Le zone 21

$$\lim_{x\to 0^+} \frac{\log(1+x) - \operatorname{sen} x + \frac{x^2}{2}}{x^3} = \frac{0}{0}$$

$$= \lim_{x\to 0^+} \frac{\overbrace{\left[x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)\right]}^{\log(1+x)} - \overbrace{\left[x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^6)\right]}^{\operatorname{sen} x} + \frac{x^2}{2}}{x^3} =$$

$$= \lim_{x\to 0^+} \frac{\frac{x^3}{3} + \frac{x^3}{6} + o(x^3)}{x^3} = \frac{1}{2}$$

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- $$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)=\infty-\infty$$

denominatore comune:

$$=\lim_{x\to 0}\frac{\sin^2 x - x^2}{x^2\sin^2 x}=\frac{0}{0}$$

$$=\lim_{x\to 0}\frac{-\frac{1}{3}x^4+o(x^4)}{x^2(x^2+o(x^2))}=-\frac{1}{3}$$

den: $\sin x = x + o(x) \longrightarrow \sin^2 x = x^2 + o(x^2)$ - - - -

$$\sin^2 x = (\sin x)^2 = \left(x-\frac{x^3}{6}+o(x^4)\right)^2 = x^2+\frac{x^6}{36}+o(x^8)+$$

$$+\,o(x^5)-\frac{1}{3}x^4+o(x^7) \Rightarrow x^2-\frac{1}{3}x^4+o(x^4)$$

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- $$\lim_{x\to 0} \frac{e^{x^2} - 1 - x^2 + \log(1+x^3)}{\operatorname{sen} x - x} =$$

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2} + o(x^4) \qquad (2^a \text{ ordine})$$

$$\log(1+x^3) = x^3 + o(x^3) \qquad (1^a \text{ ordine})$$

$$\operatorname{sen} x = x - \frac{x^3}{6} + o(x^4) \qquad (2^a \text{ ordine})$$

$$= \lim_{x\to 0} \frac{1 + x^2 + \overbrace{\frac{x^4}{2} + o(x^4)}^{\text{TRASC.}} - 1 - x^2 + x^3 + o(x^3)}{x - \frac{x^3}{6} + o(x^4) - x} =$$

$$= \lim_{x\to 0} \frac{x^3 + o(x^3)}{-\frac{x^3}{6} + o(x^4)} = -6$$

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$$\cdot \lim_{x\to 0} \frac{a \log(1+x) - 6x + 3x^2 - 2x^3}{x^4} = a\,?$$

$$= \lim_{x\to 0} \frac{a\left[x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + o(x^4)\right] - 6x + 3x^2 - 2x^3}{x^4} =$$

$$= \lim_{x\to 0} \frac{x\overset{(a-6)}{(a-6)} + x^2\overset{\frac{6-a}{2}}{\left(3-\frac{a}{2}\right)} + x^3\overset{\frac{a-6}{3}}{\left(\frac{a}{3}-2\right)} - \frac{a x^4}{4} + o(x^4)}{x^4} =$$

se $a = 6$ $\quad = \lim_{x\to 0} \frac{-\frac{6}{4}x^4 + o(x^4)}{x^4} = -\frac{3}{2}$

se $a \neq 6$ $\quad = \lim_{x\to 0} (\cdots) = \infty$

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$$\cdot \lim_{x\to 0} \left(\frac{\operatorname{sen} x}{x}\right)^{\frac{1}{x^2}} = 1^{\infty}$$

passaggio all' esponenziale

$$= \lim_{x\to 0} e^{\log\left(\frac{\operatorname{sen} x}{x}\right)^{\frac{1}{x^2}}} = \lim_{x\to 0} e^{\overbrace{\frac{1}{x^2}\cdot\log\left(\frac{\operatorname{sen} x}{x}\right)}^{(*)}} = e^{-1/6}$$

$$(*) = \lim_{x\to 0} \frac{\log\left(\frac{\operatorname{sen} x}{x}\right)}{x^2} = \lim_{x\to 0} \frac{\log\left\{\left[\underbrace{x - \frac{x^3}{6} + o(x^3)}_{x}\right]\right\}}{x^2} =$$

$$= \lim_{x\to 0} \frac{\log\left(\overbrace{1 - \frac{x^2}{6} + o(x^2)}^{1+t}\right)}{x^2} = \lim_{x\to 0} \frac{-\frac{x^2}{6} + o(x^2)}{x^2} = -\frac{1}{6}$$

