Lezione 19

STUDIO DI FUNZIONE:

$$f(x) = \sqrt[3]{x^2(x-1)}$$ dom $\forall x \in \mathbb{R}$

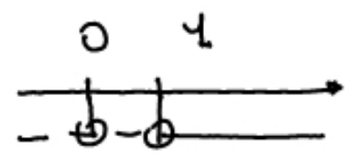
$$f(-x) = \sqrt[3]{(-x)^2(-x-1)} = \sqrt[3]{x^2(-x-1)}$$ non simmetrie

segno:

$$f(x) \geq 0$$

$$\sqrt[3]{x^2(x-1)} \geq 0 \Rightarrow \underset{\geq 0}{x^2}(x-1) \geq 0$$

$$\{ x \geq 1 \cup x = 0 \}$$



$(0, 0)$ $(1, 0)$ intersezioni

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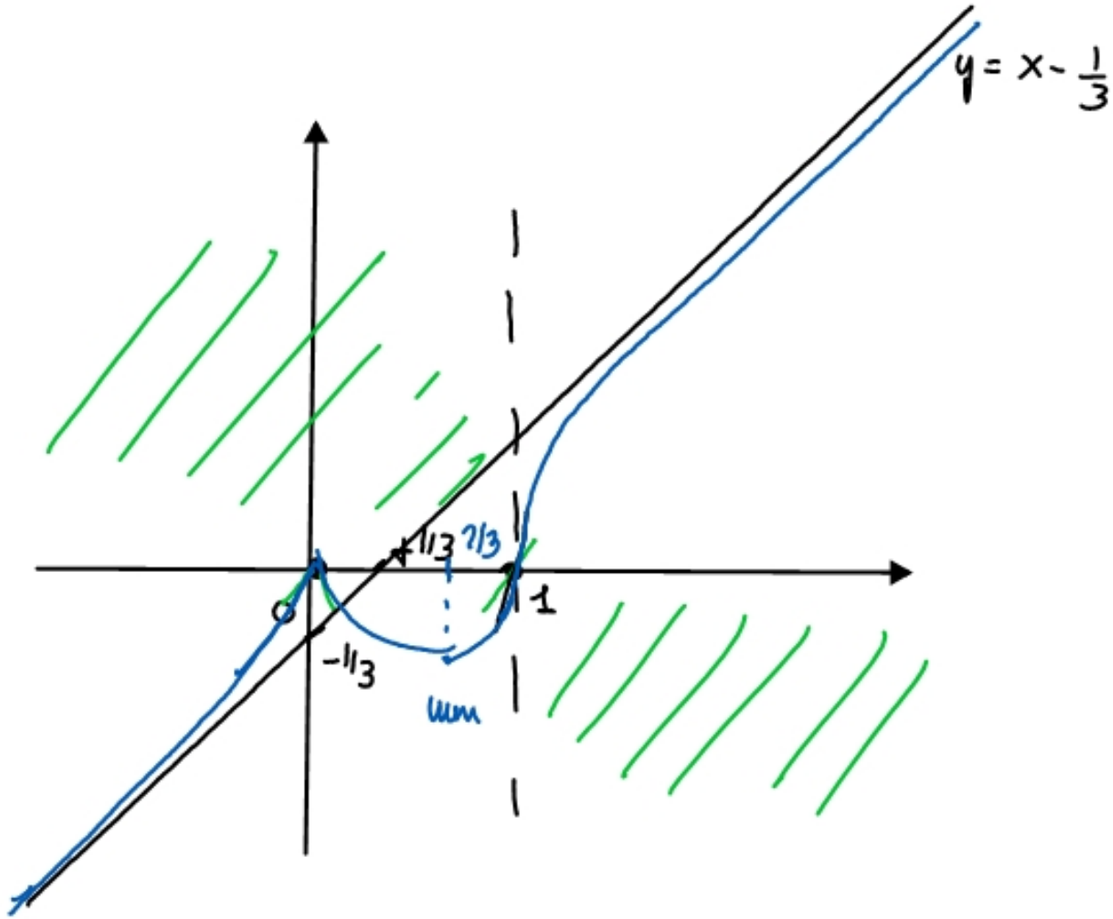
$y = x - \frac{1}{3}$



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- $y = \sqrt{1 + \frac{x^2}{2}}$ calcolare la retta tangente in $x = 2$

