

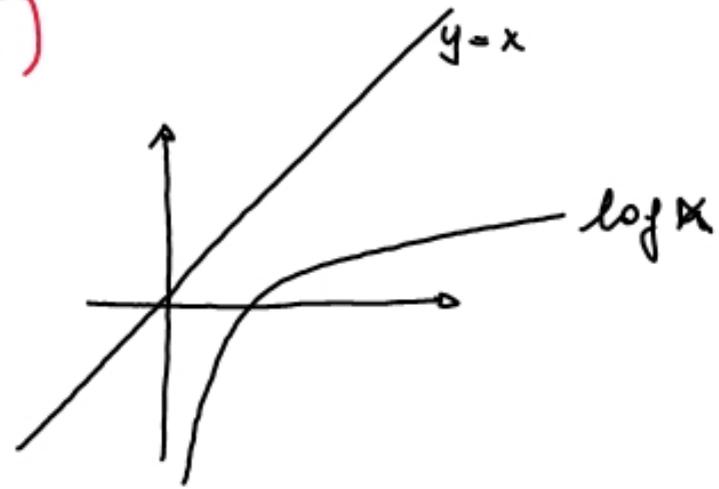
Lezione 10: Successioni

ES: $\lim_{h \rightarrow +\infty} \frac{[(h+7)! - h!] e^{\frac{\log h}{n}}}{(e^{\frac{1}{4} + 1}) (h^{\text{d} - 1}) (h! - \sqrt[3]{n})} = \text{el valore di } \alpha \in \mathbb{R}$

$(e^{\frac{1}{4} + 1})$
 $\frac{1}{2}$

$e^{\frac{\log h}{n} \rightarrow 0} \rightarrow e^0 = 1$

$e^{\frac{1}{4} + 1} \rightarrow e^0 + 1 = 1 + 1 = 2$



$\frac{1}{2} \lim_{h \rightarrow +\infty} \frac{(h+7)! \left[1 - \frac{h!}{(h+7)!} \right]}{(h^{\text{d} - 1}) n! \left(1 - \frac{\sqrt[3]{n}}{n!} \right)} =$
 $\rightarrow 0$ prevale $n!$ su $\sqrt[3]{n}$

7 prodotti di binomi

$$\frac{(n+7)!}{n!} = \frac{(n+7)(n+6)(n+5)(n+4)(n+3)(n+2)(n+1) \cancel{n!}}{\cancel{n!}} =$$

$$\approx n^7 \quad \text{COMPORTEMENTO ASINTOTICO}$$

$$\frac{n!}{(n+7)!} = \dots \approx \frac{1}{n^7} \quad \text{comp. asint.}$$

$$\approx \frac{1}{2} \lim_{n \rightarrow +\infty} \frac{n^7}{(n^{\alpha-1})} \left[1 - \frac{1}{n^{\alpha}} \right] \approx \frac{n^7}{n^{\alpha}}$$

se $\alpha > 7 \rightarrow 0$
 $\alpha = 7 \rightarrow \frac{1}{2}$
 $\alpha < 7 \rightarrow +\infty$

prevale il denominatore
 pari grado NUM e DEN
 prevale il grado del NUM

ES

$$\lim_{h \rightarrow +\infty} \frac{\left(e^{\frac{1}{2^{h!}}} - 1 \right) \left((h+1)! + 2^h \right) h^h}{(h+1)^{h+1}} =$$

$$= \lim_{h \rightarrow +\infty} \frac{e^{\frac{1}{2^{h!}}} - 1}{\frac{1}{2^{h!}} \cdot 2^{h!}} \cdot \frac{h^h}{(h+1)^h (h+1)} \cdot ((h+1)!) \left[1 + \frac{2^h}{(h+1)!} \right]$$

$$2^{n!} = 2(n!)$$

$$(2h)! = 2h(2h-1) \dots \\ \dots (2h-h+1) h!$$

$$\left(\frac{h}{h+1} \right)^h = \left(\frac{h+1}{h} \right)^{-h} = \left[\left(1 + \frac{1}{h} \right)^h \right]^{-1} \sim e^{-1}$$

$$\frac{2^h}{(h+1)!} = \frac{2^h}{(h+1)(h!)} \sim 0$$

$$= \lim_{n \rightarrow \infty} e^{-1} \frac{(n+1)!}{2(n!)^{n+1}} =$$

$$= \lim_{n \rightarrow \infty} e^{-1} \frac{\cancel{(n+1)} \cancel{n}!}{2 \cancel{(n+1)} \cancel{n}!} = \frac{1}{2} e^{-1}$$

$$= \lim_{n \rightarrow \infty} e^{-1} \frac{(n+1)!}{2(n+1)!} = \frac{1}{2} e^{-1}$$

