Lezione 5

- Determinare per quali valori del parametro reale K i seguenti numeri complessi sono numeri complessi reali

$$z = \frac{5}{4} + \frac{3}{2} i - \frac{2}{5} K i \in \mathbb{R} \Rightarrow \operatorname{Im} z = 0$$

$$= \frac{5}{4} + \left(\frac{3}{2} - \frac{2}{5} K \right) i$$

$$\operatorname{Re} z = \frac{5}{4}$$

$$\operatorname{Im} z = \frac{3}{2} - \frac{2}{5} K$$

$$\operatorname{Im} z = 0 \qquad \frac{3}{2} - \frac{2}{5} K = 0$$

$$K = \frac{3}{2} \cdot \frac{5}{2} = \frac{15}{4}$$

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- determinare il valore di k reale tale che il numero complesso sia immaginario:

$$z = k(k+2) - \left(k^2 + \frac{2}{3}\right) + ki - \frac{2}{3}i =$$

$$= \left[\cancel{k^2} + 2k - \cancel{k^2} - 2/3\right] + i\left(k - 2/3\right) =$$

$$\operatorname{Re} z = 2k - 2/3 \qquad \begin{cases} \operatorname{Re} z = 0 \\ \operatorname{Im} z \neq 0 \end{cases} \qquad \begin{cases} 2k - 2/3 = 0 \\ k = 1/3 \\ k \neq 2/3 \end{cases}$$

$$\operatorname{Im} z = k - 2/3$$

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- Calcolare a e b $\in \mathbb{R}$ in modo che

$$z_1 = (a - b + 7) + \underbrace{(2a - b)}\, i$$

$$z_2 = (4a - 3b) + (5b)\, i$$

Siano uguali $z_1 = z_2$

$$\Downarrow$$

$$\begin{cases} \mathrm{Re}\, z_1 = \mathrm{Re}\, z_2 \\ \mathrm{Im}\, z_1 = \mathrm{Im}\, z_2 \end{cases}$$

$$\begin{cases} a - b + 7 = 4a - 3b \\ 2a - b = 5b \end{cases} \quad \begin{cases} 3a - 2b - 7 = 0 \\ a = 3b \end{cases} \quad \begin{cases} a = 3 \\ b = 1 \end{cases}$$

$$9b - 2b = 7$$
$$7b = 7$$

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• Sono dati 2 numeri complessi: $z_1 = (2a-1) + (3b)i$

$z_2 = (a+b) - (a)i$

$z_1 = z_2$

$$\begin{cases} 2a-1 = a+b \\ 3b = -a \end{cases} \quad \begin{cases} a = b+1 \\ a = -3b \end{cases} \quad \begin{cases} a = \frac{3}{4} \\ b = -\frac{1}{4} \end{cases}$$

confronto $b+1 = -3b$ $\quad b = -\frac{1}{4}$

$z_1 = \overline{z_2}$

$$\begin{cases} \mathrm{Re}\, z_1 = \mathrm{Re}\, z_2 \\ \mathrm{Im}\, z_1 = -\mathrm{Im}\, z_2 \end{cases} \quad \begin{cases} 2a-1 = a+b \\ 3b = a \end{cases} \quad \begin{cases} a = b+1 \\ a = 3b \end{cases} \quad \begin{cases} a = 3/2 \\ b = 1/2 \end{cases}$$

confronto $3b = b+1$ $\quad b = \frac{1}{2}$

$z_1 = -z_2$ (opposto)

$$\begin{cases} \mathrm{Re}\, z_1 = -\mathrm{Re}\, z_2 \\ \mathrm{Im}\, z_1 = -\mathrm{Im}\, z_2 \end{cases} \quad \begin{cases} 2a-1 = -a-b \\ 3b = a \end{cases} \quad \begin{cases} a = \frac{1-b}{3} \\ a = 3b \end{cases} \quad \begin{cases} a = \frac{3}{10} \\ b = 1/10 \end{cases}$$

confronto $\frac{1-b}{3} = 3b$ $\quad 1-b = 9b$ $\quad b = \frac{1}{10}$

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• espressione di numeri complessi in notazione algebrica

$$\frac{9+7i}{2+i} + \frac{-2\overset{=1}{i^4} + 3\overset{=-i}{i^3} - 2\overset{=-1}{i^2} + 3i}{3-2i} =$$

$$= \frac{9+7i}{2+i} \cdot \frac{2-i}{2-i} + \frac{-2 \cancel{-3i+2} + 3i}{3-2i} =$$

