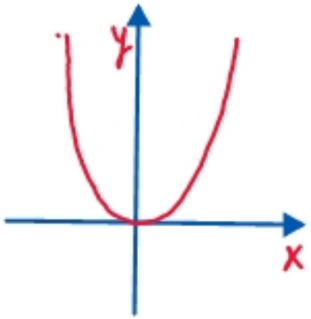
Funzioni

$f : A \to B$



$f : \mathbb{R} \to \mathbb{R}$ $\qquad y = x^2$

dominio f : $\mathbb{R}$ $\qquad D$ $\qquad x \in]-\infty, +\infty[$

codominio f : $\mathbb{R}$ $\qquad C$ $\qquad y \in [0, +\infty[$

immagini y $\mathbb{R}^+$ $\qquad \mathbb{R}^+ \subset \mathbb{R}$ $(\mathrm{im} f)$

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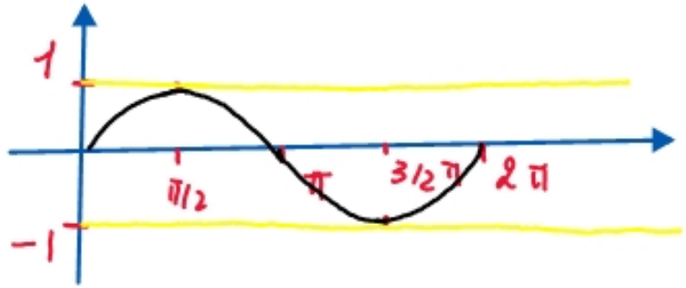
$y = \operatorname{sen} x$



$\operatorname{dom} f = \mathbb{R}$ $\qquad$ $f: \mathbb{R} \to \mathbb{R}$

$\operatorname{im} f = [-1, 1]$

$-1 \leq \operatorname{sen} x \leq 1$

limitata

periodica

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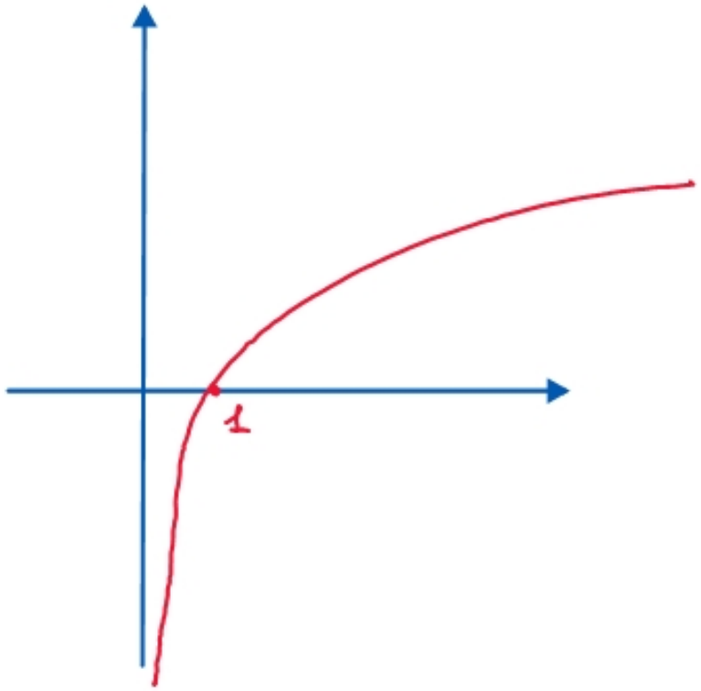
$y = \log_e x$

$(e > 1)$

$\text{dom } f \equiv \mathbb{R}_0^+ \quad (x > 0)$

$\text{cod } f = \mathbb{R} \equiv \text{im } f$



- illimitata

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Ricordarsi dei divieti:

a) nei espressione frazionarie (divisione)

non posso dividere per una quantità nulla (: 0)

$$\frac{1}{T} \rightsquigarrow T \neq 0$$

frazione : denominatore $\neq 0$

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b) in espressione con radicali di indice pari, i radicandi devono essere maggiori o uguali a 0 (≥ 0)

perché non ha significato una radice di indice pari con radicando negativo.

$$\sqrt[2n]{T} \rightsquigarrow T \geq 0 \qquad n = 1, 2 \ldots$$

$2n$ = numero pari

$2n+1$ = numero dispari

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c)

gli argomenti dei logaritmi devono essere strettamente maggiori di zero
(>0)