ES: $\lim_{h \rightarrow +\infty} \frac{2^h + h^{h+1}}{(h-1)^{7h} - h!} \quad \text{Sim} \frac{2}{h} =$

$= \lim_{h \rightarrow +\infty} \frac{h^{h+1} \left(\frac{2^h}{h^{h+1}} + 1 \right)}{(h-1)^{7h} \left(1 - \frac{h!}{(h-1)^{7h}} \right)}$

$\frac{\lim \frac{2}{h}}{\frac{2}{h} \cdot \frac{h}{2}} =$

$\frac{2^h}{h^{h+1}} = \frac{2^h}{h^h \cdot h} = \left(\frac{2}{h} \right)^h \cdot \frac{1}{h} \rightarrow 0$

esprimendo con base variabile

$a_n = \frac{2}{h^2}$

$a_1 = 2$
 $a_2 = \left(\frac{2}{2^2} \right)^2$

base < 1 $n > 2$

$$= \lim_{n \rightarrow +\infty} \frac{n^{7n+1}}{(n-1)^{7n}} \cdot \frac{2}{n} =$$

$$= \lim_{n \rightarrow +\infty} \frac{n^{7n}}{(n-1)^{7n}} \cdot \frac{\cancel{n}}{\cancel{n}} \cdot 2 =$$

$$= 2 \lim_{n \rightarrow +\infty} \left(\left(\frac{n}{n-1} \right)^n \right)^7 = 2 e^7$$

$$\left(\frac{n-1}{n} \right)^{-n} = \left(1 - \frac{1}{n} \right)^{-n} \rightarrow e$$

ES

$$\lim_{n \rightarrow +\infty} n^{2n} \left(1 + \frac{7}{n}\right)^n \sin(n^{-n}) \frac{1}{\sqrt{n^3 + n^{2n}} - \sqrt{n^3}} =$$

$$= \lim_{n \rightarrow +\infty} n^{2n} \left[\left(1 + \frac{7}{n}\right)^{\frac{n}{2}} \right]^2 \frac{\sin \frac{1}{n^n}}{\frac{1}{n^n} \cdot n^n} \frac{\sqrt{n^3 + n^{2n}} + \sqrt{n^3}}{\cancel{n^3} + n^{2n} - \cancel{n^3}} =$$

$\downarrow e^7$ $\downarrow 1$

$$= e^7 \lim_{n \rightarrow +\infty} \frac{\cancel{n^{2n}}}{\cancel{n^n}} \frac{\cancel{n^n} \left(\sqrt{\frac{n^3}{n^{2n}} + 1} + \sqrt{\frac{n^3}{n^{2n}}} \right)}{\cancel{n^{2n}}} =$$

NPO NPO

$$= e^7$$

$\sqrt{n^{2n}} = n^n$

ES

$$\lim_{n \rightarrow +\infty} \frac{n^n + 7n \log n + n \sin n}{(n+2)^n + n \log \frac{1}{n} + n \sin \frac{1}{n}} =$$

$$= \lim_{n \rightarrow +\infty} \frac{n^n}{(n+2)^n} \cdot \frac{1 + \frac{7n \log n}{n^n} + \frac{n \sin n}{n^n}}{1 + \frac{n \log \frac{1}{n}}{(n+2)^n} + \frac{n \sin \frac{1}{n}}{(n+2)^n}} =$$

$$\frac{7n \log n}{n^n} = \frac{7 \log n}{n^{n-1}} = \frac{7 \log n}{n} \cdot \frac{1}{n^{n-2}} \rightarrow 0$$

$$\frac{n \sin n}{n^n} = \frac{\sin n}{n^{n-1}} \approx \frac{[-1, 1]}{n^{n-1}} \rightarrow 0$$

$$-1 \leq \sin n \leq 1$$

Teo. CARABINIERI

$$0 < n \cdot \frac{1}{n^{n-1}} \leq \frac{\sin n}{n^{n-1}} \leq \frac{1}{n^{n-1}} \rightarrow 0$$

$$\frac{n \log \frac{1}{n}}{(n+2)^n} = \frac{n \log n^{-1}}{(n+2)^n} = -\frac{n \log n}{(n+2)^n} =$$

$$= -\frac{n}{(n+2)^{n-1}} \cdot \frac{\log n}{n+2} \approx -\frac{n}{(n+2)^{n-1}} \cdot \frac{\log n}{n}$$

