Le some e o

algebra degli $o(x)$

$o(x^n)$

① $k\, o(x^n) = o(k\, x^n) = o(x^n)$

es: $5\left(1 + x + \frac{x^2}{2} + o(x^2)\right) = 5 + 5x + \frac{5}{2}x^2 + o(x^2)$

$5\, o(x^2) = o(x^2)$

② $o(x^n) \pm o(x^n) = o(x^n)$

$2o(x^n) + 5\, o(x^n) = o(x^n)$

③ $x^m \cdot o(x^n) = o(x^{m+n})$

$x^2\left(1 + x + \frac{x^2}{2} + o(x^2)\right) = x^2 + x^3 + \frac{x^4}{2} + o(x^4)$

PAPER SHOW

$$\left(x + o(x)\right) \cdot \left(1 + x + \frac{x^2}{2} + o(x^2)\right) =$$

$$= x + \underline{o(x)} + \underbrace{x^2 + o(x^2) + \frac{x^3}{2} + o(x^3) + o(x^3) + o(x^3)}_{\text{grado} > 1 \ (o(x))} =$$

TRASCURABILI

$$= x + o(x)$$

ATTENZIONE $x^4 \sim o(x^2)$ per $x \to 0$; $I(0,\delta)$

$$f(x) = x^4 + x^3 \sim o(x^2) \qquad \lim_{x \to 0,\ I(0,\delta)} \frac{x^4}{x^2} = 0$$

$$\lim_{x \to 0} \frac{x^4 + x^3}{x^2} = 0$$

PAPER SHOW

④ $o(x^m) \cdot o(x^n) = o(x^{m+n})$

proprietà delle potenze

⑤ $o(o(x^n)) = o(x^n)$

⑥ $o(x^n + o(x^n)) = o(x^n)$

PAPER SHOW

- Calcolare lo sviluppo delle funzioni date in $x=0$ arrestandosi al 2° ordine

$$f(x) = \frac{1}{1+x} = (1+x)^{-1} \longrightarrow 1 - x + \frac{(-1)(-2)}{2} x^2 + o(x^2) =$$

$$\alpha = -1 \qquad = 1 - x + x^2 + o(x^2)$$

$$g(x) = \sqrt{1+x} = (1+x)^{\frac{1}{2}} \longrightarrow 1 + \frac{1}{2} x + \underbrace{\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}}_{-1/8} x^2 + o(x^2)$$

$$\alpha = \frac{1}{2}$$

$$(1+x)^{\alpha} = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots +$$

$$+ \frac{\alpha(\alpha-1)\dots(\alpha-n+1)}{n!} x^n + o(x^n)$$

PAPER SHOW

- $f(x) = \log(\cos x) \to$ in $x = 0$

$$\to \log\Big(\underbrace{1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)}_{1+t}\Big) =$$

$$t = -\frac{x^2}{2} + \frac{x^4}{24} + o(x^5)$$

$$= -\frac{1}{2}x^2 + \frac{x^4}{24} + o(x^5) - \frac{\left[-\frac{1}{2}x^2 + \frac{x^4}{24} + o(x^5)\right]^2}{2} + \dots =$$

$$= -\frac{1}{2}x^2 + \frac{x^4}{24} + o(x^5) - \frac{1}{8}x^4 = -\frac{1}{2}x^2 - \frac{1}{12}x^4 + o(x^5)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + o\left(x^{2n+1}\right)$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n+1}\frac{x^n}{n} + o(x^n)$$

PAPER SHOW

AITE N2 o N3

$$= T - \frac{T^2}{2} + o(T^2) = \quad T = -\frac{x^2}{2} + \frac{x^4}{24} + o(x^5)$$

$$= -\frac{x^2}{2} + \frac{x^4}{24} + o(x^5) \underbrace{-\frac{1}{2}\left(-\frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right)^2} + o\left(\left[-\frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right]^2\right) =$$

$$= -\frac{x^2}{2} + \frac{x^4}{24} + o(x^5) - \frac{1}{8}x^4 + o\Big(\underbrace{\frac{x^4}{4} + \frac{x^8}{24^2} + o(x^{10}) + o(x^9) - \frac{x^6}{24} + \dots}_{\text{min } x^4}$$

$$= -\frac{x^2}{2} - \frac{1}{12}x^4 + o(x^5) + o(x^4) =$$

$$= -\frac{x^2}{2} - \frac{1}{12}x^4 + o(x^4)$$

PAPER SHOW

$f(x) = e^{\operatorname{sen} x}$ sviluppo in $x=0$

$e^t \to 1 + \operatorname{sen} x + o(\operatorname{sen} x) = 1 + \underbrace{x - \frac{x^3}{3!} + o(x^4)}_{\operatorname{sen} x} + o\left(x - \frac{x^3}{3!} + o(x^4)\right) =$

$t = \operatorname{sen} x$

$- - - -$

$e^x = 1 + x + \frac{x^2}{2} + \dots + \frac{x^n}{n!} \quad o(x^n)$

$\operatorname{sen} x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + - + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$

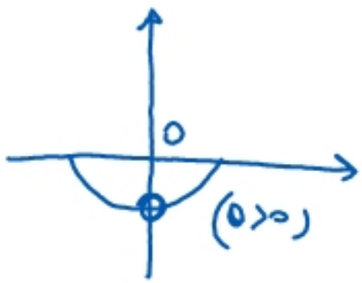
PAPER SHOW

Sia $f: \mathbb{R} \setminus \{0\} \to \mathbb{R}$ così definita: $f(x) = \dfrac{\cos x - \cosh x}{x^2}$.