1. dom $f \equiv \mathbb{R}$: $x = 2 \in$ dom f

2. coordinate del punto $P\ (2, \sqrt{3})$

$$y_P = \sqrt{1 + \frac{4}{2}} = \sqrt{3}$$

3. $m_P =$

$$f'(x) = \frac{1}{2\sqrt{1 + \frac{x^2}{2}}} \cdot \left(\frac{2x}{2}\right) = \frac{x}{2\sqrt{1 + \frac{x^2}{2}}}$$

$$m_P = \left[f'(x) \right]_{x=2} = \frac{2}{2\sqrt{1 + \frac{4}{2}}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

4. eq. retta $\quad y - y_P = m_P (x - x_P)$

$$y - \sqrt{3} = \frac{\sqrt{3}}{3}(x - 2) \quad - - - -$$

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• $f(x) = \begin{cases} 1 + \sin(ax - 7x^2) + x^2 & x \leq 0 \\ -3x^2 + (2a-1)x + a + b & x > 0 \end{cases}$ derivabilità in $x=0$

0. definita in $x=0$

1. <u>Continuità</u> in $x=0$

$$f_-(0) = 1 + \sin(a \cdot 0 - 7 \cdot 0) + 0 = 1$$

$$\lim_{x \to 0^+} \left(-3x^2 + (2a-1)x + a + b \right) = a + b$$

$\Rightarrow \quad a + b = 1$

2. derivabilità :

$$f'_-(x) = \cos(ax - 7x^2) \cdot (a - 14x) + 2x$$

$$f'_-(0) = \cos 0 \cdot a + 0 = a$$

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parte destra in $x=0$ devo calcolare il limite del rapporto incrementale $f(0+h)$ $x_0=0$ h incremento $f(0)$

$$\lim_{h\to 0} \frac{\left[-3(0+h)^3 + (2a-1)(0+h) + (a+b)\right] - \left[-3\cdot 0 + (2a-1)\cdot 0 + a+b\right]}{h} =$$

$$= \lim_{h\to 0} \frac{\left[-3h^2 + (2a-1)h + \cancel{(a+b)}\right] - \left[\cancel{a+b}\right]}{h} = \text{ divido per } h$$

$$= \lim_{h\to 0} \left\{-3h + (2a-1)\right\} = 2a-1$$

uguagliando $\boxed{2a-1 = a}$

$$\begin{cases} a+b=1 \\ 2a-1=a \end{cases} \Rightarrow \begin{cases} b=0 \\ a=1 \end{cases}$$

la funzione risulta continua e derivabile

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$$\lim_{x \to -\infty} \sqrt[3]{x^2(x-1)} = -\infty$$

$$\lim_{x \to +\infty} \sqrt[3]{x^2(x-1)} = +\infty$$

No asintoti verticali, no asintoti orizzontali

Ricerca di eventuali asintoti obliqui $\qquad y = mx + q$

$$m = \lim_{x \to \pm\infty} \frac{\sqrt[3]{x^2(x-1)}}{x} = \lim_{x \to \pm\infty} \frac{\sqrt[3]{x^3\left(1-\frac{1}{x}\right)}}{x} = 1 \qquad \left(\lim_{x \to \infty} \frac{f(x)}{x}\right)$$

$$q = \lim_{x \to +\infty} \left(\sqrt[3]{x^2(x-1)} - x\right) = f.i \qquad \left(\lim_{x \to +\infty} f(x) - mx\right)$$

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$$= \lim_{x \to +\infty} \frac{\left(\sqrt[3]{x^2(x-1)} - x\right)\left(\left(\sqrt[3]{x^2(x-1)}\right)^2 + x^2 + x\sqrt[3]{x^2(x-1)}\right)}{\left(\sqrt[3]{x^2(x-1)}\right)^2 + x^2 + x\sqrt[3]{x^2(x-1)}} =$$

Ricordam

$$A^3 \underset{+}{-} B^3 = (A-B)(A^2 + B^2 \underset{-}{+} AB)$$

$$\cancel{x^3} - x^2 - \cancel{x^3}$$

$$= \lim_{x \to +\infty} \frac{x^2(x-1) - x^3}{\left(\sqrt[3]{x^2(x-1)}\right)^2 + x^2 + x\sqrt[3]{x^2(x-1)}} =$$

$$= \lim_{x \to +\infty} \frac{-x^2}{\left(\sqrt[3]{x^2(x-1)}\right)^2 + x^2 + x\sqrt[3]{x^2(x-1)}} = -\frac{1}{3}$$

grado 2 N

$\sqrt[3]{x^3} \sim x$ $\qquad$ $1\sqrt[3]{3}$

grado: x^2 $\qquad$ x^2 $\qquad$ x^2

grado 2 D

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$y = x - \frac{1}{3}$ asintoto obliquo per $x \to +\infty$, $x \to -\infty$

