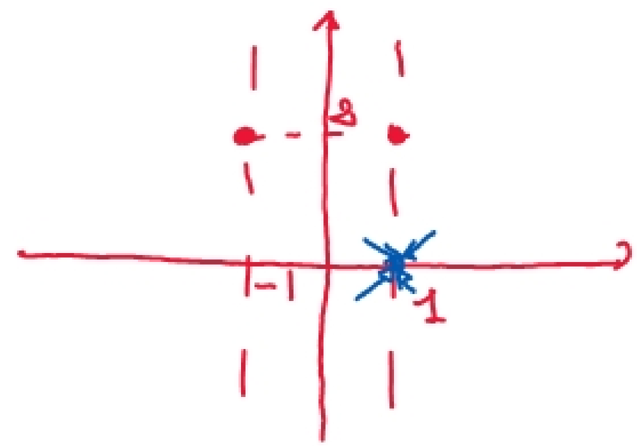
$$\lim_{x \to 3} \frac{e^{x-3}-1}{(x+1)(x-3)} = \frac{0}{0} = \frac{1}{4}$$

$$\underbrace{\frac{e^{x-3}-1}{x-3}}_{\to 1}$$

salto



3) $$f(x) = \begin{cases} (x-1)\, e^{\operatorname{sen} \frac{1}{x^2-1}} & \text{se } x \neq 1 \text{ e } x \neq -1 \\ 2 & \text{se } x = 1 \text{ o } x = -1 \end{cases}$$



$f : R \to R$

$$\lim_{x \to 1} (x-1)\, e^{\overbrace{\operatorname{sen} \frac{1}{x^2-1}}^{\nexists}} = 0 \qquad \frac{1}{x^2-1} \to \infty$$

$$-1 < \operatorname{sen}(h(x)) < 1$$

$$0 < e^{-1} < e^{\operatorname{sen}(h(x))} < e^{1}$$

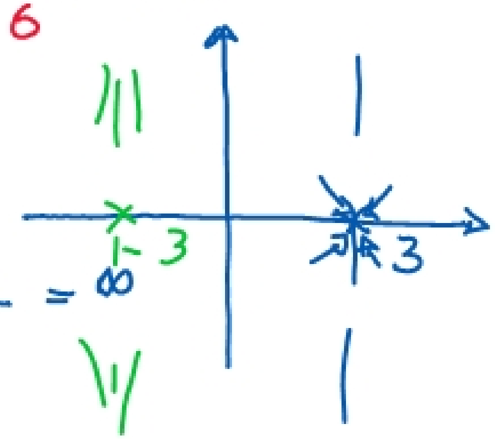
$$\lim_{x \to -1} (x-1)\, e^{\operatorname{sen} \frac{1}{x^2-1}} = (-2) \cdot (\qquad) = \nexists$$

4) $$f(x) = \frac{x^2-6x+9}{x^2-9} = \quad \text{dom.} \quad x^2-9 \neq 0 \quad x \neq \pm 3$$

$$= \frac{(x-3)^2}{x^2-9} = \frac{(x-3)^2}{(x-3)(x+3)}$$

$$\lim_{x\to 3} \frac{x^2-6x+9}{x^2-9} = \lim_{x\to 3} \frac{(x-3)^{\cancel{2}}}{\cancel{(x-3)}(x+3)} = \frac{0}{6} = 0$$

$$\lim_{x\to -3} \frac{x^2-6x+9}{x^2-9} = \lim_{x\to -3} \frac{(x-3)^{\cancel{2}}}{\cancel{(x-3)}(x+3)} = \frac{-6}{0} = \infty$$



NON E' PUNTO DI INFINITO!

ampliamento del dominio $$f(x) \begin{cases} \dfrac{x^2-6x+9}{x^2-9} & \text{se } x \neq \pm 3 \\ 0 & \text{se } x = 3 \end{cases}$$