$\log_a T \rightsquigarrow T > 0$

ATTENZIONE

$\log_a x \quad a^x$

[base $a > 0 \quad a \neq 1$]

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d) operatori trigonometrici:

$\operatorname{tg} T \rightsquigarrow T \neq \frac{\pi}{2} + k\pi$ → perioda cu multiplicate

$$\left(\begin{array}{l} \frac{\pi}{2} \leftarrow 90^\circ \\ \pi \leftarrow 180^\circ \end{array} \right)$$

$\operatorname{ctg} T \rightsquigarrow T \neq k\pi$

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FUNZIONI REALI DI VARIABILE REALE

- ALGEBRICI
  - RAZIONALI
    - POLINOMIALI
    - FRATTE
  - IRRAZIONALI
- TRASCENDENTI
  - TRIGONOMETRICHE
  - ESPONENZIALI LOGARITMICHE
  - IPERBOLICHE

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POLINOMIALI
(RAZIONALI INTERE)

$$y = a x^n + b x^{n-1} + \dots + e \qquad n \in N$$

$$a, b, \dots, e \in R$$

$$D \equiv]-\infty, +\infty[$$

$$\forall x \in R$$

RAZIONALI FRATTE

$$y = \frac{A(x)}{B(x)}$$

$A(x)$, $B(x)$ polinomi interi

$$D: \quad B(x) \neq 0$$

$$y = \frac{5x^2 - 3x + 2}{x - 3}$$

$$x - 3 \neq 0 \qquad x \neq 3$$

$$D \equiv \{ x \in R \setminus \{3\} \}$$

$$]-\infty, +3[\ \cup \ ]+3, +\infty[$$

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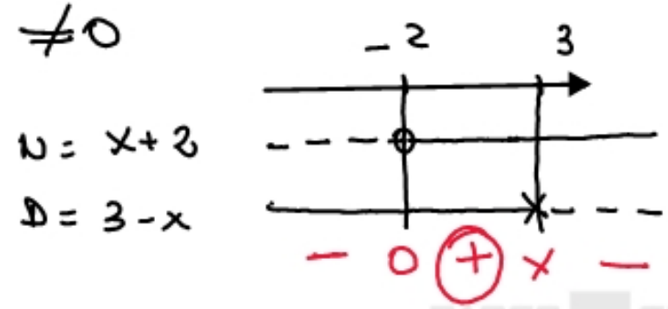
IRRAZIONALI

1) n pari $\qquad y = \sqrt[n]{A(x)} \qquad A(x) \geq 0$

2) n dispari $\qquad y = \sqrt[n]{A(x)} \qquad$ per ogni valore di $A(x)$

es: $\quad y = \sqrt{\dfrac{x+2}{3-x}}$

$$\begin{cases} \dfrac{x+2}{3-x} \geq 0 \\ 3-x \neq 0 \end{cases} \qquad -2 \leq x < 3$$



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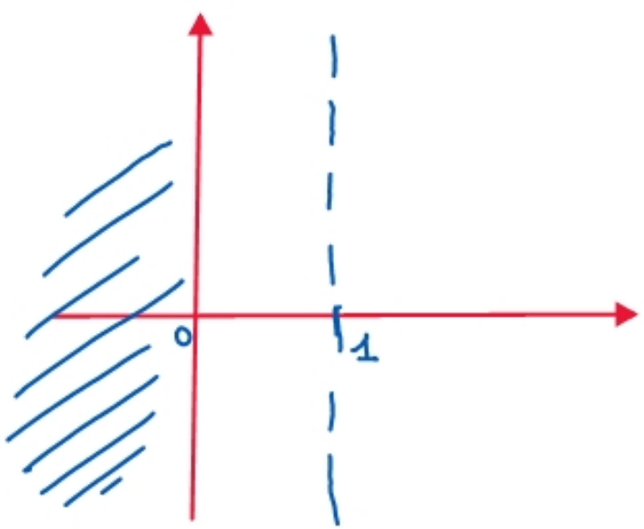
Trigonometriche

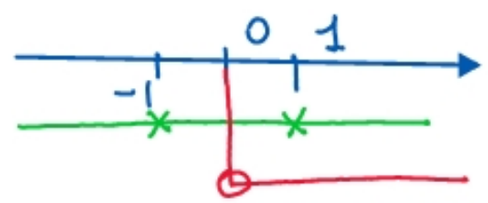
$$y = \operatorname{sen}\left(\frac{\sqrt{x}}{x^2-1}\right)$$

Argomento:

