Lezione 9

Insiemi e successioni

$$A = \left\{ (-1)^n \left(\sqrt{n+1} - \sqrt{n}\right) + 2\left(1 - (-1)^n\right),\ n \in \mathbb{N} \right\}$$

n pari $\quad (-1)^n = 1$

$$b_n = \sqrt{n+1} - \sqrt{n} + 2\underbrace{\cancel{(1-1)}}_{0} = \sqrt{n+1} - \sqrt{n}$$

$$b_0 = 1$$

$$b_2 = \sqrt{3} - \sqrt{2} < 1$$

$$\lim_{n \to +\infty} \underbrace{\left(\sqrt{n+1} - \sqrt{n}\right)}_{\text{f.i.}} = \lim_{n \to +\infty} \frac{\left(\sqrt{n+1} - \sqrt{n}\right)\left(\sqrt{n+1} + \sqrt{n}\right)}{\sqrt{n+1} + \sqrt{n}}$$

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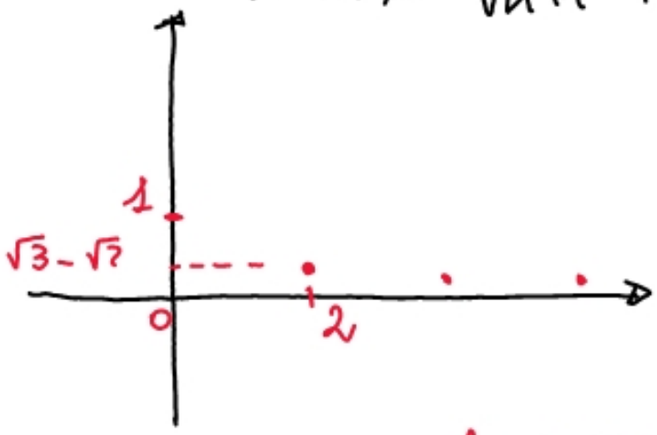
$$= \lim_{n \to +\infty} \frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} =$$

$$= \lim_{n \to +\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}} = 0$$



Sottosuccessione limitata

$\inf \{b_n\} = 0$

$\max \{b_n\} = 1$

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n dispari $\quad (-1)^n = -1$

$$c_n = -\left(\sqrt{n+1} - \sqrt{n}\right) + 2\left(1 - (-1)\right) =$$

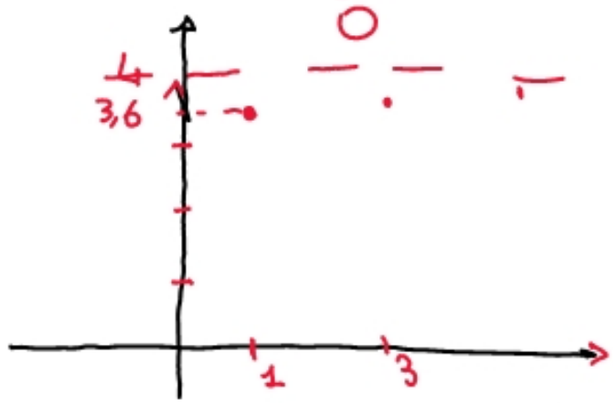
$$= -\left(\sqrt{n+1} - \sqrt{n}\right) + 4 =$$

$$= 4 - \left(\sqrt{n+1} - \sqrt{n}\right)$$

$$\lim_{n \to +\infty} \left\{4 - \left(\sqrt{n+1} - \sqrt{n}\right)\right\} = 4 - \lim_{n \to +\infty} \underbrace{\left(\sqrt{n+1} - \sqrt{n}\right)}_{\to 0} = 4$$

