# FUNZIONI (2)

ES. 1

dato la funzione $f(x) = x^2+2x-2$, risolvere la disequazione

$$f\left(\sqrt{x+1}\right) > 1$$

$$\left(\sqrt{x+1}\right)^2 + 2\sqrt{x+1} - 2 > 1$$

$x+1 \geq 0$

$x \geq -1$

pongo $t = \sqrt{x+1}$

$$t^2 + 2t - 3 > 0$$

$$(t+3)(t-1) > 0 \qquad t < -3 \cup t > 1$$

Sostituendo

$\sqrt{x+1} < -3$

MAI

$\sqrt{x+1} > 1$

$$\begin{cases} x > 0 \\ x \geq -1 \end{cases} \Rightarrow \underline{\underline{x > 0}}$$

ES 2

date le funzioni

$$y = f(x) = 4x + 3 \qquad \text{dom} \quad R \to R$$

$$y = g(x) = 2x - 1 \qquad R \to R$$

determinare l'espressione analitica di $y = f(g(x))$

$$y = f(g(x)) = 4(2x-1) + 3 = 8x - 1$$

Risolvere poi

$$f(g(x)) > f(x)$$

$$8x - 1 > 4x + 3$$

$$4x > 4 \qquad \Rightarrow \qquad x > 1$$

ES 3

Si considerino le funzioni $f: x \to \frac{2x-5}{x+1}$

$g: x \to 2x+1$

a) verificare che $f$ è una corrispondenza biunivoca tra

$$\mathbb{R} \setminus \{-1\} \to \mathbb{R} \setminus \{2\}$$

dom $f$: $y = \frac{2x-5}{x+1}$ $\quad x \neq -1$ $\quad$ dom $f = \mathbb{R} \setminus \{-1\}$