$\sqrt{x}$: $\begin{cases} x \geq 0 \\ x^2-1 \neq 0 \end{cases}$ $\begin{cases} x \geq 0 \\ x \neq \pm 1 \end{cases}$

$\frac{}{x^2-1}$:





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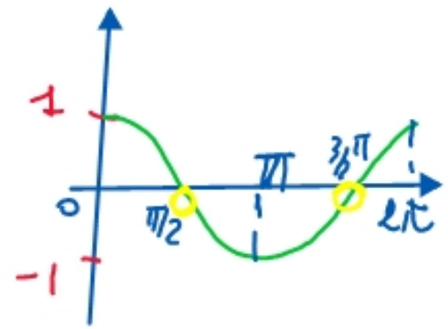
$$y = \frac{5x-3}{\cos\left(2x-\frac{\pi}{3}\right)}$$

$$\cos\left(2x-\frac{\pi}{3}\right) \neq 0$$



$$\text{Arg} \neq \frac{\pi}{2} + 2k\pi$$

$$\neq \frac{3}{2}\pi + 2k\pi$$

$\frac{\pi}{2} \longleftrightarrow \frac{3}{2}\pi$

differiscono di π

$$2x - \frac{\pi}{3} \neq \frac{\pi}{2} + k\pi$$

$$2x \neq \frac{\pi}{2} + \frac{\pi}{3} + k\pi$$

$$\frac{2x}{2} \neq \frac{\frac{5}{6}\pi + k\pi}{2}$$

$$x \neq \frac{5}{12}\pi + k\frac{\pi}{2}$$

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logaritmiche / esponenziali

$$y = e^{\frac{x+1}{\sqrt{x-2}}}$$

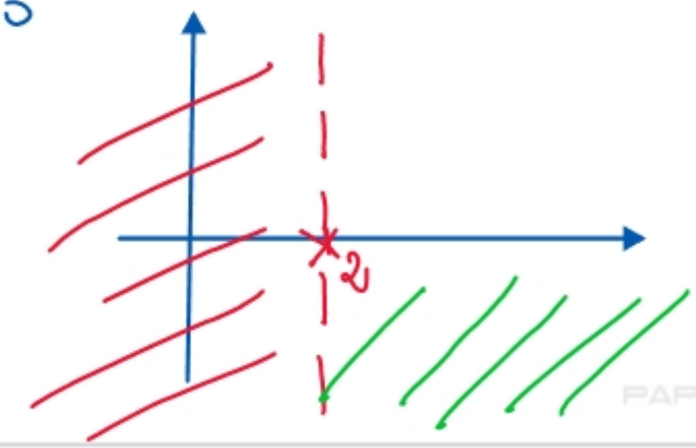
base positiva, costante, diversa da 1

esponente $\frac{x+1}{\sqrt{x-2}}$

radice $\sqrt{x-2} \neq 0$

denominatore $x-2 \geq 0$

$\Rightarrow x-2>0 \qquad x>2$



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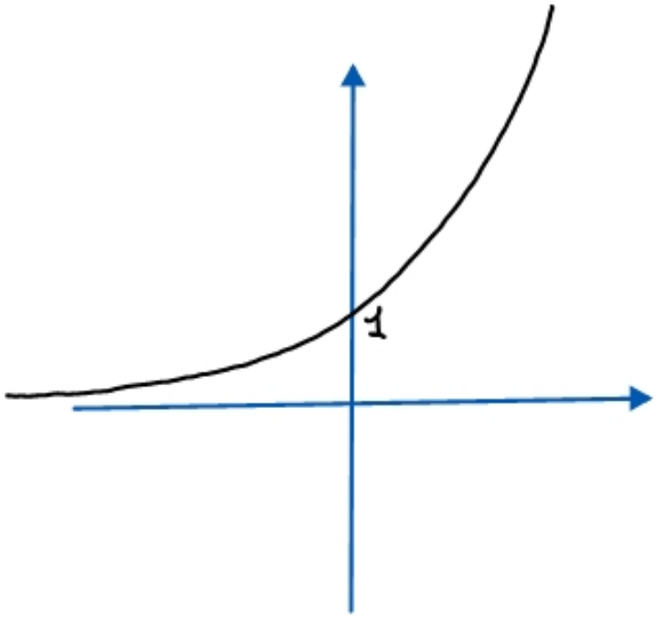
esponenziale