Il log viene sviluppato al 1° ordine

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$$\cdot \lim_{x \to +\infty} \frac{3x \cosh\left(\frac{7}{x}\right)\left(\operatorname{senh}\frac{7}{x} - \sin\frac{7}{x}\right)}{e^{7/x} - 1 - \log\left(1+\frac{7}{x}\right)}$$

$$t = \frac{1}{x} \to 0$$

$$= \lim_{t \to 0} \frac{3}{t} \frac{\cosh(7t)\left(\operatorname{senh}(7t) - \operatorname{sen}(7t)\right)}{\underbrace{e^{7t} - 1 - \log(1+7t)}} =$$

$$\cosh(7t) = 1 + \frac{t^2}{2} + o(t^2) = 1 + \frac{(7t)^2}{2} + o((7t)^2) =$$

$$= 1 + \frac{49}{2} t^2 + o(t^2)$$

$$\operatorname{senh}(7x) = 7t + \frac{7^3 t^3}{6} + o(t^3)$$

$$\operatorname{sen}(7t) = 7t - \frac{7^3 t^3}{6} + o(t^3)$$

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$$e^{7t} = 1 + 7t + \frac{(7t)^2}{2} + \frac{(7t)^3}{6} + \frac{(7t)^4}{24} + o(t^4)$$

$$\log(1+7t) = 7t - \frac{(7t)^2}{2} + \frac{(7t)^3}{3} - \frac{(7t)^4}{4} + o(t^4)$$

sostituendo:

$$= 3 \lim_{t \to 0} \frac{\left(1 + \frac{49t^2}{2} + o(t^2)\right)\left(7t + \frac{7^3t^3}{6} + o(t^3) - 7t + \frac{7^3t^3}{6} + o(t^3)\right)}{t\left(1 + 7t + \frac{49t^2}{2} + \frac{7^3t^3}{6} + \frac{7^4t^4}{24} + o(t^4) - 1 - 7t + \frac{49t^2}{2} - \frac{7^3t^3}{3} + \frac{7^4t^4}{4} + o(t^4)\right)} =$$

$$= 3 \lim_{t \to 0} \frac{\left(1 + \frac{49}{2}t^2 + o(t^2)\right)\left(\frac{2}{6}7^3t^3 + o(t^3)\right)}{t\left(49t^2 - \frac{1}{6}7^3t^3 \ldots\ldots\ldots\right)}$$

3° grado

TRASCURABILE

$$= 3 \lim_{t \to 0} \frac{\frac{1}{3} 7^3 t^3 + o(t^3) \quad - \text{———}}{7^2 t^3 + o(t^3)} = 7$$

$$3 \cdot \frac{1}{3} \cdot \frac{7^3 \; t^3}{7^2 \; t^3} = 7$$

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$$\cdot \lim_{x \to 0} \frac{4 \log\left(1 + \frac{\sin(x^2) - x^2}{x^2}\right)}{1 - 2x^2 - \cos(2x)} =$$

$$\frac{\sin(x^2) - x^2}{x^2} = \frac{x^2 - x^6/6 + o(x^6) - x^2}{x^2} = -\frac{x^4}{6} + o(x^4)$$

NUM $\log\left(1 + \frac{\sin x^2 - x^2}{x^2}\right) = \log\left(1 + \left(-\frac{x^4}{6} + o(x^4)\right)\right) = -\frac{x^4}{6} + o(x^4)$