$$= \frac{18 + 14i - 9i - 7\overset{=-1}{i^2}}{4 - \underset{=-1}{i^2}} =$$

$$= \frac{18+7+5i}{4+1} = \frac{25+5i}{5} = 5+i$$

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- $z_1 = x + iy$

$\bar{z}_1 = x - iy$

$z_1 \cdot \bar{z}_1 = (x+iy)(x-iy) = x^2 - y^2 i^2 = \underline{x^2 + y^2}$

$|z_1| = \sqrt{\underline{x^2 + y^2}}$

$z_1 \cdot \bar{z}_1 = |z_1|^2$

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- Calcolare il coniugato di

$$z = \frac{2-3i}{1+i}$$

$$z = \frac{2-3i}{1+i} \cdot \frac{1-i}{1-i} = \frac{2-3i-2i+3}{1+1} =$$

$$= \frac{-1-5i}{2} = -\frac{1}{2} - \frac{5}{2}i$$

$$\bar{z} = -\frac{1}{2} + \frac{5}{2}$$

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· da fare:

$$z = \frac{2i-2}{(1+\sqrt{3}i)(1+i)} = -\frac{1}{2} + \frac{1}{2}i$$

$$|z| = \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{1}{2}} = \sqrt{2}/2$$

$$\overline{z} = -\frac{1}{2} - \frac{1}{2}i$$

$$-z = \frac{1}{2} - \frac{1}{2}i$$

$$\frac{2i-2}{(1+\sqrt{3}i)(1+i)} = \frac{2(i-1)}{1+\sqrt{3}i+i-\sqrt{3}} = \frac{2(i-1)}{(1-\sqrt{3})+i(1+\sqrt{3})} \cdot \frac{(1-\sqrt{3})-i(1+\sqrt{3})}{(1-\sqrt{3})-i(1+\sqrt{3})} =$$

$$= \frac{2(i-1)\left[(1-\sqrt{3})-i(1+\sqrt{3})\right]}{(1-\sqrt{3})^2+(1+\sqrt{3})^2} = \frac{2\left[i(1-\sqrt{3})-(1-\sqrt{3})-(1+\sqrt{3})+(1+\sqrt{3})i\right]}{1+3-2\sqrt{3}+1+3+2\sqrt{3}} =$$

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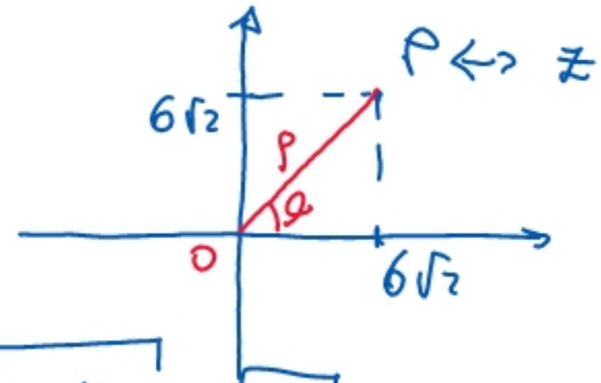
• $z = 6\sqrt{2} + 6\sqrt{2}\, i$



$$PO = \rho = \sqrt{(\mathrm{Re}\, z)^2 + (\mathrm{Im}\, z)^2} =$$

$$= \sqrt{(6\sqrt{2})^2 + (6\sqrt{2})^2} = \sqrt{72 + 72} = \sqrt{144} = 12$$

$$\mathrm{tg}\, \vartheta = \frac{y}{x} = 1 \qquad \vartheta = \mathrm{arctg}\, 1 = 45^\circ \to \pi/4$$

$(x > 0 \quad , y > 0)$

$$z_1 = -6\sqrt{2} - 6\sqrt{2}\, i$$

$$\rho = \sqrt{(-6\sqrt{2})^2 + (-6\sqrt{2})^2} = \sqrt{144} = 12$$

$$\mathrm{tg}\, \vartheta = \frac{-6\sqrt{2}}{-6\sqrt{2}} = 1 \qquad \vartheta = \mathrm{arctg}\, 1 \to \frac{\pi}{4} + \pi = \frac{5}{4}\pi$$

(3° quadrante)

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$$z = 6\sqrt{2} + 6\sqrt{2}\, i =$$

$$= 12\left(\cos\frac{\pi}{4} + \operatorname{sen}\frac{\pi}{4}\, i\right) \quad \text{scrittura trigonometrica}$$

$$= 12\, e^{i\,\pi/4} \quad \text{scrittura esponenziale}$$

<u>osservazione</u> $\quad z = x + iy \Rightarrow (x, y) \leftrightarrow (\rho, \vartheta)$