$$c_1 = 4 - \left(\sqrt{2} - 1\right) \sim 3{,}6$$

$$c_3 = 4 - \left(2 - \sqrt{3}\right) \sim 3{,}7$$



$\min\{c_n\} = c_1$

$\sup\{c_n\} = 4$

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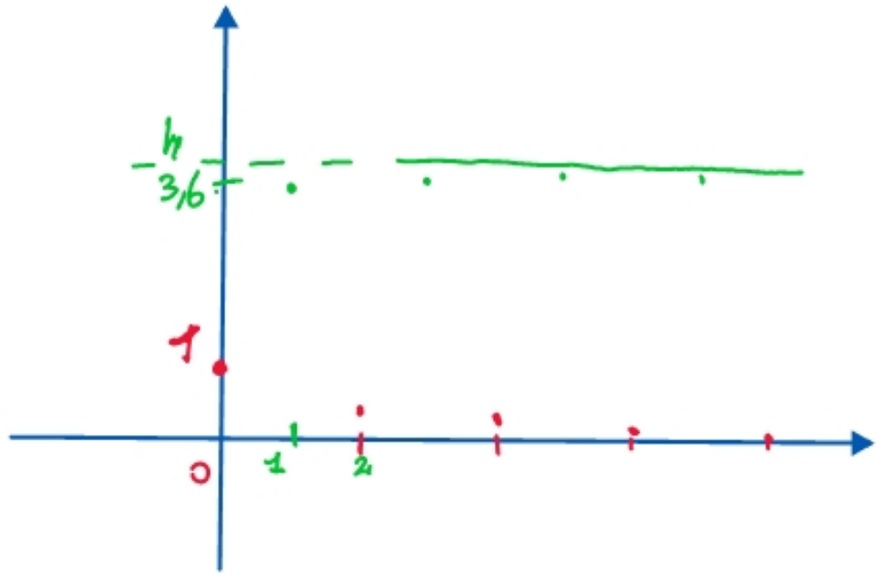

$b_n \searrow \qquad b_0 = 1 \qquad \lim b_n = 0$

$c_n \nearrow \qquad c_1 \simeq 3,6 \qquad \lim c_n = 4$

$\inf A = 0$

$\sup A = 4$

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$$\cdot\ A = \left\{ \frac{1}{2} \operatorname{arctg}\left(\frac{n + (-1)^n n - 1}{2n+1} \right) \quad n \in \mathbb{N} \right\} \qquad n \neq 0$$

n pari

$$b_n = \frac{1}{2} \operatorname{arctg}\left(\frac{n + n - 1}{2n+1} \right) =$$

$$= \frac{1}{2} \operatorname{arctg}\left(\frac{2n-1}{2n+1} \right)$$

$$\lim_{n \to +\infty} \frac{1}{2} \operatorname{arctg}\left(\frac{2n-1}{2n+1} \right) = \frac{1}{2} \operatorname{arctg} 1 = \frac{\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{8}$$

$$2/2 = 1$$

$$\lim_{n \to +\infty} \frac{2n-1}{2n+1} = 1$$

N.B. $y = \operatorname{arctg} x$

n dispari

$$c_n = \frac{1}{2} \operatorname{arctg}\left(\frac{n-n-1}{2n+1}\right) =$$

$$= \frac{1}{2} \operatorname{arctg}\left(-\frac{1}{2n+1}\right)$$

$$\lim_{n\to+\infty} \frac{1}{2} \operatorname{arctg}\Big(\underbrace{-\frac{1}{2n+1}}_{\to 0}\Big) = \frac{1}{2} \operatorname{arctg} 0 = 0$$

Argomento $\left(-\frac{1}{2n+1}\right) < 0$

$$\lim_{n\to+\infty} -\left(\frac{1}{2n+1}\right) = 0$$

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n pair

$$b_n = \frac{1}{2}\,\mathrm{arctg}\left(\frac{2n-1}{2n+1}\right) \rightsquigarrow \pi/8$$

(Arg > 0)

$$b_2 = \frac{1}{2}\,\mathrm{arctg}\left(\frac{4-1}{4+1}\right) = \frac{1}{2}\,\mathrm{arctg}\,\frac{3}{5}$$

$\rightsquigarrow 1$

n dispair

$$c_n = \frac{1}{2}\,\mathrm{arctg}\left(-\frac{1}{2n+1}\right) \rightsquigarrow 0$$

(Arg < 0)

$\searrow 0$

$$\left[\mathrm{arctg}\,(\arg < 0) < 0\right]$$

$$\left[\mathrm{arctg}\,(\arg > 0) > 0\right]$$

$$c_1 = \frac{1}{2}\,\mathrm{arctg}\left(-\frac{1}{2+1}\right) = \frac{1}{2}\,\mathrm{arctg}\left(-\frac{1}{3}\right)$$