DEN $\cos 2x = 1 - \frac{\cancel{4}^{2} x^2}{\cancel{2}_{1}} + \frac{\cancel{16}^{2} x^4}{\cancel{24}_{3}} + o(x^4) =$

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$$= \lim_{x \to 0} \frac{4\left(-\frac{x^4}{6} + o(x^4)\right)}{\cancel{1} - \cancel{2x^2} \cancel{-1} + \cancel{2x^2} - \frac{2}{3}x^4 + o(x^4)} = 1$$

$$-\frac{4}{6}\left(-\frac{3}{2}\right) = 1$$

- ATTENZIONE ALL'ORDINE DI SVILUPPO
- CONFRONTARE SEMPRE IL GRADO DEL POLINOMIO AL NUMERATORE ED AL DENOMINATORE

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$$\cdot \lim_{x \to 0} \frac{\sin(\log(1+7x)) - e^{7x} + 1}{\operatorname{arctg} x^2} = \frac{0}{0}$$

sviluppando:

NUM

$$\log(1+7x) = 7x - \frac{(7x)^2}{2} + o((7x)^2) = 7x - \frac{49x^2}{2} + o(x^2)$$

$$\sin\left(\log(1+7x)\right) = \sin\left(7x - \frac{49}{2}x^2 + o(x^2)\right) = 7x - \frac{49}{2}x^2 + o(x^2)$$

$$e^{7x} = 1 + 7x + \frac{(7x)^2}{2} + o(7x)^2 = 1 + 7x + \frac{49}{2}x^2 + o(t^2)$$

DEN

$$\operatorname{arctg}(x^2) = x^2 + o(x^2) \qquad (1^\circ \text{ ordine})$$

$$\left[t - \frac{t^3}{3} + \frac{t^5}{5} - \cdots\cdots\right]$$

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$$\lim_{x \to 0} \frac{7x - \frac{49}{2}x^2 + o(x^2) - 1 - 7x - \frac{49}{2}x^2 + o(x^2) + 1}{x^2 + o(x^2)} =$$

$$= \lim_{x \to 0} \frac{-49x^2 + o(x^2)}{x^2 + o(x^2)} = -49$$

PARI GRADO TRA NUMERATORE E DENOMINATORE

CONFRONTO DEI COEFFICIENTI

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CONVERGENZA DELLE SERIE DI TAYLOR

- $\sum_{n=1}^{+\infty} \left(e^{1/n^2} - 1\right)$

ATTENZIONE
$\sum \frac{1}{n^2}$ è l'armonica generalizzata $\alpha > 1 \Rightarrow$ CONV.

$a_n = e^{\frac{1}{n^2}} - 1 > 0 \quad \forall n \geq 1$ T.P.

(considero lo sviluppo in $x=0$ di e^x:

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$$

$$a_n = e^{\frac{1}{n^2}} - 1 \underset{\substack{\text{sviluppo} \\ x \to \frac{1}{n^2}}}{=} 1 + \frac{1}{n^2} + o\left(\frac{1}{n^2}\right) - 1 \Rightarrow \frac{1}{n^2} + o\left(\frac{1}{n^2}\right)$$

$$\lim_{n \to +\infty} \frac{a_n}{1/n^2} = \lim_{n \to +\infty} \frac{e^{\frac{1}{n^2}} - 1}{1/n^2} \overset{\text{sv.}}{=} \lim_{n \to +\infty} \frac{\frac{1}{n^2} + o\left(\frac{1}{n^2}\right)}{1/n^2} = 1$$

$\leftarrow \sum$ conv.

CONFRONTO ASINTOTICO

ATTENZIONE

CAMBIO DI VARIABILE

$$\lim_{n\to+\infty} \frac{e^{\frac{1}{n^2}}-1}{\frac{1}{n^2}} = \lim_{T\to 0} \frac{e^{T^2}-1}{T^2} = \lim_{\text{ST. } T\to 0} \frac{T^2+o(T^2)}{T^2} = 1$$

$$\frac{1}{n} = T$$

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