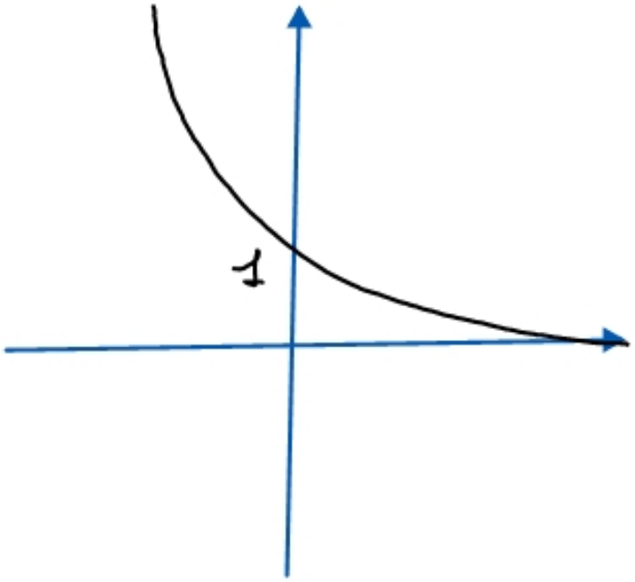
$$y = e^{x}$$

$e > 1$



$$y = e^{-x} = \left(\frac{1}{e}\right)^{x}$$

$0 < \frac{1}{e} < 1$



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$$y = \log x^2$$

$x^2 > 0 \qquad \forall x \in \mathbb{R}_0 \qquad \boxed{x \neq 0}$

Algebricamente $\qquad \log x^2 = 2 \log x$?

Attenzione

$$y = 2 \log x$$

dom: $\boxed{x > 0}$

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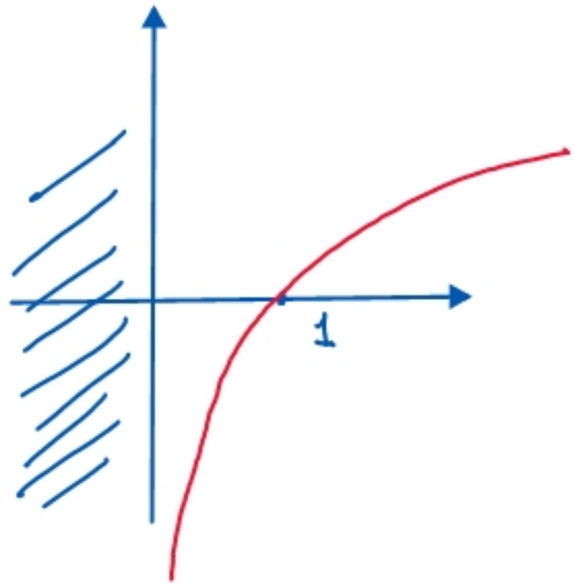
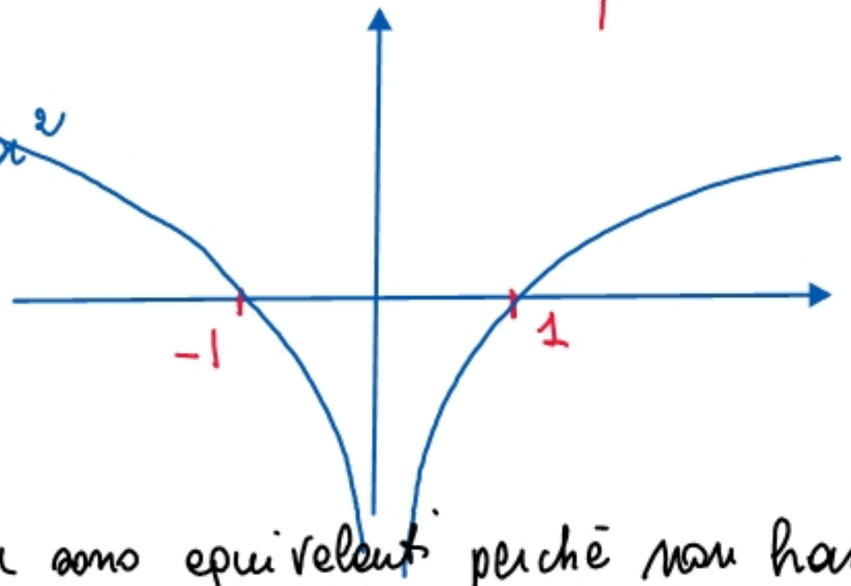
graficamente

