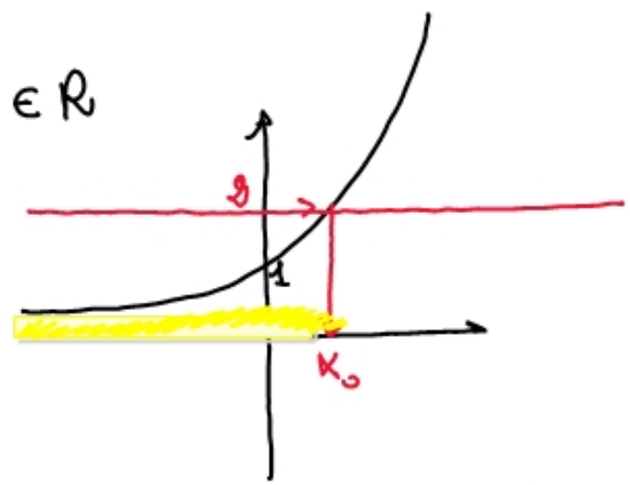
# Lezione 18

- $y = f(x) = \arcsin \left| e^{2x} - 1 \right|$

dominio $\quad -1 \le \left| e^{2x} - 1 \right| \le 1 \quad \Rightarrow \quad \left]-\infty, \frac{1}{2}\log 2\right]$

$$\begin{cases} \left| e^{2x} - 1 \right| \le 1 \\ \left| e^{2x} - 1 \right| \ge -1 \end{cases} \qquad \forall x \in \mathbb{R}$$




$$-1 \le e^{2x} - 1 \le 1$$

$$\underbrace{0 \le e^{2x} \le 2}_{\forall x \in \mathbb{R}}$$

$2x \le \log 2 \Rightarrow x \le \frac{1}{2}\log 2$

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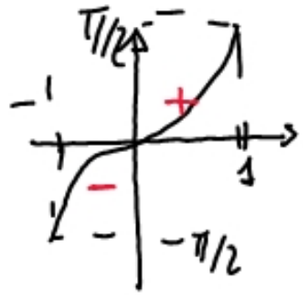
<u>Segno</u> $\quad \arcsin \underbrace{\left| e^{2x} - 1 \right|}_{+} \geq 0$




se $0 < \text{Arg} < 1 \Rightarrow 0 \leq \arcsin(\ ) \leq \pi/2$

$\Rightarrow f(x) \geq 0$ nel dominio

$$f(0) = \arcsin \left| e^{2 \cdot 0} - 1 \right| = \arcsin \left| e^{0} - 1 \right| = \arcsin |1-1| =$$

$$= \arcsin 0 = 0$$

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- continuità: la funzione è composta da funzioni continue, quindi è continua

- limiti

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \arcsin\left|\underbrace{e^{2x}}_{\to 0} - 1\right| =$$

$$= \arcsin|-1| = \arcsin 1 = \pi/2$$

$y = \frac{\pi}{2}$ asintoto orizzontale

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derivata prima

$$f(x) = \arcsin\left|e^{2x}-1\right| = \begin{cases} \arcsin\left(1-e^{2x}\right) & x<0 \\ \arcsin\left(e^{2x}-1\right) & x \geq 0 \quad \left[x \leq \frac{1}{2}\log 2\right] \end{cases}$$

$$e^{2x}-1 \geq 0 \qquad e^{2x} \geq 1 \qquad 2x \geq \log 1 \qquad x \geq 0$$

$$f'(x) = \begin{cases} \dfrac{1}{\sqrt{1-\left(1-e^{2x}\right)^2}} \cdot \left(-2e^{2x}\right) & x<0 \\ \dfrac{1}{\sqrt{1-\left(1-e^{2x}\right)^2}} \cdot \left(2e^{2x}\right) & x \geq 0 \quad \left(x \leq \frac{1}{2}\log 2\right) \end{cases}$$