cod $f$: $y = \frac{2x-5}{x+1}$

$xy + y = 2x - 5$

$xy - 2x = -y - 5$

$x(y-2) = -y-5$ $\quad y \neq 2$

$x = \frac{y+5}{2-y}$

cod $f = \mathbb{R} \setminus \{2\}$

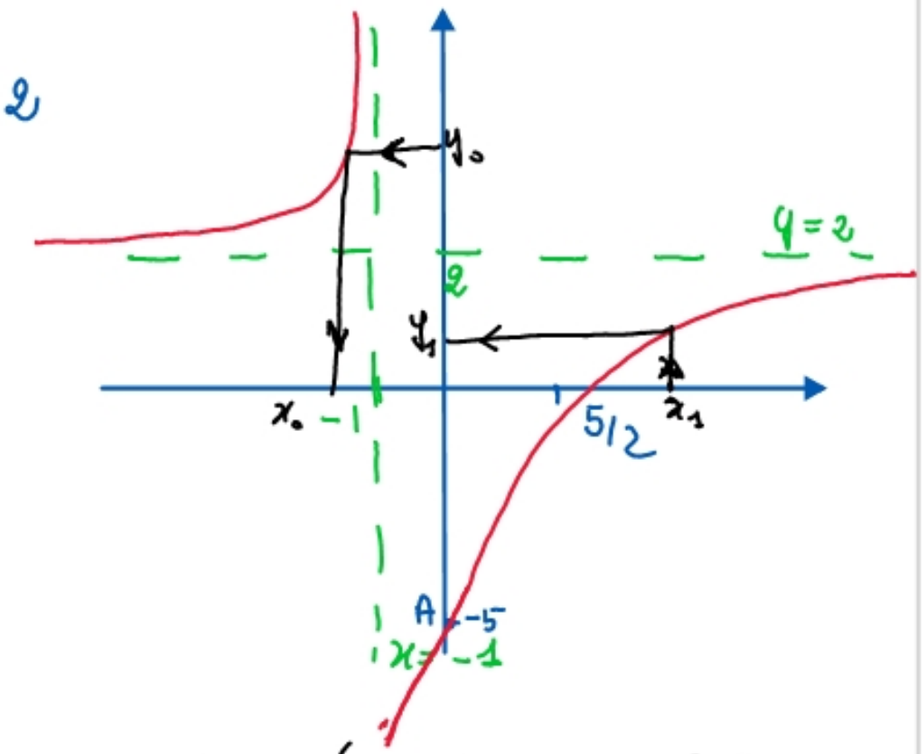
$$y = \frac{2x-5}{x+1}$$

asintoti $\quad x = -1$

$y = 2$

$$A \begin{cases} x = 0 \\ y = -5 \end{cases}$$

$$\begin{cases} x = 0 \\ y = 5/2 \end{cases}$$




N.B. $\quad y = \dfrac{ax+b}{cx+d}$ $\quad$ funzione omografica (iperbole equilatera con asintoti paralleli agli assi coordinati)