$y = 2\log x$



$y = \log x^2$



- le 2 funzioni non sono equivalenti perché non hanno lo stesso dominio

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Funzione pari

1. il grafico è simmetrico rispetto all'asse delle ordinate

2. il dominio deve essere simmetrico rispetto all'origine (zero)

$$f(-x) = f(x)$$

$(-x)$ $\quad y = \log x^2$

$$\underline{f(-x)} = \log(-x)^2 = \log x^2 = \underline{f(x)}$$

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Funzione dispari

1. simmetria rispetto all'origine del grafico

2. dominio simmetrico rispetto all'origine

$$f(-x) = -f(x)$$

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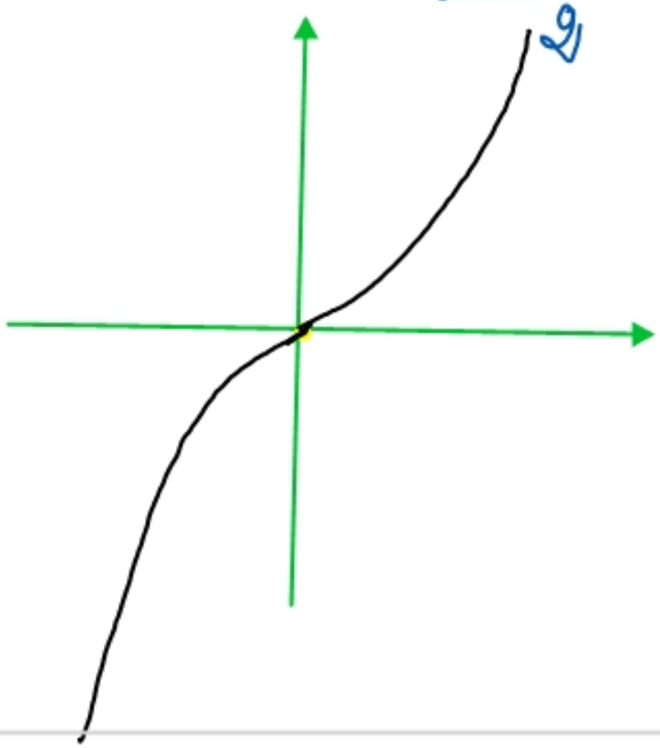
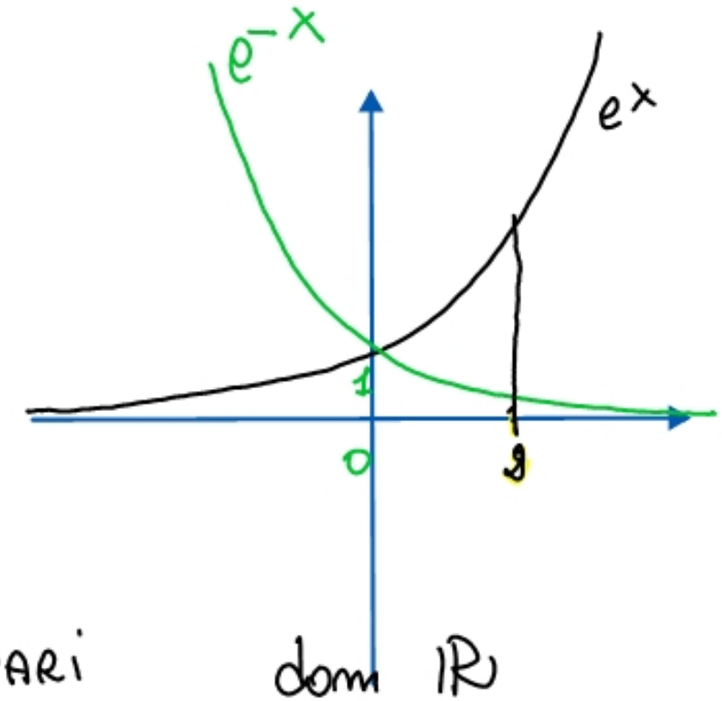
Funzioni iperboliche