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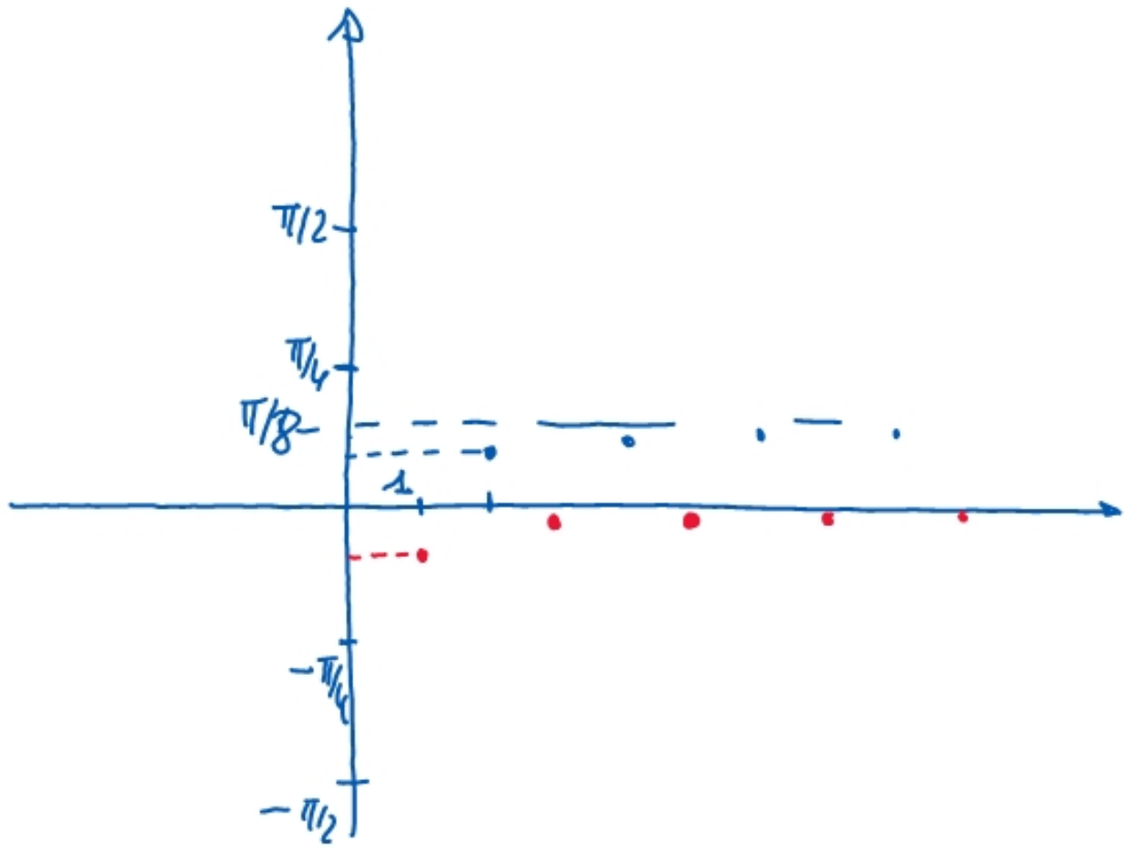

$b_n \to \pi/8$

$c_n < 0$

$0 < \mathrm{Arg} = \frac{2n+1}{2n+1} < 1 \qquad N < D$

$\sup\{A\} = \pi/8$

$\min\{A\} = \frac{1}{2}\operatorname{arctg}\left(\frac{1}{3}\right)$

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$$A = \left\{ 28 \left(\operatorname{arctg} \frac{7n}{7n+1} \right) \quad n \in \mathbb{N} \right\} \qquad n \neq 0$$

