Lezione 17

DERIVATE

$$f(x) = \frac{x|x| - 1}{x+1} + 1 = \begin{cases} \dfrac{-x^2-1}{x+1} + 1 = -\dfrac{x^2+1}{x+1} + 1 & x < 0 \text{ con } x \neq -1 \\[2ex] \dfrac{x^2-1}{x+1} + 1 & x \geq 0 \end{cases}$$

$|x|$ da sviluppare

$$\left[f'(x) = D\left[f(x)\right] \right]$$

dominio $x \neq -1$

FORMULE :

$D(k) = 0$

$D(x^n) = n\,x^{n-1}$

$$D\left\{\left[f(x)\right]^n\right\} = n\left[f(x)\right]^{n-1} \cdot f'(x)$$

$$D\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - g'(x)\, f(x)}{\left[g(x)\right]^2}$$

$$f'(x) = \begin{cases} -\dfrac{2x(x+1)-(x^2+1)}{(x+1)^2} = -\dfrac{x^2+2x-1}{(x+1)^2} & x<0 \cup x \neq -1 \\[2ex] \dfrac{2x(x+1)-(x^2-1)}{(x+1)^2} = \dfrac{x^2+2x+1}{(x+1)^2} & x \geq 0 \end{cases}$$

$$D(x^2+1) = D(x^2)+D(1) = 2x+0$$

$$D(x+1) = 1$$

$$D(x^2-1) = 2x$$

$$D[|x|] = 1 \cdot \underline{\operatorname{sgn}(x)} = \begin{cases} +1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

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$x > 0$

$$y = \frac{x^2-1}{x+1} + 1 = \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} + 1 = x - 1 + 1 = x$$

rette con $x \neq -1$

$$\lim_{x \to -1} f(x)$$

attenzione al dominio:

quando c'è il valore assoluto, conviene "1 doppiando"

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$$f(x) = \sqrt{e^{4x} - e^{6x}} = \left(e^{4x} - e^{6x}\right)^{1/2}$$

$$f'(x) = \frac{1}{2}\left(e^{4x} - e^{6x}\right)^{\overbrace{\frac{1}{2}-1}^{-1/2}} \cdot \left(e^{4x} \cdot 4 - e^{6x} \cdot 6\right) =$$

$$= \frac{\cancel{2}\left(2e^{4x} - 3e^{6x}\right)}{\cancel{2}\sqrt{e^{4x} - e^{6x}}} = \frac{2e^{4x} - 3e^{6x}}{\sqrt{e^{4x} - e^{6x}}}$$

$$D\left(e^{x}\right) = e^{x}$$

$$D\left(e^{f(x)}\right) = e^{f(x)} \cdot f'(x)$$

$$D\left(a^{x}\right) = a^{x} \cdot \log_e a$$

$$D\left(e^{4x}\right) = e^{4x} \cdot D(4x) =$$

$$= e^{4x} \cdot 4$$

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dominio

$$f(x) = \sqrt{e^{4x} - e^{6x}} \longrightarrow e^{4x} - 6^{6x} \geq 0$$

$$e^{4x} \geq e^{6x}$$

$$4x \geq 6x$$

$$2x \leq 0 \qquad \boxed{x \leq 0}$$

$$f'(x) = \frac{2e^{4x} - 3e^{6x}}{\sqrt{e^{4x} - e^{6x}}} \longrightarrow e^{4x} - e^{6x} > 0$$

$$\vdots \qquad \boxed{x < 0}$$

$$D(f(x)) \supset D(f'(x))$$

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Attenzione: $D(f(x))$ & $D(f'(x))$

