Lez 10

FORMULA DI DE MOIVRE - STIRLING

$$\boxed{n! = n^n e^{-n} \sqrt{2\pi n}\; e^{\vartheta_n/12n}}$$

con $\vartheta_n \in (0,1)$

$\downarrow$

$\frac{\vartheta_n}{12n} \to 0$

$e^{\frac{\vartheta_n}{12n}} \to 1$

- $\lim_n \frac{(2e)^n \sqrt{n^2+1}\; n!}{(2n)^n n^2} =$

$\sqrt{n^2\left(1+\frac{1}{n^2}\right)}$

$$= \lim_n \frac{\cancel{2^n}\, \cancel{e^n}\; n \sqrt{1+\frac{1}{n^2}}^{\to 1} \left(\cancel{n^n}\, \cancel{e^{-n}} \sqrt{2\pi n}\; e^{\vartheta_n/12n}\right)^{\to 1}}{\cancel{2^n}\; \cancel{n^n}\; n^2}$$

$\sqrt{n} = n^{\frac{1}{2}}$

$$= \lim_n \frac{n \cdot \sqrt{2\pi n}}{n^2} \sim \sqrt{2\pi} \lim_n \frac{n^{3/2}}{n^2} = 0$$

$$\cdot \lim_{n} \frac{-7\left((2(n+1))!\right)^{n}}{2^{n}\, n^{n} \left((2n+1)!\right)^{n}} =$$

$$(2n+2)! = (2n+2)\,(2n+1)!$$

$$= -7 \lim_{n} \frac{\left[(2n+2)!\right]^{n}}{2^{n}\, n^{n} \left[(2n+1)!\right]^{n}} =$$

$$= -7 \lim_{n} \frac{\left\{(2n+2)\left[(2n+1)!\right]\right\}^{n}}{2^{n}\, n^{n} \left[(2n+1)!\right]^{n}} =$$

$$= -7 \lim_{n} \frac{(2n+2)^{n} \cancel{\left[(2n+1)!\right]^{n}}}{2^{n}\, n^{n} \cancel{\left[(2n+1)!\right]^{n}}} =$$

$$= -7 \lim_{n} \frac{\left(2(n+1)\right)^n}{2^n \, n^n} =$$

$$= -7 \lim_{n} \frac{\cancel{2^n} \, (n+1)^n}{\cancel{2^n} \, n^n} =$$

$$= -7 \lim_{n} \left(\frac{n+1}{n}\right)^n =$$

$$= -7 \lim_{n} \left(1 + \frac{1}{n}\right)^n = -7e$$

$$\bullet \quad \lim_{n} \frac{(n!)^{\frac{1}{n^2}} - [(n+2)!]^{1/n^2}}{\frac{1}{n^2} \cdot \log(n^3+1)} =$$

$$= \lim_{n} \frac{(n!)^{\frac{1}{n^2}} \left[1 - \frac{[(n+2)!]^{\frac{1}{n^2}}}{(n!)^{1/n^2}}\right]}{\frac{1}{n^2} \log(n^3+1)} =$$

$$\left[\frac{(n+2)!}{n!}\right]^{1/n^2} =$$

$$= \left[\frac{(n+2)(n+1)\cancel{n!}}{\cancel{n!}}\right]^{1/n^2} =$$

$$= [(n+2)(n+1)]^{\frac{1}{n^2}}$$

$$= \lim_{n} \frac{(n!)^{\frac{1}{n^2}} \left[1 - [(n+2)(n+1)]^{\frac{1}{n^2}}\right]}{\frac{1}{n^2} \log(n^3+1)} =$$

① $$\lim_n (n!)^{\frac{1}{n^2}} = \lim_n e^{\log(n!)^{\frac{1}{n^2}}} =$$

$$= e^{\lim_n \frac{1}{n^2}\cdot \log(n!)} \to e^0 = 1$$

② $$1 - \left[(n+2)(n+1)\right]^{1/n^2} = 1 - e^{\log[(n+2)(n+1)]^{\frac{1}{n^2}}}$$

$$\frac{e^{\overbrace{\frac{1}{t}}^{\to 0}} - 1}{\underbrace{1/t}_{\to 0}} \to 1$$

$$\lim_n \frac{1 - e^{\frac{\log[(n+2)(n+1)]}{n^2}}}{\frac{1}{n^2}\log(n^3+1)} = \underbrace{\frac{\log[(n+2)(n+1)]}{n^2}}_{\to 0}$$

$$= -\lim_n \frac{e^{\frac{\log[(n+2)(n+1)]}{n^2}} - 1}{\frac{1}{n^2}\log(n^3+1)}$$

STUDIO DI INSIEMI MEDIANTE LE SUCCESSIONI

- $A = \left\{ (-1)^n \left(\sqrt{n+1} - \sqrt{n} \right) + 2\left(1-(-1)^n\right) \ ; \ n \in N \right\}$