CALCOLO DELLE DERIVATE

$$y = \frac{x}{\sqrt{2-x^2}} =$$

$$= \frac{x}{(2-x^2)^{1/2}} =$$

$$= x \cdot (2-x^2)^{-1/2}$$

N.B. ove è possibile fattorizzare
⇓
studio del segno per lo studio di funzione

FORMULE

$$D\left[\frac{N(x)}{D(x)}\right] = \frac{D[N(x)] \cdot D(x) - D[D(x)] \cdot N(x)}{[D(x)]^2}$$

$$D[t^n] = n\, t^{n-1}$$

$$D[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

$$D[(f(x))^n] = n[f(x)]^{n-1} \cdot D[f(x)]$$

$$D[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$f(x) = \frac{x}{(2-x^2)^{1/2}}$$

$$f'(x) = \frac{(1)\cdot(2-x^2)^{1/2} - \overbrace{\frac{1}{\cancel{2}}(2-x^2)^{-1/2}\cdot(-\cancel{2}x)}^{D\ (\text{denominatore})}\cdot x}{\left((2-x^2)^{1/2}\right)^2} =$$

$$= \frac{\sqrt{2-x^2} + x^2\frac{1}{\sqrt{2-x^2}}}{(2-x^2)} =$$

$$= \frac{2-x^2+x^2}{\sqrt{2-x^2}\,(2-x^2)} = \frac{2}{\sqrt{2-x^2}\,(2-x^2)} =$$

$$= \frac{2}{\sqrt{(2-x^2)^3}}$$

$$f(x) = \frac{x}{\sqrt{(2-x^2)}} \qquad f'(x) = \frac{2}{(2-x^2)\sqrt{2-x^2}}$$

dom f $\quad 2-x^2 > 0$

$-\sqrt{2} < x < \sqrt{2}$

dom f' $\quad 2-x^2 \neq 0$

$2-x^2 > 0$

$-\sqrt{2} < x < \sqrt{2}$

in $x = \pm\sqrt{2}$ valuto la derivabilità ?

NO!!

per valutare la derivabilità di una funzione in un punto x_0

$x_0 \in \text{dom } f$ (definita)

e

in x_0 la $f(x)$ deve essere continua

$f(x) = \sqrt{2-x^2} = \quad \text{dom} f \quad 2-x^2 \geq 0 \quad -\sqrt{2} \leq x \leq \sqrt{2}$

$= (2-x^2)^{1/2}$

$f'(x) = \dfrac{1}{2\sqrt{2-x^2}} \cdot (-2x) = -\dfrac{x}{\sqrt{2-x^2}}$

$\text{dom} f' \quad 2-x^2 > 0 \quad -\sqrt{2} < x < \sqrt{2}$

$\boxed{\text{dom} f' \subset \text{dom} f}$

in $x = \pm\sqrt{2}$ $\qquad f(\pm\sqrt{2}) = 0$

valuto il comportamento di $f(x)$ in $x = \pm\sqrt{2}$

$\lim_{h \to 0} \dfrac{f(x_0+h) - f(x_0)}{h}$

$y = \log x$ dom $x > 0$

$y' = \frac{1}{x}$ dom $x \neq 0$

dom $f' \supset$ dom f

$[x \neq 0]$ $[x > 0]$

per i valori $x < 0$ non interessano al fine dello studio della funzione

lo stesso in $x = 0$ dove la $f(x)$ non è definita

- $y = \dfrac{1}{\operatorname{arctgh} x} = (\operatorname{arctgh} x)^{-1} =$

$D(\operatorname{arctgh} x) = \dfrac{1}{1-x^2}$ $\quad (-1 < x < 1)$

$$y' = -1 \cdot (\operatorname{arctgh} x)^{-2} \cdot \frac{1}{1-x^2} = \frac{1}{(\operatorname{arctgh} x)^2 (x^2-1)}$$

- $y = \dfrac{1}{\operatorname{arctg} x} = (\operatorname{arctg} x)^{-1}$

$D(\operatorname{arctg} x) = \dfrac{1}{1+x^2}$

$$y' = -1\, (\operatorname{arctg} x)^{-2} \cdot \frac{1}{1+x^2} = -\frac{1}{\operatorname{arctg}^2 x\, (x^2+1)}$$