(7) $y = \sqrt{x}$

dom: $x \geq 0$

derivata

$y' = \dfrac{1}{2\sqrt{x}}$

dom: $x > 0$

$D(f(x)) \supset D(f'(x))$

il punto $x=0$ è di non derivabilità

(8) $y = \log x$

dom: $x > 0$

derivata

$y' = \dfrac{1}{x}$

dom: $x \neq 0$

$D(f(x)) \subset D(f'(x))$

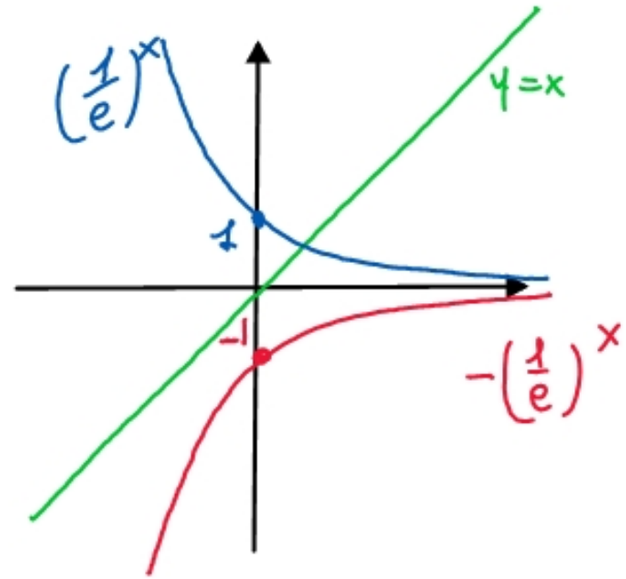
dove la funzione non è definita, non parlo di derivabilità

$$f(x) = \frac{7x^2}{x + e^{-x}}$$

dom : $x + e^{-x} \neq 0$

$x \neq -e^{-x} = -\frac{1}{e^x} = -\left(\frac{1}{e}\right)^x$

$\forall x \in \mathbb{R}$



$$f'(x) = 7\,\frac{2x\left(x+e^{-x}\right) - x^2\left(1-e^{-x}\right)}{\left(x+e^{-x}\right)^2} =$$

$$= 7\,\frac{x\left[2x + 2e^{-x} - x + xe^{-x}\right]}{\left(x+e^{-x}\right)^2} =$$

$$= 7\,\frac{x\left[x + 2e^{-x} + xe^{-x}\right]}{\left(x+e^{-x}\right)^2}$$

$+$

dom f' = dom f

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$$f(x) = 3 \operatorname{arctg} x + \frac{1}{x}$$

dom f : $x \neq 0$

$$f'(x) = 3 \frac{1}{1+x^2} - \frac{1}{x^2} =$$

dom f' : $x \neq 0$

$$D\left(\frac{1}{x}\right) = D\left(x^{-1}\right) = - x^{-2} = - \frac{1}{x^2}$$

$$= \frac{3x^2 - 1 - x^2}{x^2(1+x^2)} = \frac{2x^2 - 1}{x^2(1+x^2)}$$

$$D(\operatorname{arctg} x) = \frac{1}{1+x^2}$$

$$D(\operatorname{arctg}(f(x))) = \frac{1}{1+f^2(x)} \cdot f'(x)$$

$$D(\operatorname{arcsen} x) = \frac{1}{\sqrt{1-x^2}}$$

$$D(\operatorname{arcsen}(f(x))) = \frac{1}{\sqrt{1-f^2(x)}} \cdot f'(x)$$

$$D(\operatorname{arcctg} x) = - \frac{1}{1+x^2}$$

$$D(\arccos x) = - \frac{1}{\sqrt{1-x^2}}$$

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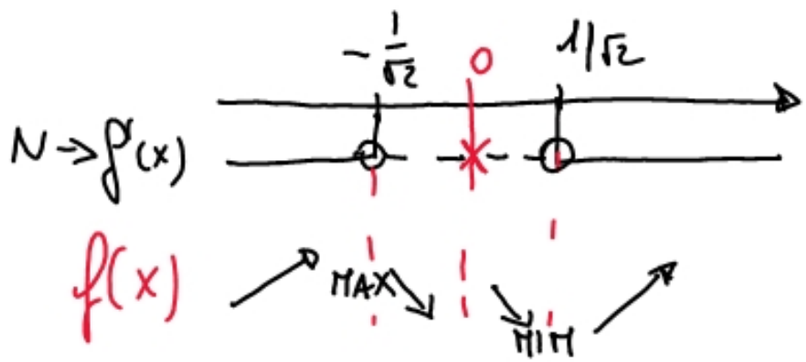
$$f(x) = \frac{2x^2-1}{\underset{+}{x^2}\underset{+}{(1+x^2)}} \qquad x \neq 0$$

Studio del segno considero solo il numeratore

$$2x^2-1 \geq 0 \qquad x \leq -\frac{1}{\sqrt{2}} \quad \cup \quad x \geq \frac{1}{\sqrt{2}}$$