$(c \neq 0)$ $\qquad x = -\dfrac{d}{c} \qquad y = \dfrac{a}{c}$

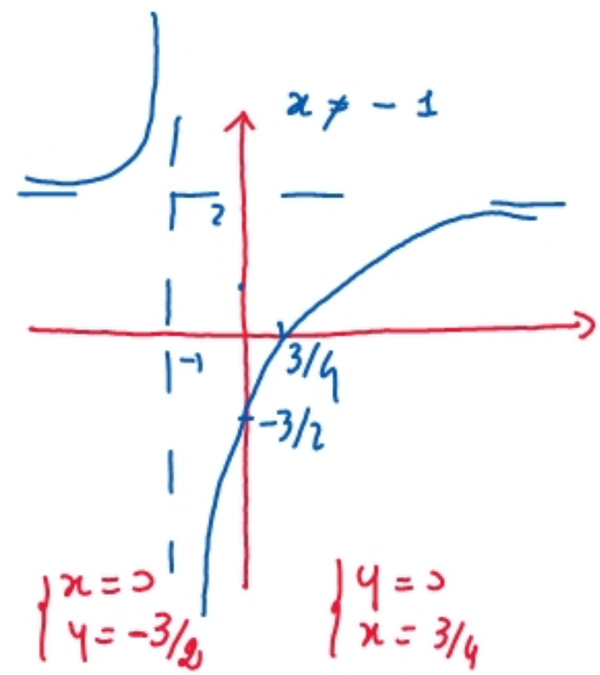
b) determinare la composta

$h = g \circ f$

$$y = f(g(x)) = \frac{2(2x+1) - 5}{(2x+1) + 1} =$$

$$= \frac{4x - 3}{2(x+1)}$$

DISEGNARE $y = \dfrac{4x-3}{2(x+1)}$




FUNZIONE OMOGRAFICA

$x = -1$
$y = 2$

$\begin{cases} x = 0 \\ y = -3/2 \end{cases}$

$\begin{cases} y = 0 \\ x = 3/4 \end{cases}$

$$y = g(f(x)) = 2\left(\frac{2x-5}{x+1}\right) + 1 =$$

$$= \frac{4x - 10 + 1}{x+1} = \frac{4x-9}{x+1}$$

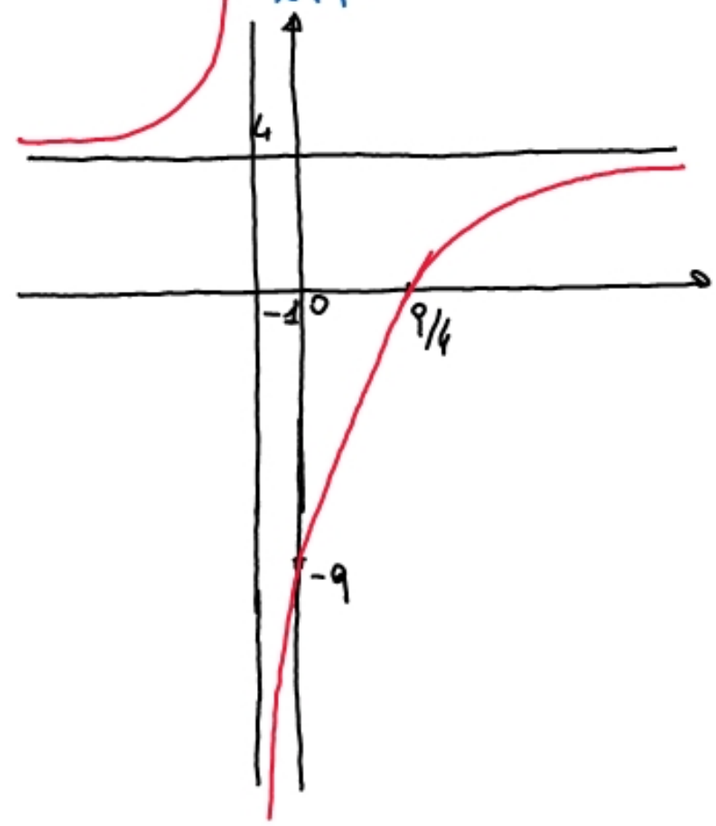
FUNZIONE OMOGRAFICA

asintoti $x=-1$

$y=4$

$\begin{cases} x=0 \\ y=-9 \end{cases}$ $\begin{cases} y=0 \\ x=9/4 \end{cases}$

DISEGNARE




come trasformata si:

Raddoppia $f(x)$ $[2 \cdot f(x)]$

si trasla verso l'alto di $(+1)$

c) Risolvere la disequazione :

$$h(|x|) > 1$$

$$\frac{4|x| - 3}{2(|x|+1)} > 1$$

$$\frac{4|x| - 3 - 2|x| - 2}{2(|x| + 1)} > 0$$

$$\frac{2|x| - 5}{2(|x| + 1)} > 0 \Rightarrow 2|x| - 5 > 0 \qquad |x| > 5/2$$

$$\boxed{x < -5/2 \quad \cup \quad x > 5/2}$$

$+$ Purché $|x| \neq -1$ (sempre)

Risolvere la disequazione

$$f(|x-1|) < 1$$

$$\frac{2|x-1| - 5}{|x-1| + 1} < 1$$

$$\frac{2|x-1| - 5 - |x-1| - 1}{|x-1| + 1} < 0$$

Denominatore

$|x-1|+1 \neq 0 \quad \forall x$

$(|x-1| \neq -1)$

$\Rightarrow$ la disequazione è soddisfatta, ponendo

la condizione solo al numeratore:

$$|x-1| - 6 < 0 \quad \Rightarrow \quad |x-1| < 6$$

$-6 < x-1 < 6$

$-5 < x < 7$

ES 4

Per quali valori del parametro k la funzione

$$y = f(x) = \frac{3k + x}{5} - \frac{2k + x}{2}$$

assume in $x = 2$ valore $\left(-\frac{9}{5}\right)$

(passa per il punto di coordinate $\left(2, -\frac{9}{5}\right)$

il punto di coordinate $\left(2, -\frac{9}{5}\right)$ appartiene alla funzione)

Sostituisco le coordinate $\left(2, -\frac{9}{5}\right)$ nella espressione analitica della funzione:

$$f(2) = -\frac{9}{5}$$

$$\frac{3k + 2}{5} - \frac{2k + 2}{2} = -\frac{9}{5}$$

$$6k + 4 - 10k - 10 = -18$$

$$-4k = -12 \Rightarrow k = 3$$

ES 5

Per quali valori di $x$ le due funzioni:

$$f(x) = \frac{1}{3}(x+1)^2 - \frac{x^2+3}{3} - 1$$

$$g(x) = (2x-1)^2 - 4(x-2)^2$$

hanno le stesse immagine (stesso ordinato, si intersecano

$\Rightarrow$ Risolvere il sistema ... )

