$$\sum_{n=2}^{+\infty}\left(\frac{e^{\alpha n}}{n^2}+\frac{\log(n+1)-\log n}{n^{\alpha+2}\log n}\right)$$ = serie a termini positivi

$\alpha\in\mathbb{R}$

$$=\underbrace{\sum_{n=2}^{+\infty}\frac{e^{\alpha n}}{n^2}}_{1}+\underbrace{\sum_{n=2}^{+\infty}\frac{\log(n+1)-\log n}{n^{\alpha+2}\log n}}_{2}$$

CONVERGENZA IN TOTO $\rightarrow$ se converge 1 e 2

STUDIO 1 :

$$\sum_{n=2}^{+\infty}\frac{e^{\alpha n}}{n^2}$$

C.N. $\lim_{n\to+\infty}\frac{e^{\alpha n}}{n^2}=0 \iff \alpha\le 0$

per $\alpha \leq 0$ $\qquad \dfrac{e^{\alpha n}}{n^2} < \dfrac{1}{n^2}$

convergente perché è la serie armonica generalizzata ($\alpha > 1$)

converge per il criterio del confronto

$$\Rightarrow \sum_{n=2}^{+\infty} \frac{e^{\alpha n}}{n^2} \text{ converge } \Leftrightarrow \boxed{\alpha \leq 0}$$

②
$$\sum_{n=2}^{+\infty} \frac{\log(n+1) - \log n}{n^{\alpha+2} \log n} =$$

$$= \sum_{n=2}^{+\infty} \frac{\log\left(\frac{n+1}{n}\right)}{n^{\alpha+2} \log n} = \sum_{n=2}^{+\infty} \frac{\log\left(1+\frac{1}{n}\right)}{n^{\alpha+2} \log n}$$

C.N.
$$\lim_{n\to+\infty} \underbrace{\frac{\log\left(1+\frac{1}{n}\right)}{n^{\alpha+2} \log n}}_{b_n} = 0 \quad ?$$

$$b_n = \frac{\log\left(1+\frac{1}{n}\right)}{n^{\alpha+2} \log n} \simeq$$

$$\simeq \frac{\frac{1}{n}}{n^{\alpha+2} \log n} = \frac{1}{n^{\alpha+3} \log n}$$

attenzione:
$$\lim_{n\to+\infty} \frac{\log\left(1+\frac{1}{n}\right)}{1/n} = 1$$

$$\log\left(1+\frac{1}{n}\right) \simeq \frac{1}{n}$$
per $n\to+\infty$

ATTENZIONE

SERIE CAMPIONE PER CONFRONTI

$$\sum_{n=1}^{+\infty} q^n \quad \text{convergente} \iff |q| < 1 \implies -1 < q < 1$$

serie geometrica di ragione q

$$\sum_{n=1}^{+\infty} \frac{1}{n^\alpha} \quad \text{convergente} \iff \alpha > 1$$

serie armonica generalizzata

$$\sum_{n=2}^{+\infty} \frac{1}{n \log^\beta n} \quad \text{convergente} \iff \beta > 1$$

$$\sum_{n=2}^{+\infty} \frac{1}{n^\alpha \log^\beta n} \quad \text{convergente} \iff \begin{cases} \forall \beta \\ \alpha > 1 \end{cases} \quad \begin{cases} \beta > 1 \\ \alpha = 1 \end{cases}$$

$$\sum_{n=2}^{+\infty} \frac{\log\left(1+\frac{1}{n}\right)}{n^{\alpha+2}\log n}$$

$$\downarrow$$

$$\sum_{n=2}^{+\infty} \frac{1}{n^{\alpha+3}\log n}$$ nel confronto con la $\displaystyle\sum_{n=2}^{+\infty} \frac{1}{n^{p}\log^{\lambda} n}$

esp. log = 1

esp. n > 1

esp. log = 1

esp n > 1 $\Rightarrow$ $\alpha+3>1$ $\Rightarrow$ $\alpha>-2$

$\displaystyle\sum_{n=2}^{+\infty} \frac{1}{n^{\alpha+3}\log n}$ converge $\Rightarrow$ confronto $\displaystyle\sum_{n=2}^{+\infty} \frac{\log\left(1+\frac{1}{n}\right)}{n^{\alpha+2}\log n}$ conv.

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serie

1. conv. $\begin{cases} \alpha \le 0 \\ \alpha > -2 \end{cases}$ $\rightarrow$ $-2<\alpha\le 0$
2. conv.