0 = nulla

x = non esiste

—— = positive

- - - = negative

$$D\left(f(x)^{g(x)}\right) = D\left[e^{\log\left(f(x)\right)^{g(x)}}\right] = D\left(e^{g(x)\cdot\log f(x)}\right) =$$

$$= e^{g(x)\cdot\log(f(x))}\cdot\underbrace{\left(g'(x)\cdot\log\left(f(x)\right) + g(x)\cdot\frac{1}{f(x)}\cdot f'(x)\right)}_{\text{derivate dell' esponente}} =$$

$$f(x)^{g(x)}$$

$$D\left(f(x)\cdot g(x)\right) = f'(x)\,g(x) + g'(x)\,f(x)$$

$$D\left(\log x\right) = \frac{1}{x} \qquad D\left(\log\left(f(x)\right)\right) = \frac{1}{f(x)}\cdot f'(x)$$

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$$f(x) = 5\,\frac{\log x}{\underbrace{1+\log^2 x}_{>0}} + 3\,\mathrm{arctg}\,(\log x) \qquad \text{dom} \quad x>0$$

$$f'(x) = 5\,\frac{\frac{1}{x}\left(1+\log^2 x\right) - \log x\left(\frac{2}{x}\log x\right)}{\left(1+\log^2 x\right)^2} + 3\,\frac{1}{1+\log^2 x}\cdot\frac{1}{x}$$

$$D\left(\log^2 x\right) = 2\log x\cdot\frac{1}{x}$$

$$D\left(\mathrm{arctg}\,x\right) = \frac{1}{1+x^2}$$

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$$= 5\,\frac{1}{x}\,\frac{1+\log^2 x - 2\log^2 x}{(1+\log^2 x)^2} + 3\cdot\frac{1}{x}\cdot\frac{1}{1+\log^2 x} = =$$

$$= \frac{1}{x}\cdot\frac{1}{1+\log^2 x}\left(5\,\frac{1-\log^2 x}{1+\log^2 x} + 3\right) =$$

$$= \frac{1}{x(1+\log^2 x)}\cdot\frac{5 - 5\log^2 x + 3 + 3\log^2 x}{1+\log^2 x} =$$

$$= \frac{1}{x(\underset{+}{1+\log^2 x})}\cdot\frac{8 - 2\log^2 x}{\underset{+}{1+\log^2 x}}$$

Studio del segno:

$(8 - 2\log^2 x) \geq 0$

$x > 0$

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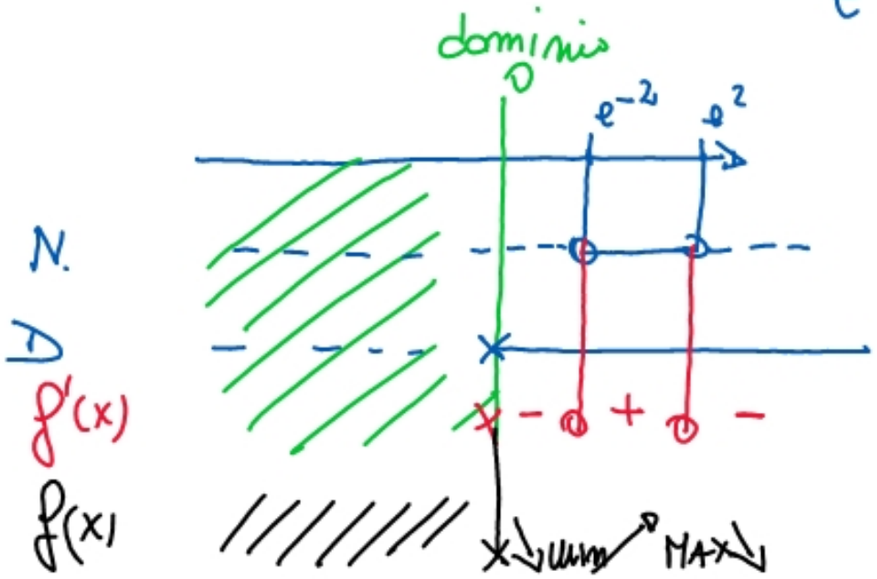
STUDIO DEL SEGNO:

N: $8 - 2\log^2 x \geq 0$ $\quad 4 - \log^2 x \geq 0$ $\quad x > 0$

D: $x > 0$

$-2 \leq \log x \leq 2$

$e^{-2} \leq x \leq e^{2}$



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Derivate delle funzioni trigonometriche "dirette"