$x$: $f(x) = g(x)$

$$\frac{1}{3}(x+1)^2 - \frac{x^2+3}{3} - 1 = (2x-1)^2 - 4(x-2)^2$$

$$\cancel{x^2} + 1 + 2x - \cancel{x^2} - 3 - 3 = 3(4x^2 + 1 - 4x) - 12(x^2 + 4 - 4x)$$

$$2x - 5 = \cancel{12x^2} + 3 - 12x \cancel{- 12x^2} - 48 + 48x$$

$$-34x = -40 \qquad x = 20/17$$

ES 6

Si consideri la funzione

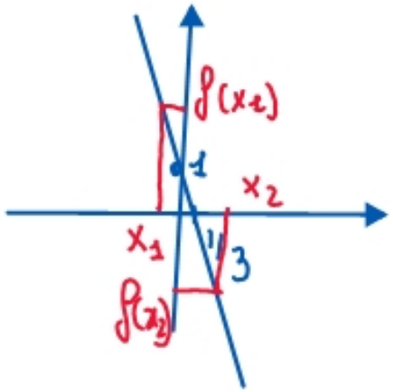
$$f: x \to 1-3x$$

Si verifichi che è biunivoca e decrescente.

Detta $y = f^{-1}(x) = g(x)$ l'espressione analitica della funzione inversa, risolvere la seguente disequazione

$$\frac{f(|x|) + f(2x) + 6x}{3(g|x|) + 4|x|} > 1$$

biunivoca e decrescente




$y = 1 - 3x$

$\forall\ x_1, x_2 \in \text{dom } f$ con $x_1 \leq x_2 \Rightarrow f(x_1) \geq f(x_2)$

$g(x) = f^{-1}(x)$ (inversa se monotona)

$y = 1 - 3x$

$$\begin{cases} x = \dfrac{1-y}{3} \\ x = y \end{cases}$$ TRASFORMATA

$$y = g(x) = f^{-1}(x) = \frac{1-x}{3} = \frac{1}{3} - \frac{1}{3}x$$

Calcolare:

$f(|x|) = 1 - 3|x|$

$f(2x) = 1 - 6x$

$g|x| = \frac{1}{3} - \frac{1}{3}|x|$

$\Rightarrow$ la disequazione diventa

$$\frac{1 - 3|x| + 1 - \cancel{6x} + \cancel{6x}}{\cancel{3}\left(\frac{1}{\cancel{3}} - \frac{1}{\cancel{3}}|x|\right) + 4|x|} > 1$$

$$\frac{2 - 3|x|}{1 - |x| + 4|x|} > 1$$

$[1 + 3|x| > 0 \quad \forall x \in \mathbb{R}]$

$$\frac{2 - 3|x| - 1 - 3|x|}{1 + 3|x|} > 0$$

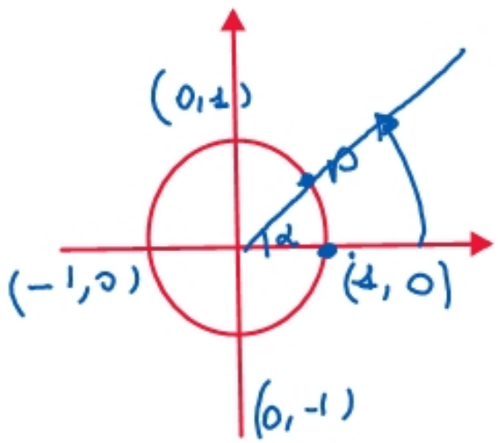
$$\frac{1 - 6|x|}{\underset{+}{1 + 3|x|}} > 0 \quad \text{se} \quad 1 - 6|x| > 0$$