$$\sum_{n=0}^{+\infty} \frac{n^2}{e^{6n\alpha}} \qquad \alpha \in \mathbb{R}$$

* serie a termini positivi

C.N. $\lim_{n \to +\infty} \frac{n^2}{e^{6n\alpha}} = 0$ se $\alpha > 0$

se $\alpha \leq 0$

CN non rispetta

$$\lim_n \frac{n^2}{e^{6n\alpha}} = +\infty$$

criterio della radice

$$\sqrt[n]{a_n} = \frac{\sqrt[n]{n^2}}{\sqrt[n]{e^{6n\alpha}}} =$$

$$= \frac{n^{2/n}}{e^{\frac{6n\alpha}{n}}} =$$

$$= \frac{n^{\frac{1}{n}} \cdot n^{\frac{1}{n}}}{e^{6\alpha}} = \frac{\overset{\to 1}{\sqrt[n]{n}} \cdot \overset{\to 1}{\sqrt[n]{n}}}{e^{6\alpha}} \to \frac{1}{e^{6\alpha}} < 1$$

$$\alpha > 0 \Updownarrow \frac{1}{e^{6\alpha}} < 1$$

$$\sum_{n=1}^{+\infty} \frac{4^n \left(n^2 + \sin e^n\right)}{3^{2n}}$$

serie a termini positivi

argomento: $-1 \le \sin e^n \le 1$

$$n^2 + \sin e^n \geqslant \underbrace{n^2 - 1}_{\inf\{\sin e^n\}} \geqslant 0 \quad \forall n \geq 1$$

$$\lim_{n\to+\infty} \frac{n^2 + \sin e^n}{n^2} = 1 \qquad n^2 + \sin e^n \simeq n^2$$

$$a_n = \frac{4^n \left(n^2 + \sin e^n\right)}{3^{2n}} \simeq \frac{4^n}{3^{2n}} n^2 = b_n$$

$$\lim_{n\to+\infty} b_n = \lim_{n\to+\infty} \left(\frac{4}{9}\right)^n n^2 =$$

$$= \lim_{n\to+\infty} \left(\frac{4}{9}\right)^{n} n^2 = \lim_{n\to+\infty} \left(\frac{9}{4}\right)^{-n} n^2 =$$

$$= \lim_{n\to+\infty} \frac{n^2}{\left(\frac{9}{4}\right)^{n}} = 0 < 1$$

$\Rightarrow$ conv. $\underline{\sum b_n \Rightarrow \text{conv.}} \sum a_n$

criterio della radice:

$$\lim_{n\to+\infty} \sqrt[n]{\frac{4^n}{3^{2n}} n^2} = \lim_{n\to+\infty} \left(\frac{4}{3^2}\right)^{n/n} \cdot \sqrt[n]{n^2} =$$

$$= \frac{4}{9} \lim_{n\to+\infty} \underbrace{\sqrt[n]{n}}_{1} \cdot \underbrace{\sqrt[n]{n}}_{1} = 4/9 < 1$$

$$\left[\lim_{n\to+\infty} \sqrt[n]{n} = \lim_{n\to+\infty} n^{1/n} = \lim_{n\to+\infty} e^{\log n^{1/n}} = \lim_{n\to+\infty} e^{\overset{\to 0}{\frac{\log n}{n}}} = e^0 = 1 \right]$$

$$\sum_{n=1}^{+\infty} \frac{\overset{+}{(n+1)} \; \overset{+/-}{\sin n}}{\underbrace{n^{7/3} + \log n}_{+}}$$

$-1 < \sin n < 1$

serie a segni alterni

① CONVERGENZA ASSOLUTA

$\sum a_n$ c. a. se $\sum |a_n|$ converge

② CONVERGENZA SEMPLICE

$\sum a_n$ converge ma non $\sum |a_n|$

③ TEOREMA

$\sum |a_n|$ conv. $\Rightarrow$ $\sum a_n$ conv.

ATTENZIONE

CONV. ASS. $\Rightarrow$ CONV. SEMPL.

$\not\Leftarrow$ NO

CRITERIO DI LEIBNIZ

Sia $\{a_n\}$ reale non crescente e infinitesima

$\Rightarrow \sum_{n=0}^{+\infty} (-1)^n a_n$ converge

non crescente $\quad a_n \geq a_{n+1} \quad \forall n$

$$\lim_{n \to +\infty} a_n = 0$$

$$\sum_{n=1}^{+\infty} \frac{(n+1)\sin n}{n^{7/3}+\log n}$$

$$|a_n| = \left|\frac{(n+1)\sin n}{n^{7/3}+\log n}\right| = \frac{n+1}{n^{7/3}+\log n}\underbrace{|\sin n|}_{[0,1]} \leq$$

$$\leq \frac{n+1}{n^{7/3}+\log n} \simeq$$

$$\lim_{n\to+\infty}\left(n^{7/3}+\log n\right) = \lim_{n\to+\infty} n^{7/3}\left(1+\frac{\log n}{n^{7/3}}\right) = \lim_{n} n^{7/3 \geq 0}$$