$$D\left(\arcsin x\right) = \frac{1}{\sqrt{1-x^2}}$$

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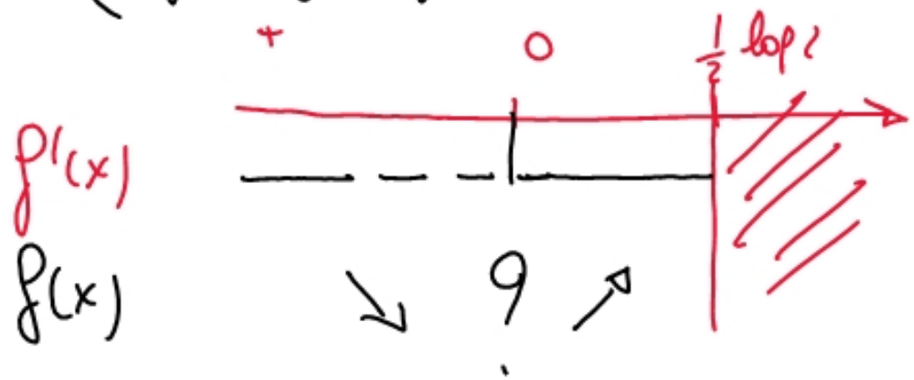
STUDIO DEL SEGNO

$$f'(x) = \begin{cases} \dfrac{-2e^{2x}}{\sqrt{1-(1-e^{2x})^2}} & x < 0 \\[2ex] \dfrac{2e^{2x}}{\sqrt{1-(1-e^{2x})^2}} & 0 \le x \le \frac{1}{2}\log 2 \end{cases}$$

$f'(x) < 0$

$f'(x) > 0$




$f'_-(0) \to -2$

$f'_+(0) = 2$

punto angoloso in $x=0$ è di non derivabilità

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Derivata seconda:

$$f''(x) = \pm 2\left\{ \underbrace{e^{2x}\cdot 2}_{D\,e^{2x}} \sqrt{1-(1-e^{2x})^2} + e^{2x} \underbrace{\qquad (*) \qquad}_{D\sqrt{1-(1-e^{2x})^2}} \right.$$

$$D\left(\sqrt{1-(1-e^{2x})^2}\right) = \frac{1}{2\sqrt{1-(1-e^{2x})^2}} \cdot \left(-2(1-e^{2x})\cdot(-e^{2x}\cdot 2)\right) =$$

$$= \frac{1}{2\sqrt{1-(1-e^{2x})^2}} \cdot \left(4\, e^{2x}\,(1-e^{2x})\right) \quad (*)$$