DERIVATA PRIMA

$$f(x) = \sqrt[3]{x^2(x-1)} = \left(x^3 - x^2\right)^{1/3}$$

$$f'(x) = \frac{1}{3}\left(x^3 - x^2\right)^{-2/3}\left(3x^2 - 2x\right) = \frac{3x^2 - 2x}{3\sqrt[3]{\left[x^2(x-1)\right]^2}}$$

$$\text{dom } f' = \text{dom } f \setminus \{0, 1\}$$

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$$f'(x) = \frac{3x^2 - 2x}{3\sqrt[3]{x^4(x-1)^2}} = \frac{x\,(3x-2)}{3x\sqrt[3]{x(x-1)^2}} = \frac{3x-2}{3\sqrt[3]{x(x-1)^2}}$$

limiti della f'

$$\lim_{x \to 0^{\pm}} \frac{3x-2}{3\sqrt[3]{x(x-1)^2}} = \text{"}\frac{-2}{0}\text{"} = \mp\infty$$

CUSPIDE

$$\lim_{x \to 1^{\pm}} \frac{3x-2}{3\sqrt[3]{x(x-1)^2}} = \frac{1}{0^{+}} = +\infty$$

pt. a Tangente Verticale

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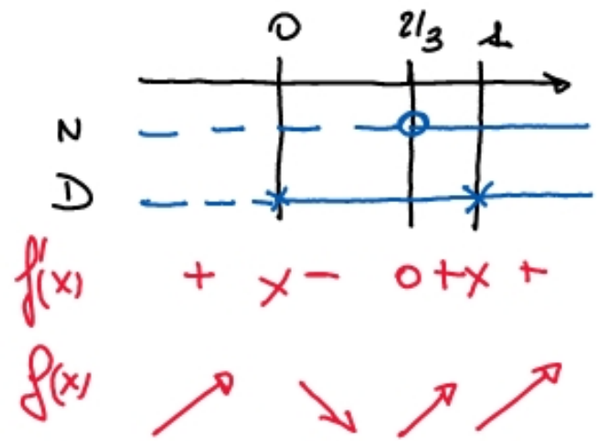
studio del segno di $f'(x) = \dfrac{3x-2}{3\sqrt[3]{x(x-1)^2}}$

N: $3x-2 \geq 0 \quad x \geq 2/3$

D: $(x-1)^2 > 0 \quad \forall x \neq 1$

$x > 0$



$x = 2/3$ minimo

$f\left(\frac{2}{3}\right) = \sqrt[3]{\frac{4}{9}\left(\frac{2}{3}-1\right)} = \sqrt[3]{\frac{4}{9}\cdot\left(-\frac{1}{3}\right)} = \sqrt[3]{\frac{-4}{27}} = -\frac{\sqrt[3]{4}}{3}$

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DERIVATA SECONDA

$$f'(x) = \frac{3x-2}{3\sqrt[3]{x(x-1)^2}} = \frac{1}{3}(3x-2)\left(x^3-2x^2+x\right)^{-2/3}$$

x^2-2x+1

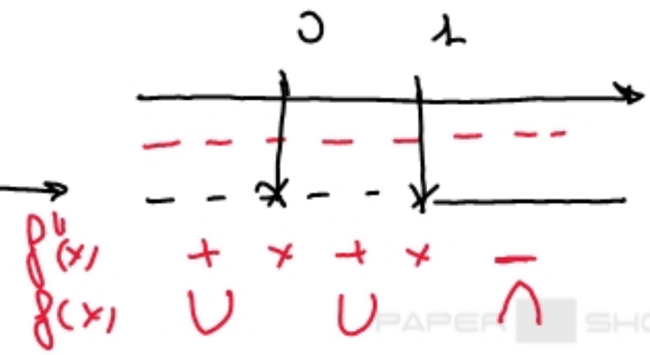
$$f''(x) = \frac{1}{3}\left\{3\left(x^3-2x^2+x\right)^{-2/3} + (3x-2)\left(-\frac{2}{3}\right)\left(x^3-2x^2+x\right)^{-5/3}\left(3x^2-4x+1\right)\right\} =$$