$a_n = (-1)^n \left(\sqrt{n+1} - \sqrt{n} \right) + 2\left(1-(-1)^n\right)$

estraggo 2 sottosuccessioni

$n = 2m$ pari $\qquad (-1)^n = 1$

$b_{n} = \left(\sqrt{n+1} - \sqrt{n} \right) + 2(1-1) = \sqrt{n+1} - \sqrt{n}$

$b_0 = \sqrt{0+1} - \sqrt{0} = 1$

$b_2 = \sqrt{3} - \sqrt{2} < 1$

confronto

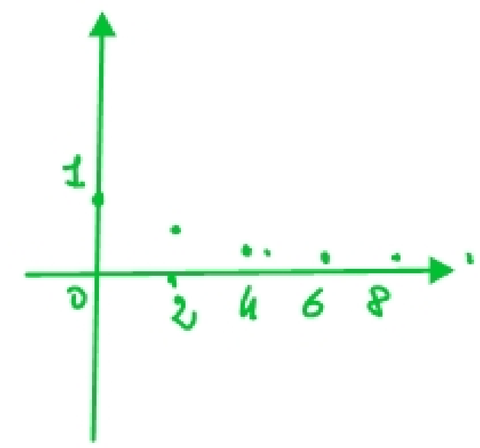
$$0 < \sqrt{3} - \sqrt{2} \overset{?}{<} 1$$

$$3 + 2 - 2\sqrt{6} < 1$$

$$3 + 2 - 1 < 2\sqrt{6}$$

$$4^2 < (2\sqrt{6})^2$$

$$16 < 24$$



$$\lim_n \left(\sqrt{n+1} - \sqrt{n}\right) = \lim_n \frac{\cancel{n}+1 - \cancel{n}}{\sqrt{n+1} + \sqrt{n}} = 0$$

$b_n > 0$ $\qquad$ $0 < b_n < 1$ $\qquad$ n pari

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$$\lim_{n} \frac{\log (n+2)(n+1)}{n^2} = \lim_{n} \frac{\log\{n^2(1+\frac{2}{n})(1+\frac{1}{n})\}}{n^2} =$$

$$= \lim_{n} \frac{\log n^2 + \log\overbrace{\left(1+\frac{2}{n}\right)}^{\to 0} + \log\overbrace{\left(1+\frac{1}{n}\right)}^{\to 0}}{n^2} =$$

$$\simeq \lim_{n} \frac{\log n^2}{n^2} = \lim_{n} \frac{2\log n}{n^2} = 0$$

$$= -\lim_{n} \underbrace{\frac{e^{\frac{\log\{(n+2)(n+1)\}}{n^2}} - 1}{\frac{\log\{(n+2)(n+1)\}}{n^2}}}_{\substack{\wr \\ 1}} \cdot \underbrace{\frac{\frac{\log\{(n+2)(n+1)\}}{n^2}}{\frac{1}{n^2}\log(n^3+1)}}_{(*)}$$

$(*)$

$$= -\lim_{n} \frac{\log\{\overbrace{(n+2)(n+1)}^{n^2\left(1+\frac{2}{n}\right)\left(1+\frac{1}{n}\right)}\}}{\cancel{n^2}\,\log(\underbrace{n^3+1}_{n^3\left(1+\frac{1}{n^3}\right)})} \cdot \cancel{n^2} \;\sim -\lim_{n} \frac{2\log n}{3\log n} =$$

$$= -2/3$$

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$n = 2m+1$ dispari

$$c_n = -\left(\sqrt{n+1} - \sqrt{n}\right) + 2\left(1 - (-1)\right) =$$

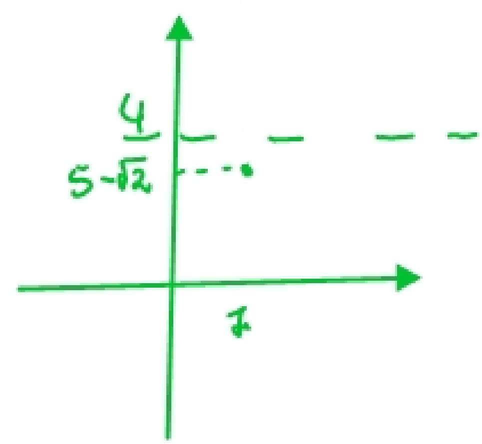
$$= 4 - \left(\sqrt{n+1} - \sqrt{n}\right)$$

$$\lim_n c_n = \lim_n \left[4 - \underbrace{\left(\sqrt{n+1} - \sqrt{n}\right)}_{\to 0}\right] = 4$$

$$c_1 = 4 - \left(\sqrt{2} - \sqrt{1}\right) = 5 - \sqrt{2}$$

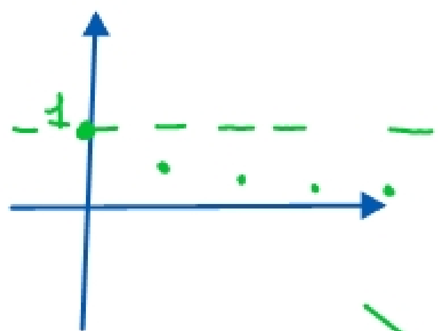
$$c_3 = 4 - \left(\sqrt{4} - \sqrt{3}\right) =$$