$$\simeq \frac{n+1}{n^{7/3}} \sim \frac{n}{n^{7/3}} = \frac{1}{n^{4/3}}$$

$\Rightarrow$ serie armonica generalizzata con esponente $\alpha = 4/3 > 1$

convergenza assoluta

$\Downarrow$

convergenza semplice

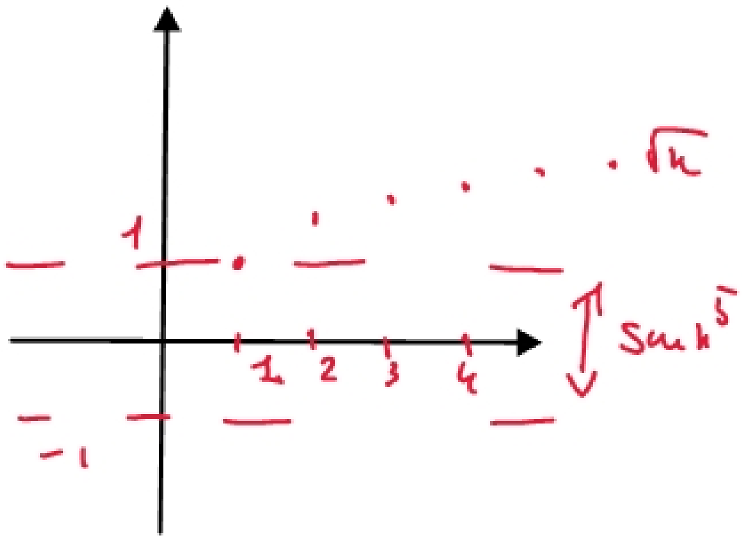
$$\sum_{n=1}^{+\infty} \frac{\sin n^5 - \sqrt{n}}{\sqrt{n+\log n}\,\left(\log\left(n^n + n!\right)\right)}$$

$+ \qquad\qquad +$

NUM: $\sin n^5 - \sqrt{n}$

$\sin n^5 \quad ? \quad \sqrt{n}$

$[-1, 1]$



NUM < 0

la serie è a termini negativi

$$\sum_{n=1}^{+\infty} \frac{\sin n^5 - \sqrt{n}}{\sqrt{n+\log n}\,\left(\log(n^n+n!)\right)} =$$

$$= - \sum_{n=1}^{+\infty} \frac{\sqrt{n} - \sin n^5}{\underbrace{\sqrt{n+\log n}\,\left(\log(n^n+n!)\right)}_{b_n > 0}}$$

$$-a_n = b_n \quad > 0$$

con fronti asintotici :

NUM: $\sqrt{n} - \sin n^5 \simeq \sqrt{n}$ perché $\displaystyle\lim_{n\to+\infty} \frac{\sqrt{n} - \overset{[-1,1]}{\sin n^5}}{\sqrt{n}} = 1$

DEN:

$\sqrt{n+\log n} \simeq \sqrt{n}$ perché $\lim_{n\to+\infty} \frac{\sqrt{n+\log n}}{\sqrt{n}} = 1$

$\sqrt{n} \cdot \sqrt{1+\frac{\log n}{n}}$, $\frac{\log n}{n} \to 0$

$\log(n^n+n!) \simeq n\log n$ perché $\lim_{n\to+\infty} \frac{\log(n^n+n!)}{n\log n} = 1$

$\Rightarrow b_n \simeq \frac{\sqrt{n}}{\sqrt{n}\cdot n\log n} \simeq \frac{1}{n\log n}$

$\log(n^n+n!) = \log\left[n^n\left(1+\frac{n!}{n^n}\right)\right] \simeq \log n^n = n\log n$, $\frac{n!}{n^n} \to 0$, $n\to+\infty$

esp $\log = 1$

esp. $n = 1$

nel confronto con la serie campione è divergente

$$\sum_n b_n \quad \text{diverge a } +\infty$$ (serie a termini positivi)

$\Downarrow$

$$\sum_n a_n \quad \text{diverge a } -\infty$$ (serie a termini negativi)

$$\sum_{n=0}^{+\infty} \frac{n!}{(2+(n+1)!)^{\alpha}} \qquad \alpha \in \mathbb{R} \qquad \text{termini positivi}$$

$$\frac{n!}{(2+(n+1)!)^{\alpha}} = \frac{n!}{[(n+1)!]^{\alpha}\left[1+\underbrace{\frac{2}{[(n+1)!]}}_{\to 0}\right]^{\alpha}} \sim \frac{n!}{((n+1)!)^{\alpha}} =$$

per $n \to +\infty$

proprietà fattoriale

$$\sim \frac{n!}{(n+1)^{\alpha}(n!)^{\alpha}} = \underbrace{\frac{1}{(n+1)^{\alpha}(n!)^{\alpha-1}}}_{b_n}$$

C.N. $\quad \lim_{n\to+\infty} \frac{1}{(n+1)^{\alpha}(n!)^{\alpha-1}} = 0 \iff \begin{cases} \alpha-1 \geq 0 \\ \alpha > 0 \end{cases} \Rightarrow \alpha \geq 1$

se $\alpha = 1$ $\quad b_n = \dfrac{1}{(n+1)\,(n!)^0} = \dfrac{1}{n+1}$

$a_n = \dfrac{n!}{\left(2 + (n+1)!\right)} \simeq \dfrac{1}{n+1} \sim \dfrac{1}{n}$ serie armonica $\Downarrow$ diverge

se $\alpha > 1$ $\quad b_n = \dfrac{1}{(n+1)^{\alpha}\,(n!)^{\alpha-1}} < \dfrac{1}{(n+1)^{\alpha}} \simeq \dfrac{1}{n^{\alpha}}$

serie armonica generalizzata $\Downarrow$ $\alpha > 1$

$\sum a_n$ conv. per $\alpha > 1$