$$= \cdots\cdots = -\frac{2}{9\sqrt[3]{x^4(x-1)^5}} \qquad x \neq 0,\ x \neq 1$$

STUDIO DEL SEGNO

$x-1>0$

$x>1$



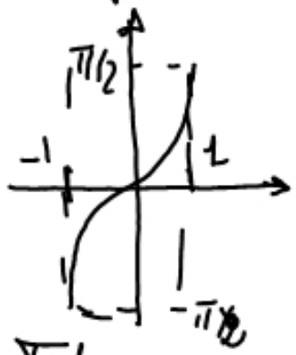
N.B. Teoremi ed applicabilità :

VERIFICARE SEMPRE LE IPOTESI

- Teo. di Lagrange $f(x) = \arcsin x$ $[a, b]$ dominio

determinare i punti di Lagrange : c : $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$f(x) = \arcsin x$



dom f: $[-1, 1]$ intervallo chiuso e limitato

$[-1, 1]$ f continua

$]-1, 1[$ è derivabile ($x = \pm 1$ pt. a tangente verticale)

c ?

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$f(b) = \pi/2$

$f(a) = -\pi/2$

$a = -1$

$b = 1$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} \qquad (\Leftarrow D(\arcsen x))$$

sostituendo

$$\frac{1}{\sqrt{1-c^2}} = \frac{f(1) - f(-1)}{1-(-1)} = \frac{\frac{\pi}{2} + \frac{\pi}{2}}{2} = \frac{\pi}{2}$$

$$\frac{1}{\sqrt{1-c^2}} = \frac{\pi}{2}$$

$$\frac{1}{1-c^2} = \frac{\pi^2}{4} \qquad 1 - c^2 = \frac{4}{\pi^2} \qquad c = \pm\sqrt{1 - \frac{4}{\pi^2}}$$

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TEOREMA DI L'HÔPITAL

1. forme di indeterminazione $\frac{0}{0}$, $\frac{\infty}{\infty}$

   con $x \to x_0$ oppure $x \to \pm\infty$

2. fornisce una condizione sufficiente per l'esistenza del limite

3. attenzione: il limite di f/g può esistere anche se non esiste il limite di f'/g'

4. attenzione alle verifiche di tutte le ipotesi

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esempi

- $$\lim_{x\to 0} \frac{\log(1+\sin x)}{\sin 2x} = \frac{0}{0} \overset{H}{=} \lim_{x\to 0} \frac{\overbrace{\frac{1}{1+\underbrace{\sin x}_{\to 0}} \cdot \cos x}^{f'(x)}}{2\cos 2x} = \frac{1}{2}$$

- $$\lim_{x\to 0} \frac{\sin(\pi\, 3^x)}{x} = \frac{0}{0} \overset{H}{=} \lim_{x\to 0} \frac{\overbrace{\cos(\pi\, \overset{1}{3^x}) \cdot \pi \cdot \overbrace{3^x \log 3}^{D(3^x)}}^{g'(x)\ \neq 0}}{1} =$$

$$= \cos\pi \cdot \pi \cdot 3^0 \cdot \log 3 = -\pi \log 3$$

- $$\lim_{x\to 0} (e^x + x)^{\frac{1}{x}} = 1^\infty = \lim_{x\to 0} e^{\log(e^x+x)^{\frac{1}{x}}} =$$

passaggio all' esponenziale

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$$= e^{\lim_{x\to 0} \frac{\log(e^x+x)}{x}} \to \frac{0}{0} \overset{H}{=}$$

$$D(e^x+x)$$

$$= e^{\lim_{x\to 0} \frac{e^x+1}{e^x+x}} =$$

$$D(\log(e^x+x))$$

$$= e^{2/1} = e^2$$

- $\lim_{x\to+\infty} x\left(\frac{\pi}{2} - \operatorname{arctg} x\right) = \infty \cdot 0$

$$= \lim_{x\to+\infty} \frac{\frac{\pi}{2} - \operatorname{arctg} x}{\frac{1}{x}} = \frac{0}{0} \overset{H}{=} \lim_{x\to+\infty} \frac{-\frac{1}{1+x^2}}{-1/x^2} =$$

$$= \lim_{x\to+\infty} \frac{x^2}{1+x^2} = 1$$

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