N.B. Si procede nei calcoli per lo studio del segno

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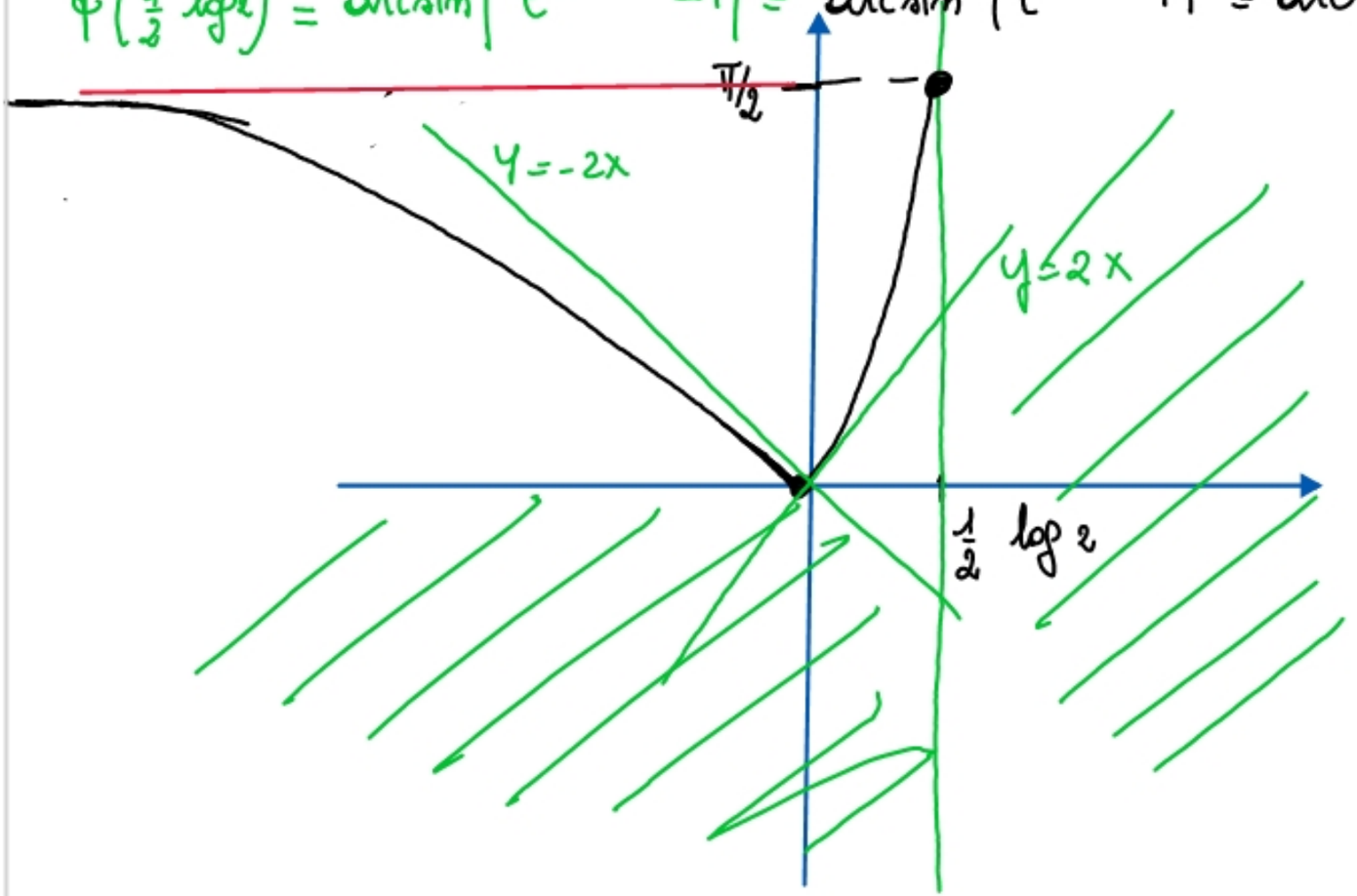
$$f\left(\tfrac{1}{2}\log 2\right) = \arcsin\left|e^{2\cdot\frac{1}{2}\log 2}-1\right| = \arcsin\left|e^{\log 2}-1\right| = \arcsin|2-1| = \arcsin 1 =$$

$$= \pi/2 \sim 1{,}5$$




$$\frac{1}{2}\log 2 = \log 2^{\frac{1}{2}} = \log\sqrt{2} \approx \log(1{,}4) < 1$$

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$$f(x) = \left| \log \frac{x-2}{x+3} - 1 \right|$$

$f(x) \geq 0$ nel suo dominio

dominio $\quad \frac{x-2}{x+3} > 0 \quad \Rightarrow \quad x < -3 \ \cup \ x > 2$

$$]-\infty, -3[ \ \cup \ ]2, +\infty[$$

$$\lim_{x \to -\infty} \left| \log \frac{x-2}{x+3} - 1 \right| = \left| \overset{=0}{\log 1} - 1 \right| = 1$$

$$\lim_{x \to +\infty} \left| \log \frac{x-2}{x+3} - 1 \right| = \left| \log 1 - 1 \right| = 1$$

$$\lim_{x \to -3^-} \left| \log \underset{(+0)}{\overset{\perp 5}{\frac{x-2}{x+3}}} - 1 \right| = +\infty$$

$$\lim_{x \to 2^+} \left| \underbrace{\log \overset{+0}{\frac{x-2}{x+3}}}_{\log 0^+ \rightsquigarrow -\infty} - 1 \right| = +\infty$$