$$= 4 - 2 + \sqrt{3} = 2 + \sqrt{3}$$

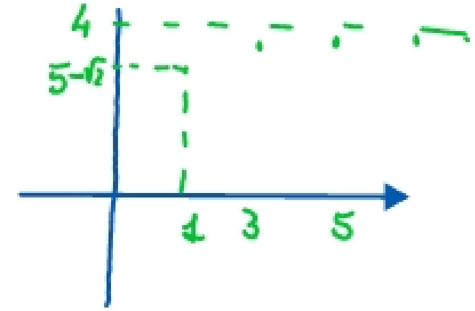


$c_1 \; ? \; c_3$

n pair



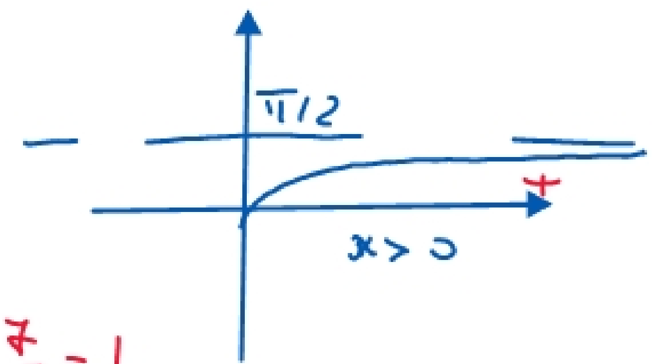
n impair



$\inf A = 0$

$\sup A = 4$

• $A = \left\{ 28 \operatorname{arctg}\left(\underbrace{\frac{7n}{7n+1}}_{b_n}\right), \ \underline{n \in \mathbb{N}} \right\}$



$b_n > 0$

$\lim_n b_n = \lim_n \frac{7n}{7n+1} = \frac{7}{7} = 1$

$\lim_n 28 \operatorname{arctg} \frac{7n}{7n+1} = 28 \operatorname{arctg} 1 = 28 \cdot \frac{\pi}{4} = 7\pi$

$a_0 = 28 \operatorname{arctg} 0 = 0$

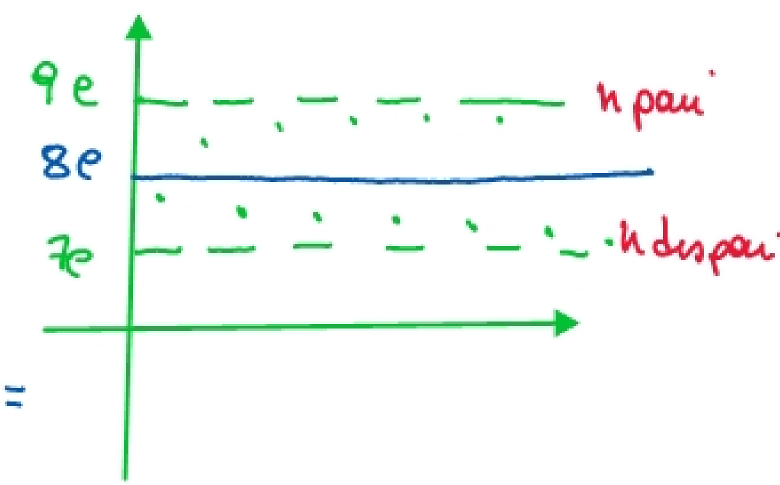
a_n crescente, limitata, positiva

$\sup A = 7\pi$

$\min A = 0$

• $A = \left\{ \underbrace{8}_{\text{cost.}} e + (-1)^n \, e^{\frac{n^2}{n^2+3}} \; ; \; n \in N \right\}$

$$\lim_n e^{\frac{n^2}{n^2+3}} = e$$



n pari

$$b_n = 8e + e^{\frac{n^2}{n^2+3}}$$

$$\lim_n b_n = \lim_n \left\{ 8e + e^{\frac{n^2}{n^2+3}} \right\} = 9e$$

n dispari

$$c_n = 8e - e^{\frac{n^2}{n^2+3}}$$

$$\lim_n c_n = \lim_n \left(8e - e^{\frac{n^2}{n^2+3}} \right) = 8e - e = 7e$$

$$a_0 = 8e + (-1)^0 \, e^{\frac{0}{0+3}} = 8e + 1 \qquad < 9e$$

$$a_1 = 8e + (-1)^1 \, e^{\frac{1}{4}} = 8e - e^{1/4} \qquad > 7e$$

$$\sup A = 9e$$

$$\inf A = 7e$$

Conclusione limite log g:

$$\dots = \lim_{n} \frac{3}{49}\, n^2 \underbrace{\log\left(\frac{e^{\frac{14}{n}}+1}{2e^{7/n}}\right)}_{g}$$

$$\downarrow$$

$$\log\left[\frac{1}{2}\left(e^{\frac{7}{n}} + e^{-7/n}\right)\right] =$$

$$\log\left[\frac{1}{2}\, e^{7/n}\left(1 + e^{-\frac{14}{n}}\right)\right] =$$

$$\left\{\log e^{7/n} - \cancel{\log 2} + \log\left(\cancel{1 + e^{-\frac{14}{n}}}\right)\right\}$$