$6|x| > 1$

$6x < -1 \quad \cup \quad 6x > 1$

$x < -\frac{1}{6} \quad \cup \quad x > 1/6$

# FUNZIONI GONIOMETRICHE




## CIRCONFERENZA GONIOMETRICA

$x^2 + y^2 = 1$ $\quad (0,0)$ centro

$r = 1$

$P(x, y) \equiv (\cos\alpha, \operatorname{sen}\alpha)$

$$\begin{cases} x = \cos\alpha \\ y = \operatorname{sen}\alpha \end{cases}$$

$R = 1$

## RELAZIONE FONDAMENTALE

$x^2 + y^2 = 1$

$$\boxed{\cos^2\alpha + \operatorname{sen}^2\alpha = 1}$$

GLI OPERATORI GONIOMETRICI

$-1 \leq \operatorname{sen} \alpha \leq 1$ sono limitati

$-1 \leq \cos \alpha \leq 1$

LE FUNZIONI GONIOMETRICHE

$y = \operatorname{sen} x$ $\forall x \in \mathbb{R}$ <u>sono limitate</u>

$\operatorname{cod} f = [-1, 1]$

$y = \cos x$ sono periodiche

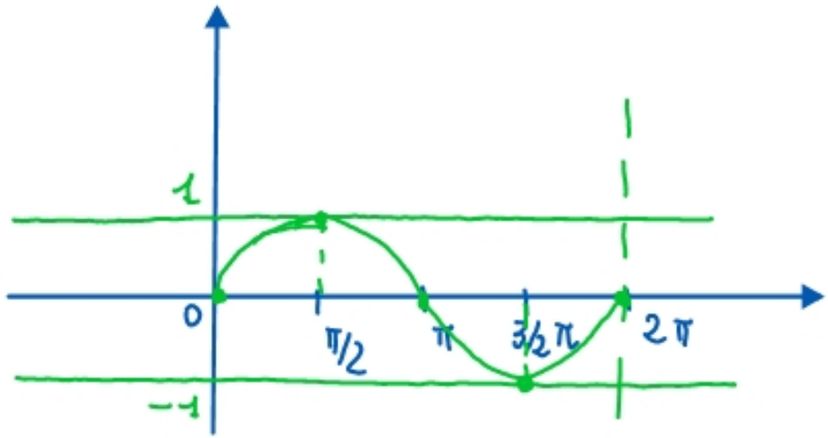
$T = 2\pi$

LE FORMULE DEVONO ESSERE NOTE!

$$\operatorname{sen} 2x = 2 \operatorname{sen} x \cos x$$

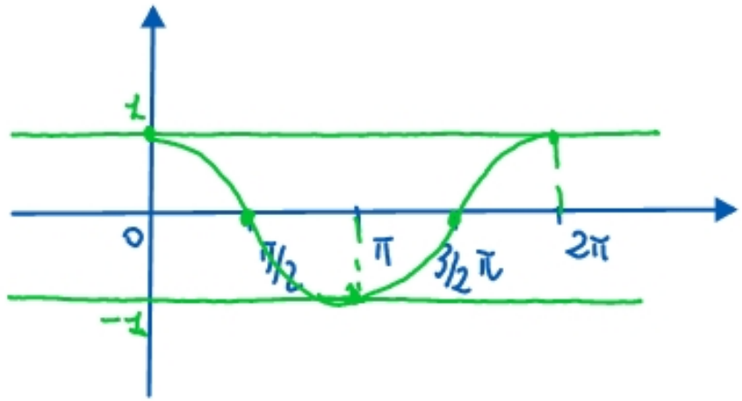
$$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha \cos\beta + \cos\alpha \operatorname{sen}\beta$$

$y = \operatorname{sen} x$




$[0, 2\pi]$

$y = \cos x$




$[0, 2\pi]$

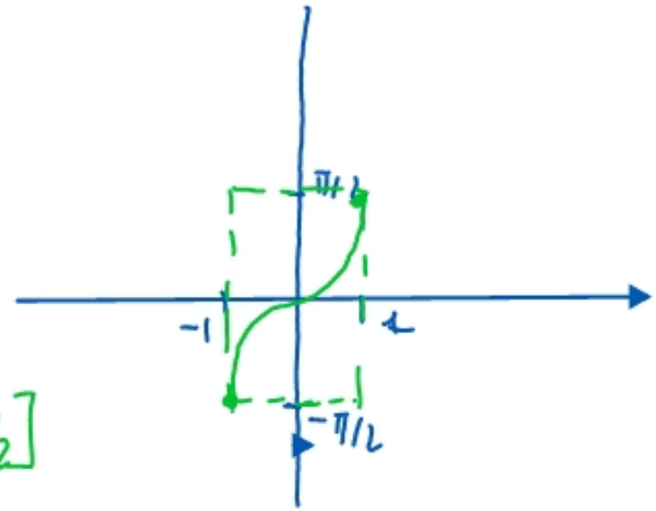
# FUNZIONI GONIOMETRICHE INVERSE

[SIMMETRIA BISETTRICE 1° - 3° quadrante]