$5$

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asintoto orizzontale $y=1$ per $x \to \pm\infty$

intersezione con l'asse delle ascisse

$$\begin{cases} y=0 \\ \left| \log \frac{x-2}{x+3} - 1 \right| = 0 \end{cases}$$

$$\log \frac{x-2}{x+3} - 1 = 0 \Rightarrow$$

$$\log \frac{x-2}{x+3} = 1 = \log_e e$$

$$\frac{x-2}{x+3} = e$$

$$x-2 = ex+3e$$

$$x(e-1) = -(2+3e)$$

$$x = -\frac{2+3e}{e-1}$$

$$A\left(-\frac{2+3e}{e-1}, 0\right)$$

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Derivata prima

A punto di non derivabilità

$$f(x) = \left| \log \frac{x-2}{x+3} - 1 \right| = \begin{cases} -\left(\log \frac{x-2}{x+3} - 1\right) & \text{Arg} < 0 \\ \log \frac{x-2}{x+3} - 1 & \text{Arg} \geq 0 \end{cases}$$

$$f'(x) = \frac{\log \frac{x-2}{x+3} - 1}{\left| \log \frac{x-2}{x+3} - 1 \right|} \cdot \underbrace{\left\{ \frac{1}{\cancel{\frac{x-2}{x+3}}} \; \frac{x+3-x+2}{(x+3)^{\cancel{2}}} \right\}}_{D(\text{arg.})} =$$

$$= \frac{\log \frac{x-2}{x+3} - 1}{\left| \log \frac{x-2}{x+3} - 1 \right|} \cdot \left\{ \frac{5}{(x-2)(x+3)} \right\}$$

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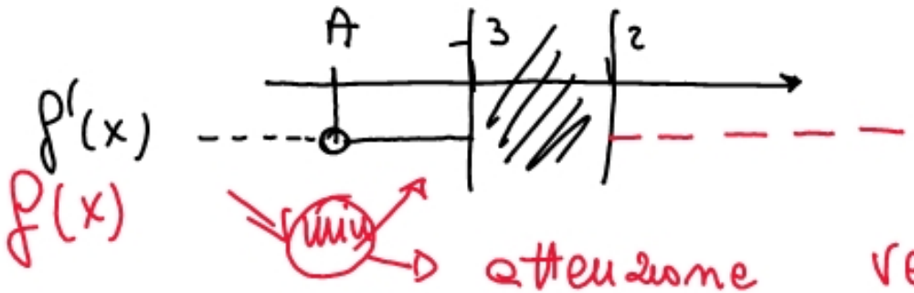
studio del segno di $f'(x)$

① $\log \frac{x-2}{x+3} - 1 > 0$

② $(x-2)(x+3) > 0$ $\Rightarrow \forall x \in$ dominio $[x < -3 \cup x > 2]$




attenzione verificare !

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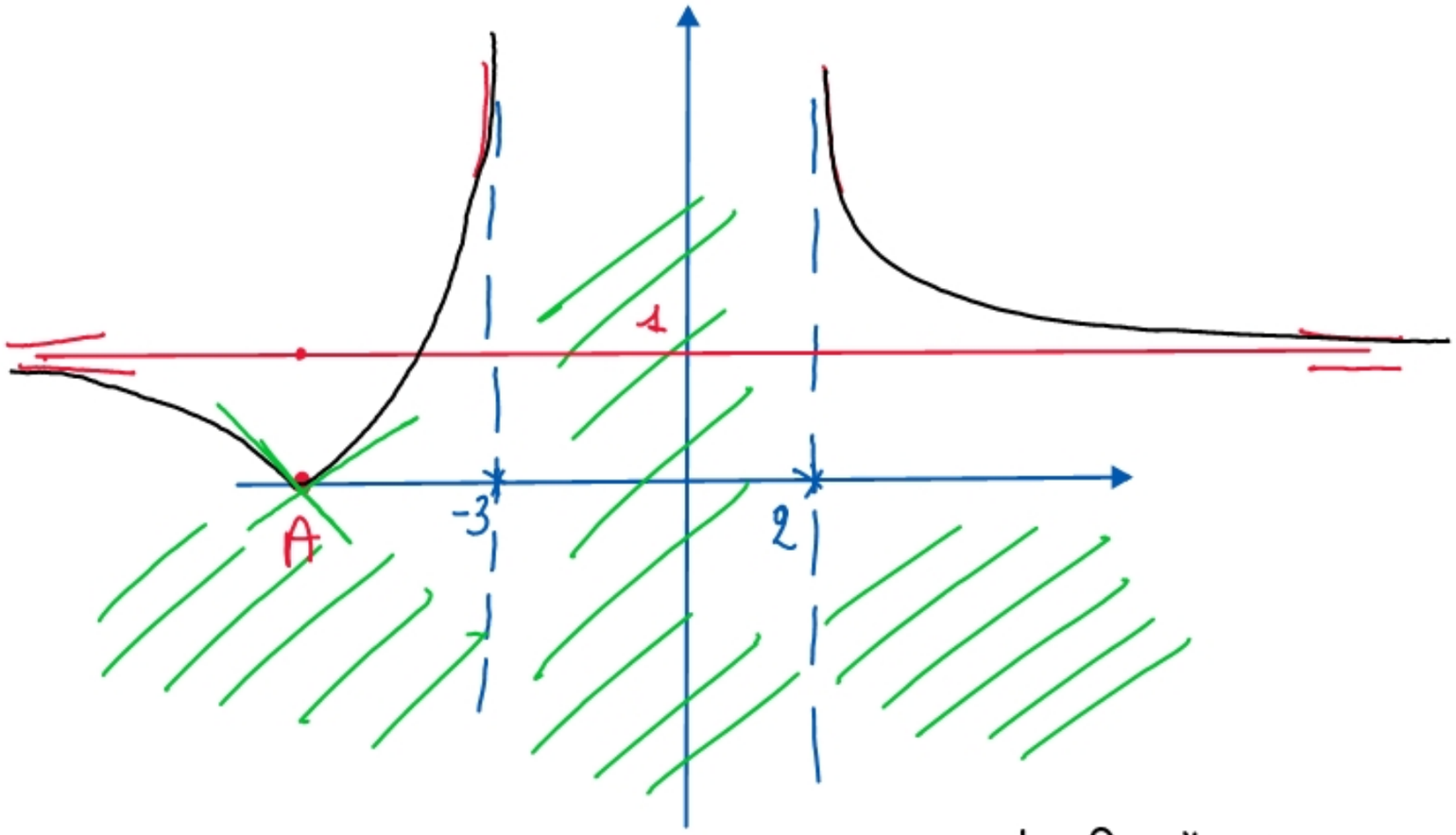