Trasformazioni:

$$\begin{cases} x = \rho\cos\vartheta \\ y = \rho\operatorname{sen}\vartheta \end{cases} \leftrightarrow \begin{cases} \rho = \sqrt{x^2+y^2} \\ \cos\vartheta = \dfrac{x}{\sqrt{x^2+y^2}} \\ \operatorname{sen}\vartheta = \dfrac{y}{\sqrt{x^2+y^2}} \end{cases}$$

$$\operatorname{arctg}\frac{y}{x} = \vartheta$$

Operazioni con i numeri complessi

$$z_1 = x_1 + i y_1 \rightarrow (\rho_1, \vartheta_1) \rightarrow$$

$$z_2 = x_2 + i y_2 \rightarrow (\rho_2, \vartheta_2) \rightarrow$$

$$z_1 = x_1 + i y_1 = \rho_1 (\cos\vartheta_1 + i \operatorname{sen}\vartheta_1) = \rho_1 e^{i\vartheta_1}$$

$$z_2 = x_2 + i y_2 = \rho_2 (\cos\vartheta_2 + i \operatorname{sen}\vartheta_2) = \rho_2 e^{i\vartheta_2}$$

PRODOTTO $\quad z_1 \cdot z_2 = \rho_1 \cdot \rho_2 \left[\cos(\vartheta_1 + \vartheta_2) + i \operatorname{sen}(\vartheta_1 + \vartheta_2)\right] = \rho_1 \rho_2 e^{i(\vartheta_1 + \vartheta_2)}$

QUOZIENTE $\quad z_1 : z_2 = \rho_1 / \rho_2 \left[\cos(\vartheta_1 - \vartheta_2) + i \operatorname{sen}(\vartheta_1 - \vartheta_2)\right] = \frac{\rho_1}{\rho_2} e^{i(\vartheta_1 - \vartheta_2)}$

POTENZA $\quad (z_1)^n = \rho_1^n \left[\cos(n\vartheta_1) + i \operatorname{sen}(n\vartheta_1)\right] = \rho_1^n e^{in\vartheta_1}$

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Potenza con esponente negativo

$$(z_1)^{-n} = \rho_1^{-n} \left[\cos(-n\vartheta_1) + i \operatorname{sen}(-n\vartheta_1) \right] =$$

$$= \frac{1}{\rho_1^{n}} \left[\cos(n\vartheta_1) - i \operatorname{sen}(n\vartheta_1) \right]$$

$$= \rho_1^{-n} e^{-in\vartheta_1}$$

Radice n-esime di un numero complesso

$$z = \rho e^{i\vartheta} = \rho(\cos\vartheta + i \operatorname{sen}\vartheta)$$

$$\sqrt[n]{z} = \sqrt[n]{\rho} \left(\cos \frac{\vartheta + 2k\pi}{n} + i \operatorname{sen} \frac{\vartheta + 2k\pi}{n} \right)$$

$$= \sqrt[n]{\rho}\, e^{i \frac{\vartheta + 2k\pi}{n}} \qquad k \in \mathbb{N} \qquad k = 0, 1, 2, \dots, n-1$$

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- $z = 8\, e^{i\,\pi/3} = 8\left(\cos\frac{\pi}{3} + i \operatorname{sen}\frac{\pi}{3}\right) = 8\left(\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right) =$

$= 4 + 4\sqrt{3}\, i =$

$= 4\,(1 + \sqrt{3}\, i)$

$\sqrt[4]{z}$ $\qquad n = 4 \qquad \sqrt[4]{8} = \sqrt[4]{2^3} = 2^{3/4}$

$$\frac{\vartheta + 2k\pi}{n} \rightarrow \frac{\frac{\pi}{3} + 2k\pi}{4}$$

$k = 0 \qquad \vartheta_0 = \dfrac{\pi/3}{4} = \dfrac{\pi}{12} \qquad z_0 = \sqrt[4]{8}\; e^{i\,\pi/12}$

$k = 1 \qquad \vartheta_1 = \dfrac{\frac{\pi}{3} + 2\pi}{4} = \dfrac{7}{12}\pi \qquad z_1 = \sqrt[4]{8}\; e^{i\,\frac{7}{12}\pi}$

$k = 2 \qquad \vartheta_2 = \dfrac{\frac{\pi}{3} + 4\pi}{4} = \dfrac{13}{12}\pi \qquad z_2 = \sqrt[4]{8}\; e^{i\,\frac{13}{12}\pi}$

$k = 3 \qquad \vartheta_3 = \dfrac{\frac{\pi}{3} + 6\pi}{4} = \dfrac{19}{12}\pi \qquad z_3 = \sqrt[4]{8}\; e^{i\,\frac{19}{12}\pi}$

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