- $y = \cos\left((x^2+x)^5\right)$

$$y' = -\operatorname{sen}\Big[\underbrace{(x^2+x)^5}_{t^5}\Big] \cdot \underbrace{5\cdot(x^2+x)^4}_{D(t^5)}\ \underbrace{(2x+1)}_{D(\text{base})} =$$

$$= -5\,(2x+1)\,(x^2+x)^4\ \operatorname{sen}\left[(x^2+x)^5\right]$$

$D(\operatorname{sen} x) = \cos x$

$D(\cos x) = -\operatorname{sen} x$

$D(\operatorname{tg} x) = 1+\operatorname{tg}^2 x = \dfrac{1}{\cos^2 x}$

$D(\operatorname{ctg} x) = -1-\operatorname{ctg}^2 x = -\dfrac{1}{\operatorname{sen}^2 x}$

- $y = \dfrac{1}{x} + \operatorname{sen}\dfrac{1}{x}$

$\dfrac{1}{x} \Rightarrow x^{-1} \quad D(x^{-1}) = -x^{-2} = -\dfrac{1}{x^2}$

$$y' = -\frac{1}{x^2} + \cos\frac{1}{x}\ \underbrace{\left(-\frac{1}{x^2}\right)}_{D(\text{ang. del sen})} = \underbrace{-\underset{+}{\frac{1}{x^2}}\cdot\Big(\underset{+\ \Rightarrow}{\cos\frac{1}{x}+1}\Big)}_{-}$$

$$\cdot\ y = \arcsin\sqrt{1-x^2} =$$

$$= \arcsin\left[(1-x^2)^{1/2}\right]$$

$$y' = \frac{1}{\sqrt{1-\left((1-x^2)^{1/2}\right)^2}} \cdot \underbrace{\frac{1}{\cancel{2}}\frac{1}{\sqrt{1-x^2}} \cdot (-\cancel{2}x)}_{\text{D argomento}} =$$

$$= \frac{-x}{\sqrt{1-x^2}\ \sqrt{1-1+x^2}} = \frac{-x}{\sqrt{x^2}\ \sqrt{1-x^2}} = \frac{-x}{|x|\ \sqrt{1-x^2}} =$$

$$f'(x) = \begin{cases} -\dfrac{1}{\sqrt{1-x^2}} & \text{se } x > 0 \\ \dfrac{1}{\sqrt{1-x^2}} & \text{se } x < 0 \end{cases}$$

$$D(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

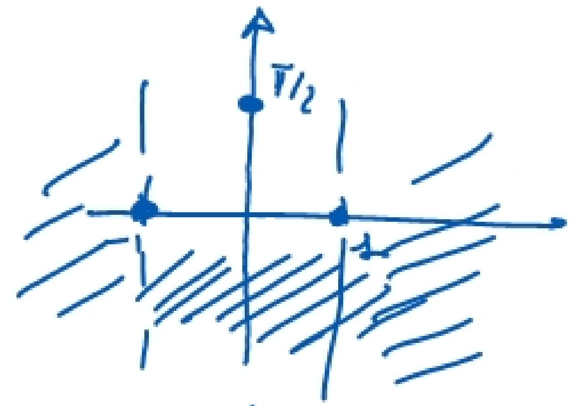
$$D(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$$

$$D(\operatorname{arctg} x) = \frac{1}{1+x^2}$$

$y = f(x) = \arcsen \sqrt{1-x^2}$ dom $f(x) \Rightarrow 1-x^2 \geq 0$

$-1 \leq x \leq 1$

segno $f(x) \geq 0$



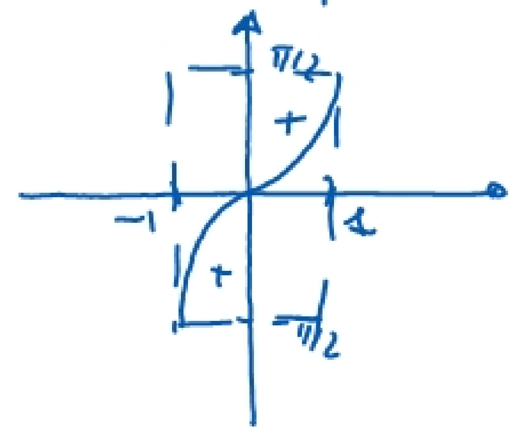
$$y' = \begin{cases} -\dfrac{1}{\sqrt{1-x^2}} & \text{se } x > 0 \\ \dfrac{1}{\sqrt{1-x^2}} & \text{se } x < 0 \end{cases}$$