$$b_n = \frac{7n}{7n+1} > 0 \qquad 0 < b_n < 1$$

$$\lim_{n \to +\infty} \frac{7n}{7n+1} = 1$$

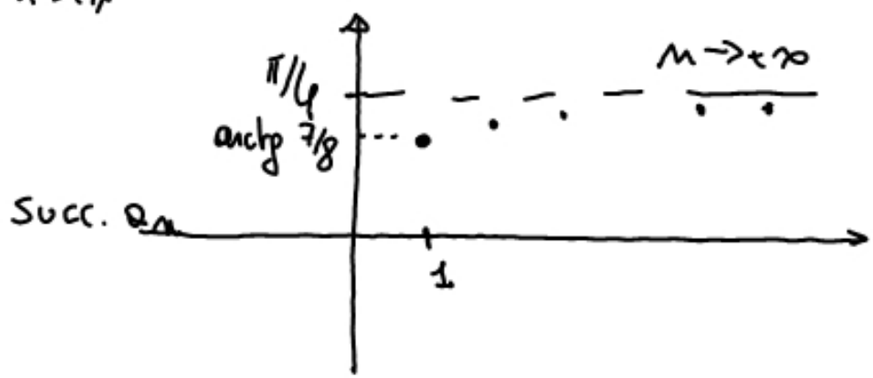
per monotonia

$$b_1 = \frac{7}{8}$$

$$a_n = \operatorname{arctg} \frac{7n}{7n+1}$$

$$\left\{ b_n \right\}_{n \to +\infty} = 1$$

$$a_1 = \operatorname{arctg} 7/8 < \pi/4$$



$$a_n \xrightarrow{n \to +\infty} \operatorname{arctg} 1 = \pi/4$$

$$\underline{\underline{\min A}} = \inf A = 28 \operatorname{arctg} \frac{7}{8}$$

$$\sup A = 28 \cdot \frac{\pi}{4} = 7\pi$$

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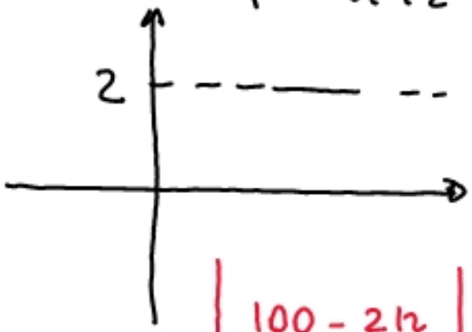
$$A = \left\{ \left| \frac{100-2n}{n+2} \right| \quad , \; n \in \mathbb{N} \right\}$$

$n \neq 0$

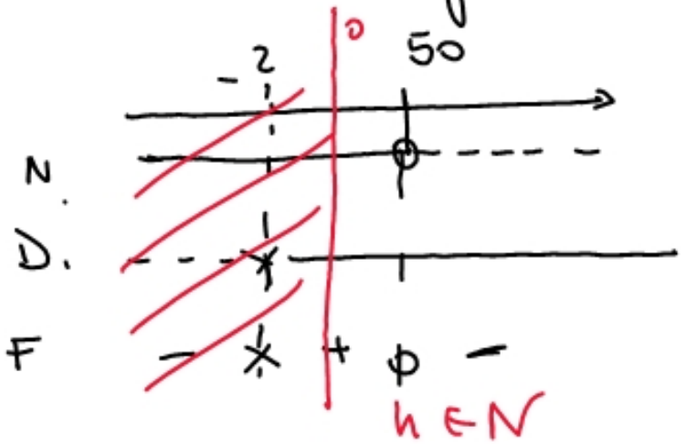
$$a_n = \left| \frac{100-2n}{n+2} \right|$$

$$\lim_{n \to +\infty} \left| \frac{100-2n}{n+2} \right| = |-2| = 2$$



$$\frac{100-2n}{n+2}$$

studio del segno



$$\left| \frac{100-2n}{n+2} \right| =$$

$$= \begin{cases} \dfrac{100-2n}{n+2} & 0 < n < 50 \\[2ex] \dfrac{2n-100}{n+2} & n \geq 50 \end{cases}$$

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$0 < n < 50 \qquad a_n = \dfrac{100-2n}{n+2}$