$y = \arcsin x$

dom $\quad x \in [-1, 1]$

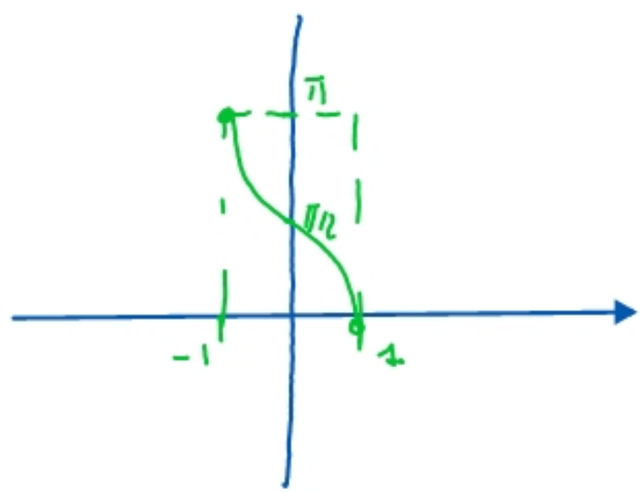
cod $\quad y \in [-\pi/2, \pi/2]$




$y = \arccos x$

dom $\quad x \in [-1, 1]$

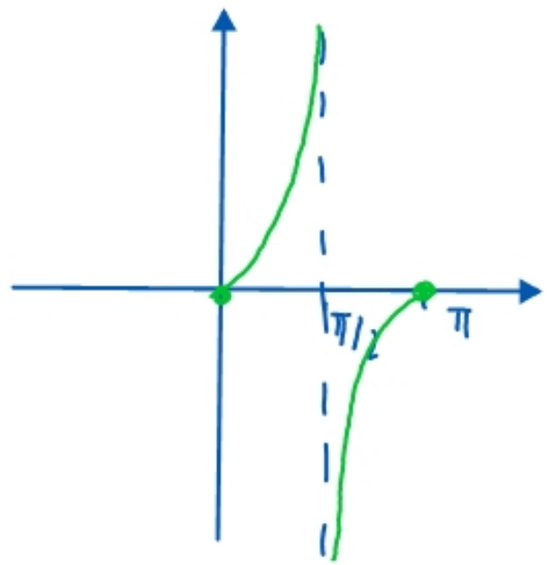
cod. $\quad y \in [0, \pi]$



$\operatorname{Tg} x = \dfrac{\operatorname{sen} x}{\cos x}$

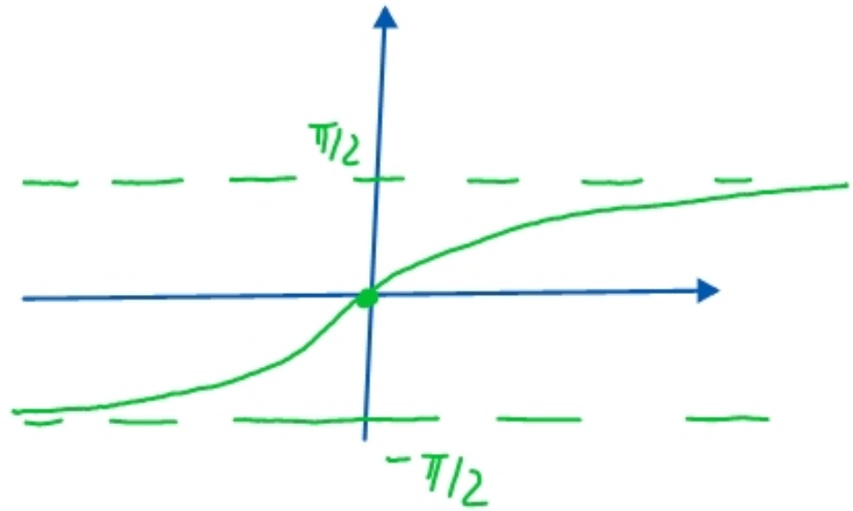
$\operatorname{dom} f \Rightarrow \cos x \neq 0 \qquad x \neq \frac{\pi}{2} + k\pi$