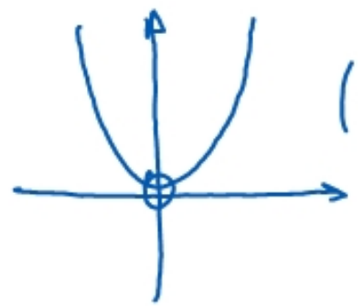
Il suo grafico approssimativo

in un $I(0,\delta)$ è dato da:

A

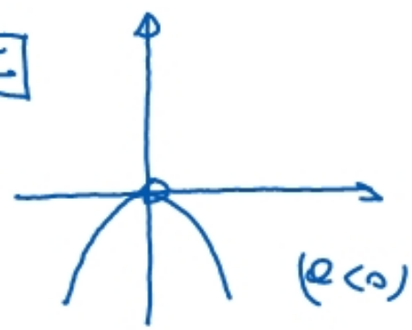


B



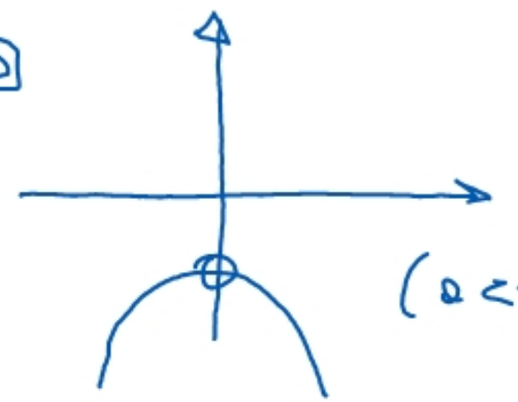
(a>0

C



(a<0)

D



(a<0)

PAPER SHOW

$$f(x) = \frac{\cos x - \cosh x}{x^2}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + o(x^7)$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + o(x^7)$$

considero lo sviluppo di Taylor centrato nell'origine:

$$f(x) = p(x) = \frac{\overbrace{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}}^{\cos x} - \overbrace{1 - \frac{x^2}{2!} - \frac{x^4}{4!} - \frac{x^6}{6!}}^{\cosh x} + o(x^7)}{x^2} =$$

$$= \frac{-x^2 - 2\frac{x^6}{6!} + o(x^7)}{x^2} = -1 - \frac{x^4}{360} + o(x^5)$$

polinomio di 4 grado, rappresenta una "parabola" rivolta verso il basso con $V(0,-1)$ $\longrightarrow$ D

PAPER SHOW

$$\cdot \lim_{x \to 0} \frac{\log(1+\sin x)}{\sin 2x} = \frac{0}{0}$$

$$\underset{T}{=} \lim_{x \to 0} \frac{\sin x - \frac{\operatorname{sen}^2 x}{2} + o(\operatorname{sen}^2 x)}{2x - \frac{(2x)^3}{3!} + o((2x)^4)} =$$

$$= \lim_{x \to 0} \frac{\overset{\operatorname{sen} x}{\overbrace{x + o(x)}} - \overbrace{\frac{(x+o(x))^2}{2} + \cdots}^{\text{TRASCUR.}}}{\underbrace{2x - \frac{(2x)^3}{3!} + o(x^4)}_{\text{TRASC.}}} =$$

$$\log(1+t) = t - \frac{t^2}{2} + o(t^2)$$

$$\sin(t) = t - \frac{t^3}{3!} + o(t^4)$$

$$= \lim_{x \to 0} \frac{x + o(x)}{2x + o(x)} =$$

$$= \lim_{x \to 0} \frac{x}{2x} = \frac{1}{2}$$

PAPER SHOW

- $$\lim_{x \to 0} \frac{1 - \cos x + \log(\cos x)}{x^4} =$$

$$\log(\cos x) = \log\left[1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right] = -\frac{x^2}{2} - \frac{x^4}{12} + o(x^4)$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)$$

$$= \lim_{x \to 0} \frac{1 - \left[1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right] + \left[-\frac{x^2}{2} - \frac{x^4}{12} + o(x^4)\right]}{x^4} =$$

$$= \lim_{x \to 0} \frac{-\frac{x^4}{24} - \frac{x^4}{12} + o(x^4)}{x^4} = -1/8$$

PAPER SHOW

$$\lim_{x \to 0^+} \frac{\sqrt{x} + \sin x}{\operatorname{tg} x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\sqrt{x} + o(\sqrt{x})}{x + o(x)} \simeq \lim_{x \to 0} \frac{\sqrt{x}}{x} = +\infty$$

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{\sqrt{x}} = \lim_{x \to 0} \sqrt{x} \underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1} = 0$$

$$\operatorname{sen} x \simeq o(\sqrt{x})$$

$$\operatorname{Tg}(x) = x + \frac{1}{3} x^3 + o(x^4)$$

PAPER SHOW

PAPER SHOW®