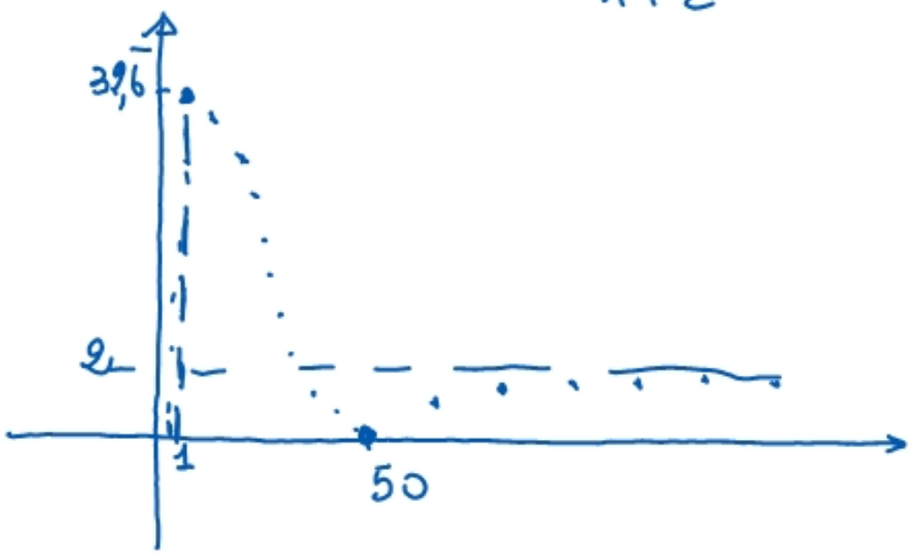
$a_1 = \dfrac{100-2}{3} = \dfrac{98}{3} = 32,\overline{6}$

$a_2 = \dfrac{100-4}{4} = \dfrac{96}{4} = 24$

$\vdots$

$a_{50} = \dfrac{100-100}{52} = 0$

$n > 50 \qquad 0 < a_n = \dfrac{2n-100}{n+2} \rightsquigarrow 2$



$MIN\{A\} = 0$

$MAX\{A\} = a_1$

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$$\lim_{n\to+\infty} n^{2n}\left(1+\frac{7}{n}\right)^{n}\cdot \underbrace{\sin(n^{-n})}_{0}\cdot\frac{1}{\sqrt{n^3+n^{2n}}-\sqrt{n^3}}=$$

$$\left(1+\frac{7}{n}\right)^{n}=\left[\left(1+\frac{7}{n}\right)^{\frac{n}{7}}\right]^{7}\rightsquigarrow e^{7}$$

$$n^{-n}=\left(\frac{1}{n^{n}}\right)=e^{\log n^{-n}}=e^{-\overbrace{n\log n}^{\to+\infty}}=e^{-\infty}=0$$

$$=e^{7}\lim_{n\to+\infty} n^{2n}\cdot\frac{\sin\left(\frac{1}{n^{n}}\right)}{\frac{1}{n^{n}}}\cdot\frac{1}{n^{n}}\,\frac{1}{\sqrt{n^{2n}\left(\underbrace{\frac{n^3}{n^{2n}}}_{\to 0}+1-\underbrace{\frac{\sqrt{n^3}}{n^{2n}}}_{\to 0}\right)}}=$$

$$\frac{\sin\left(\frac{1}{n^{n}}\right)}{\frac{1}{n^{n}}}\to 1$$

$$\frac{1}{\sqrt{n^{2n}}}=n^{n}$$

$$=e^{7}$$

$$\frac{1}{2}\operatorname{arctg}\frac{3}{5} \rightsquigarrow \frac{1}{2}\operatorname{arctg} 1$$

$$\frac{2n-1}{2n+1} \nearrow \qquad n=2 \qquad \frac{3}{5}$$

$$\operatorname{arctg}\left(\frac{2n-1}{2n+1}\right) \nearrow \qquad n=4 \qquad \frac{7}{9}$$

$$\frac{1}{2}\operatorname{arctg}\left(\frac{2n-1}{2n+1}\right) \nearrow \qquad n\to+\infty \qquad 1$$

$$\frac{3}{5} < \frac{7}{9}$$

$$\frac{3\times 9}{5\times 9} < \frac{7\times 5}{5\times 9}$$

$$\frac{27}{45} < \frac{35}{45}$$

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