$y = \operatorname{senh} x = \operatorname{sh} x$ seno iperbolico di x

$$\operatorname{sh} x = \frac{e^{x} - e^{-x}}{2}$$





DISPARI

dom $\mathbb{R}$

CRESCENTE

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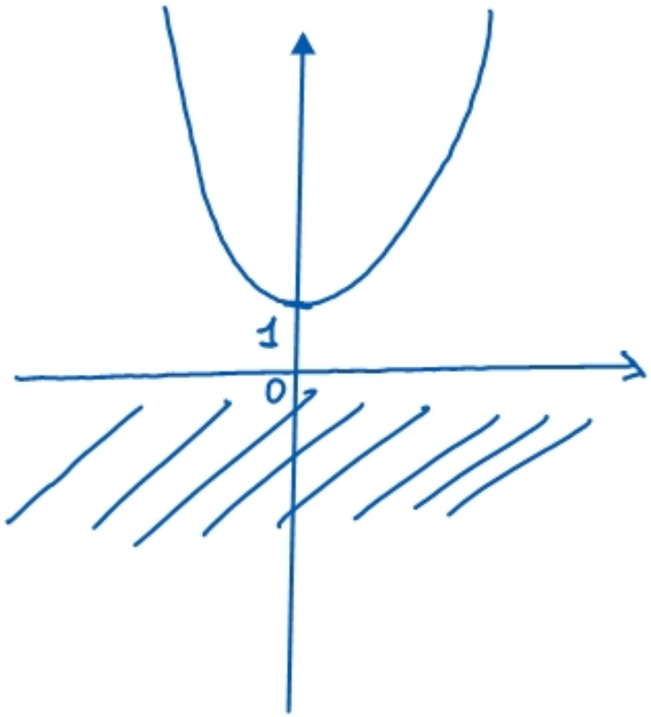
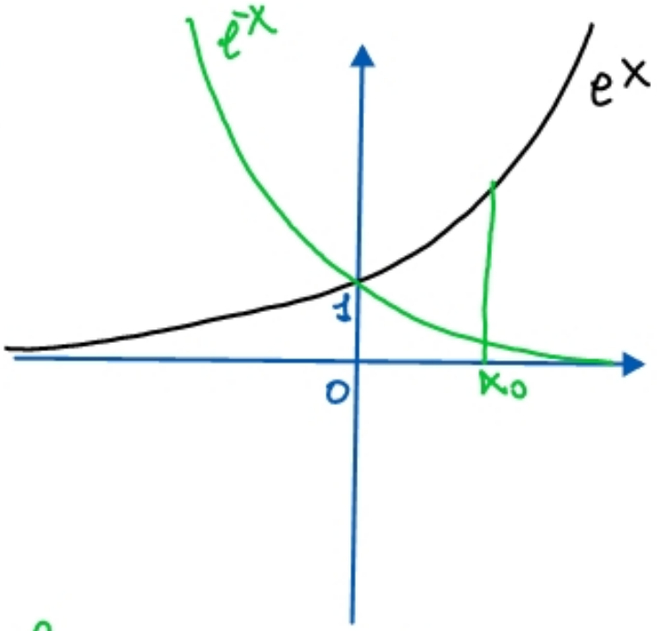
$y = \cosh x = \mathrm{ch}\, x$ coseno iperbolico x

$$\mathrm{ch}\, x = \frac{e^{x} + e^{-x}}{2}$$





funzione pari

positiva

$]-\infty, 0]$ decrescente

$]0, +\infty[$ crescente

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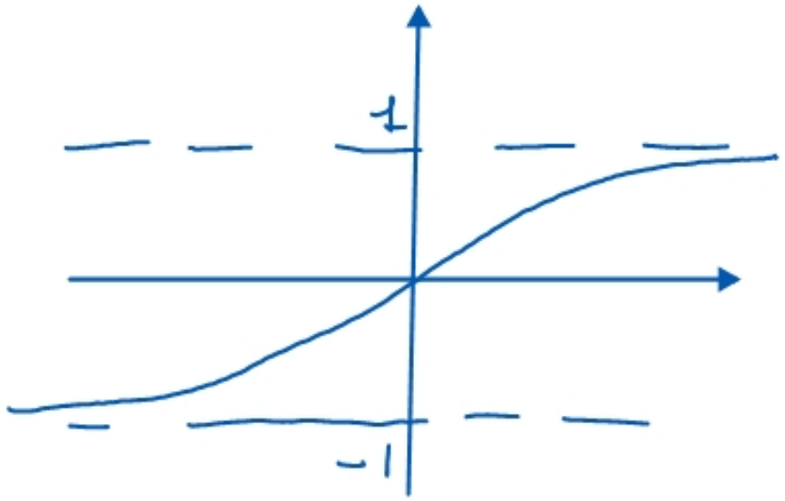
Relazione fondamentale:

$$\cosh^2 x - \operatorname{senh}^2 x = 1$$

$$\operatorname{Tgh} x = \frac{\operatorname{sh} x}{\operatorname{ch} x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$ dispari



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