$y = \operatorname{sen} x$ $\qquad D(\operatorname{sen} x) = \cos x$

$y = \cos x$ $\qquad D(\cos x) = -\operatorname{sen} x$

$D(\operatorname{tg} x) = 1 + \operatorname{tg}^2 x = \dfrac{1}{\cos^2 x}$

$D(\operatorname{ctg} x) = -\dfrac{1}{\operatorname{sen}^2 x} = -(1 + \operatorname{ctg}^2 x)$

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$$f(x) = \frac{x}{2} + \operatorname{arctg} \frac{1}{x+2} =$$

dom $x \neq -2$

$$= \frac{x}{2} + \operatorname{arctg}\left[(x+2)^{-1}\right]$$

$$\frac{1}{x+2} = (x+2)^{-1}$$

$$D\left(\frac{1}{x}\right) \Rightarrow -\frac{1}{x^2}$$

$$D(x+2) = 1$$

$$f'(x) = \frac{1}{2} + \frac{1}{1+\left(\frac{1}{x+2}\right)^2} \cdot \left(-(x+2)^{-2} \cdot 1\right) =$$

$$= \frac{1}{2} + \frac{\cancel{(x+2)^2}}{x^2+4+4x+1} \cdot \left(-\frac{1}{\cancel{(x+2)^2}}\right) = \frac{1}{2} - \frac{1}{x^2+4x+5} =$$

$$= \frac{x^2+4x+5-2}{2(x^2+4x+5)} = \frac{x^2+4x+3}{2(x^2+4x+5)}$$

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$$f(x) = \frac{1}{2x\sqrt{2x-1}} = \frac{1}{2}\, x^{-1}(2x-1)^{-1/2}$$

$$f'(x) = \frac{1}{2}\left\{ -\frac{1}{x^2}(2x-1)^{-1/2} - \frac{1}{\cancel{2}}\frac{1}{(2x-1)^{3/2}}\cdot \cancel{2}\cdot x^{-1} \right\} =$$

$$= \frac{1}{2}\left\{ -\frac{1}{x^2\sqrt{(2x-1)}} - \frac{1}{x\sqrt{2x-1}\,(2x-1)} \right\} =$$

$$\sqrt{(2x-1)^3} = (2x-1)\sqrt{2x-1}$$

$$= -\frac{1}{2x}\left\{ \frac{1}{x\sqrt{2x-1}} + \frac{1}{(2x-1)\sqrt{2x-1}} \right\} =$$

$$= -\frac{1}{2x}\left\{ \frac{2x-1+x}{x\sqrt{2x-1}\,(2x-1)} \right\} = -\frac{1}{2x^2}\,\frac{3x-1}{(2x-1)\sqrt{2x-1}}$$

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$$f(x) = \frac{3}{\sqrt[3]{e^x - 2}} + \log|e^x - 2|$$

$$3\,(e^x - 2)^{-1/3}$$

$$-\frac{1}{3} - 1 = \frac{-1-3}{3} = -4/3$$

$$f'(x) = 3\left(-\frac{1}{3}\right)(e^x - 2)^{-4/3} \cdot e^x + \frac{1}{|e^x - 2|} \cdot e^x \cdot \operatorname{sgn}|e^x - 2| =$$

$$= -\frac{e^x}{(e^x - 2)\sqrt[3]{e^x - 2}} + \frac{1}{|e^x - 2|}\, e^x \operatorname{sgn}(e^x - 2) = \text{----}$$

attenzione $\quad |e^x - 2| \begin{cases} e^x - 2 & e^x \geq 2 \quad x \geq \log 2 \\ -(e^x - 2) & e^x < 2 \quad x < \log 2 \end{cases}$

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$$f(x) = \sqrt{|x-1|} - 3\log\left(1+\sqrt{|x-1|}\right) =$$

$$= \begin{cases} \sqrt{x-1} - 3\log\left(1+\sqrt{x-1}\right) & \text{per } x \geq 1 \\ \sqrt{1-x} - 3\log\left(1+\sqrt{1-x}\right) & \text{per } x < 1 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{1}{2\sqrt{x-1}} - \dfrac{3}{1+\sqrt{x-1}} \cdot \dfrac{1}{2\sqrt{x-1}} & \text{per } x > 1 \\ \dfrac{1}{2\sqrt{1-x}}(-1) - \dfrac{3}{1+\sqrt{1-x}} \cdot \dfrac{1}{2\sqrt{1-x}}(-1) & \text{per } x < 1 \end{cases}$$

$x = 1$ punto di non derivabilità

si può procedere nei calcoli per lo studio del segno di $f'(x)$

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