$y = \operatorname{Tg} x$




- ✓ illimitata
- crescente
- ✓ periodica $(\pi)$

$y = \operatorname{arctg} x$




crescente

limitata

asintoti orizzontali $y = -\pi/2$

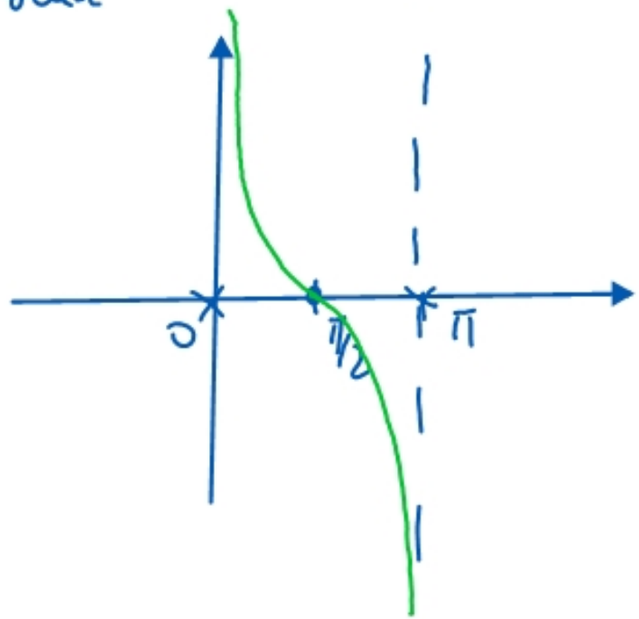
$y = \pi/2$

dom $f$ : $\mathbb{R}$

cod $f$ : $]-\pi/2, \pi/2[$

$y = \operatorname{ctg} x = \operatorname{Tg}^{-1} x = \dfrac{\cos x}{\operatorname{sen} x}$ $\qquad x \neq k\pi$

$y = \operatorname{ctg} x$

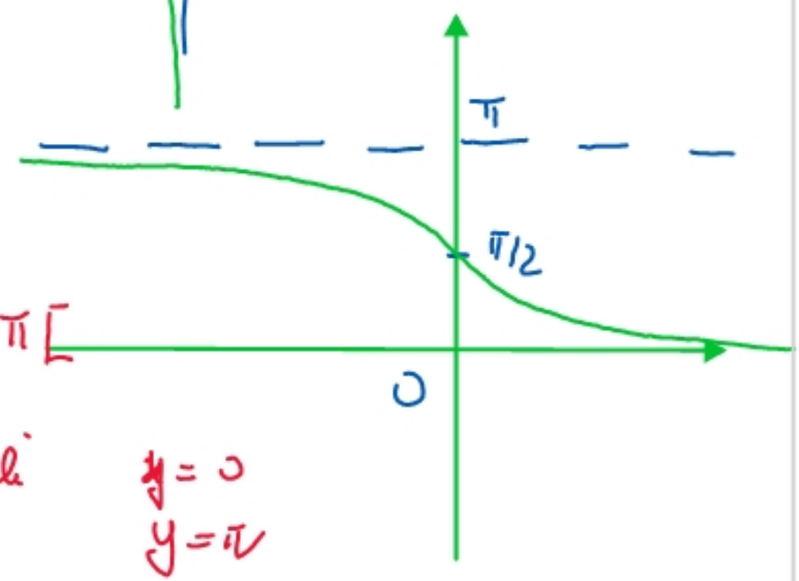



decrescente

illimitata

periodica $(\pi)$

$y = \operatorname{arcctg} x$




limitata $]0, \pi[$

positiva

asintoti orizzontali $y = 0$, $y = \pi$

$\operatorname{dom} f = \mathbb{R}$