$f(0) = \arcsen \sqrt{1-0} = \arcsen 1 = \pi/2$

in $x = 0$ $f'(x)$ non esiste

$$\lim_{x \to 0^-} f'(x) = 1$$

$$\lim_{x \to 0^+} f'(x) = -1$$



limite del rapporto incrementale da sinistra e da destra: $x_0 = 0$

$$\frac{f(x_0+h)-f(x_0)}{h} = \frac{\operatorname{arcsen}\sqrt{1-(0+h)^2} - \operatorname{arcsen}\sqrt{1-0}}{h} = \frac{\operatorname{arcsen}\sqrt{1-h^2} - \operatorname{arcsen} 1}{h}$$

$$\lim_{h\to 0} \frac{\operatorname{arcsen}\sqrt{1-h^2} - \overbrace{\operatorname{arcsen} 1}^{=\pi/2}}{h} = \frac{0}{0}$$

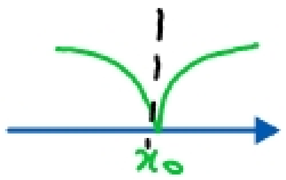
$$\underset{\substack{\text{D.H.}\\ \text{derivate}}}{=} \lim_{h\to 0} \frac{1}{\sqrt{1-\left(\sqrt{1-h^2}\right)^2}} \cdot \frac{1}{2\sqrt{1-h^2}} \cdot (-2h) =$$

$$= \lim_{h\to 0} \frac{-\cancel{2}h}{\cancel{2}\sqrt{1-h^2}\cdot\sqrt{h^2}} = \lim_{h\to 0^{\pm}} -\frac{h}{|h|\underbrace{\sqrt{1-h^2}}_{>0}} =$$

$= \mp 1$ ← coeff. angolare rette tangenti

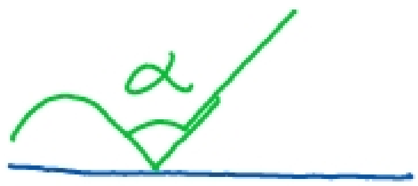
PUNTI DI NON DERIVABILITÀ

CUSPIDI



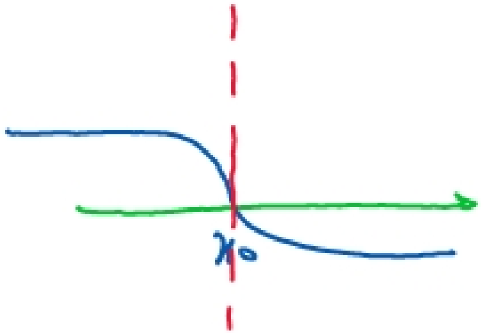
x_0 è punto di raccordo, ove la funzione è continua ma 1. m_- e/o m_+ è ∞

ANGOLOSI



x_0 è punto di raccordo, ove la funzione è continua ma 1. $m_- \neq m_+$ e sono finiti
2. formano un angolo α

TANGENZA VERTICALE



es. $y = \sqrt{x}$ dom $x \geq 0$

$y' = \dfrac{1}{2\sqrt{x}}$

la tangente in $x=0$ è l'asse delle ordinate

$m = +\infty$

$$\cdot\ y = \frac{\operatorname{arctg} \overbrace{(x^4 - x^2)}^{\text{Arg.}}}{x} \qquad x \neq 0$$

$$y' = \frac{\frac{1}{1+(x^4-x^2)^2} \cdot \overbrace{(4x^3 - 2x)}^{D(\text{arg})}\, x - \operatorname{arctg}(x^4 - x^2)}{x^2} =$$

$$= \frac{x^2(4x^2-2) - [1 + (x^4-x^2)^2]\operatorname{arctg}(x^4-x^2)}{\underset{+}{(1+(x^4-x^2)^2)}\quad \underset{+}{x^2}}$$