punto A "minimo assoluto"
per la funzione

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considerando il punto A

è un minimo assoluto, cioè la quota minima

è un punto di non derivabilità

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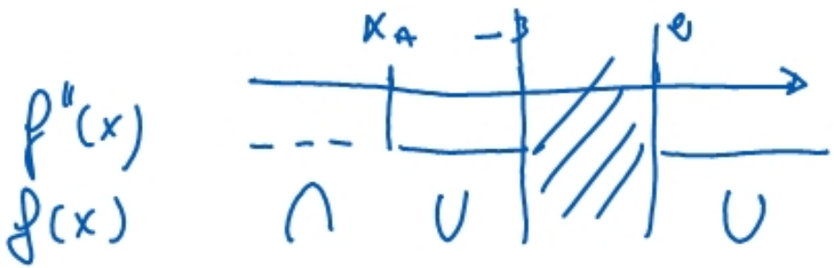
$$f''(x) = -\frac{5(2x+1)}{(x-2)^2(x+3)^2} \cdot \frac{\log\frac{x-2}{x+3} - 1}{\left|\log\frac{x-2}{x+3} - 1\right|}$$




provare a fare i calcoli

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- Scrivere l'equazione della retta tangente nel punto di ascissa $x = 2$ al grafico

$$f(x) = \frac{x+2}{x^2-1} - \log(2x-3)$$

dom : $\begin{cases} x \neq \pm 1 \\ x > 3/2 \end{cases} \Rightarrow x > 3/2$

$$f(2) = \frac{4}{3} - \underbrace{\log(4-3)}_{0} = 4/3$$

$$P\left(2, 4/3\right)$$

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$$f(x) = \frac{x+2}{x^2-1} - \log(2x-3)$$

$$f'(x) = \frac{x^2-1-2x(x+2)}{(x^2-1)^2} - \frac{1}{2x-3}\,2$$

$$m = f'(2) = \frac{4-1-4(2+2)}{(4-1)^2} - \frac{2}{4-3} =$$

$$= \frac{3-16}{9} - 2 =$$

$$= -\frac{13}{9} - \frac{18}{9} = -\frac{31}{9}$$

$$y - y_P = m(x - x_P)$$

$$\boxed{y - \frac{4}{3} = -\frac{31}{9}(x-2)}$$

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