$\downarrow \approx 0$ $\downarrow \approx 0$

$$\frac{n \sin \frac{1}{n}}{(n+2)^n} \approx \frac{1}{(n+2)^n} \rightarrow 0$$

$$n \sin \frac{1}{n} = \frac{\sin \frac{1}{n}}{\frac{1}{n}} \rightarrow 1$$

$$= \lim_{n \rightarrow +\infty} \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow +\infty} \left(\frac{n+2}{n} \right)^{-n} = \lim_{n \rightarrow +\infty} \left[\left(1 + \frac{2}{n} \right)^{\frac{n}{2}} \right]^{-2} = e^{-2}$$

ES:

$$\lim_{n \rightarrow \infty} \sqrt[n]{(1 - \cos \pi n) 3^n + 7^n} =$$

$$= \lim_{n \rightarrow \infty} \sqrt[n]{7^n} \cdot \left[(1 - \cos \pi n) \frac{3^n}{7^n} + 1 \right] =$$

$$= 7 \lim_{n \rightarrow \infty} \sqrt[n]{1 + (1 - \cos \pi n) \frac{3^n}{7^n}} =$$

$$= 7 \lim_{n \rightarrow \infty} \left[1 + (1 - \cos \pi n) \frac{3^n}{7^n} \right]^{\frac{1}{n}} = 7$$

$\underbrace{\hspace{10em}}_{\approx 1}$

$$(1 - \cos \pi n) \left(\frac{3}{7}\right)^n \approx 0$$

$$(1 - \cos \pi n) = \{0, 2\}$$

$$n = 2m$$

$$1 + (1 - \cos \pi n) \left(\frac{3}{7}\right)^n \rightarrow 1 + 0 \left(\frac{3}{7}\right)^n = 1$$

$$n = 2m+1$$

$$1 + (1 - \cos \pi n) \left(\frac{3}{7}\right)^n \rightarrow 1 + 2 \cdot \underbrace{\left(\frac{3}{7}\right)^n}_{> 0} \approx 1$$

NB che la base valga \pm (per n pari) o \pm tende a \pm (per n dispari)
 elevandola allo zero \Rightarrow vale sempre \pm

ES paraggio all'espressione per le basi naturali:

$$= \lim_{n \rightarrow \infty} \left[(1 - \cos \pi n) \left(\frac{3}{7} \right)^n + 1 \right]^{\frac{1}{n}} =$$

$$= \lim_{n \rightarrow \infty} e^{\frac{\log \left\{ (1 - \cos \pi n) \left(\frac{3}{7} \right)^n + 1 \right\}}{n}} =$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \log \left\{ (1 - \cos \pi n) \left(\frac{3}{7} \right)^n + 1 \right\}$$

$$= e$$

ES

 $\lim_{h \rightarrow +\infty}$

$$\frac{\sqrt{h^4 + \frac{14}{h^2}} - h^2}{\log\left(1 + \frac{7}{h^2}\right)^{8n}} \cdot h^3$$

$$= \lim_{h \rightarrow +\infty} \frac{\cancel{h^4} + \frac{14}{h^2} - \cancel{h^4}}{\sqrt{h^4 + \frac{14}{h^2}} + h^2} \cdot \frac{h^3}{\log\left[\left(1 + \frac{7}{h^2}\right)^{\frac{7}{h^2}}\right]^{\frac{7}{h^2} \cdot 8n}}$$

$$= \lim_{h \rightarrow +\infty} \frac{14}{h^2} \cdot \frac{h^3}{h^2(\sqrt{1 + \frac{14}{h^4}})} \cdot \frac{1}{\frac{7}{h^2} \cdot 8n}$$

$$= \lim_{h \rightarrow +\infty} \frac{14}{h^2} \cdot \frac{h^3}{2h^2} \cdot \frac{h^2}{56n} = \frac{14}{2 \cdot 56 \cdot 4} \frac{h^5}{h^5} = \frac{1}{8}$$