con $x \neq 0$

- $y = \dfrac{x}{3} + \sqrt[3]{2-x} = \dfrac{x}{3} + (2-x)^{1/3}$

dom $\forall x \in \mathbb{R} \Rightarrow (-\infty, +\infty)$

$f(x) \geq 0$ $\qquad \dfrac{x}{3} + \sqrt[3]{2-x} \geq 0$

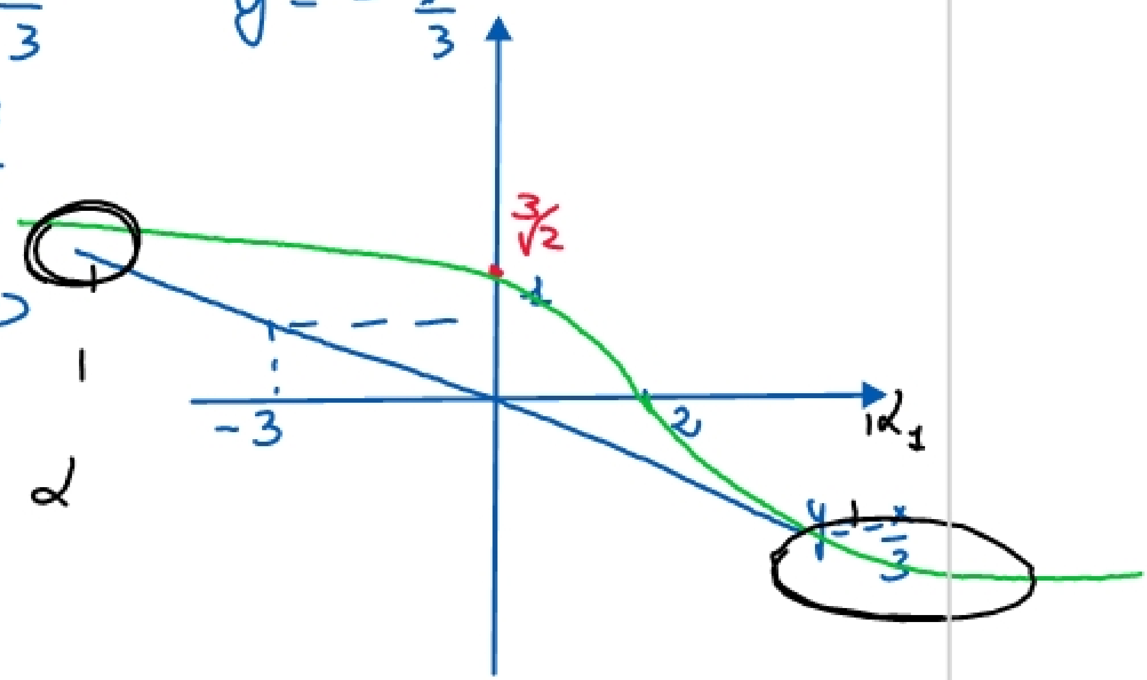
$\sqrt[3]{2-x} \geq -\dfrac{x}{3}$

$2 - x \geq -\dfrac{x^3}{27}$

$\dfrac{x^3}{27} - x + 2 \geq 0$

$y = \sqrt[3]{2-x}$

$y = -\dfrac{x}{3}$



$\begin{cases} x = 0 \\ y = \sqrt[3]{2} \end{cases}$

$\sqrt[3]{2} > 1$

$2 > 1^3$

$$y = \frac{x}{3} + (2-x)^{1/3}$$

$$y' = \frac{1}{3} + \frac{1}{3}(2-x)^{-2/3} \cdot (-1) =$$

$$= \frac{1}{3} - \frac{1}{3\sqrt[3]{(2-x)^2}} = \frac{\sqrt[3]{(2-x)^2} - 1}{\underset{+}{3\sqrt[3]{(2-x)^2}}}$$

dom f'

$x \neq 2$

(studiarlo)

$f'(x) \geq 0$ $\qquad \sqrt[3]{(2-x)^2} - 1 \geq 0$

$$\sqrt[3]{(2-x)^2} \geq 1$$

$$4 + x^2 - 4x - 1 \geq 0$$

$$x^2 - 4x + 3 \geq 0 \longrightarrow (x-3)(x-1) \geq 0$$

		1		2		3	
$f'(x)$	+	0	−	×	−	0	+
$f(x)$	↗	0	↘	×	↘	0	↗

quindi $\quad x = 1 \quad$ pt. di massimo $\quad f(1) = \frac{1}{3} + \sqrt[3]{2-1} =$

$= \frac{1}{3} + 1 = 4/3$

$x = 3 \quad$ pt. di minimo $\quad f(3) = \frac{3}{3} + \sqrt[3]{2-3} =$

$= 1 - 1 = 0$