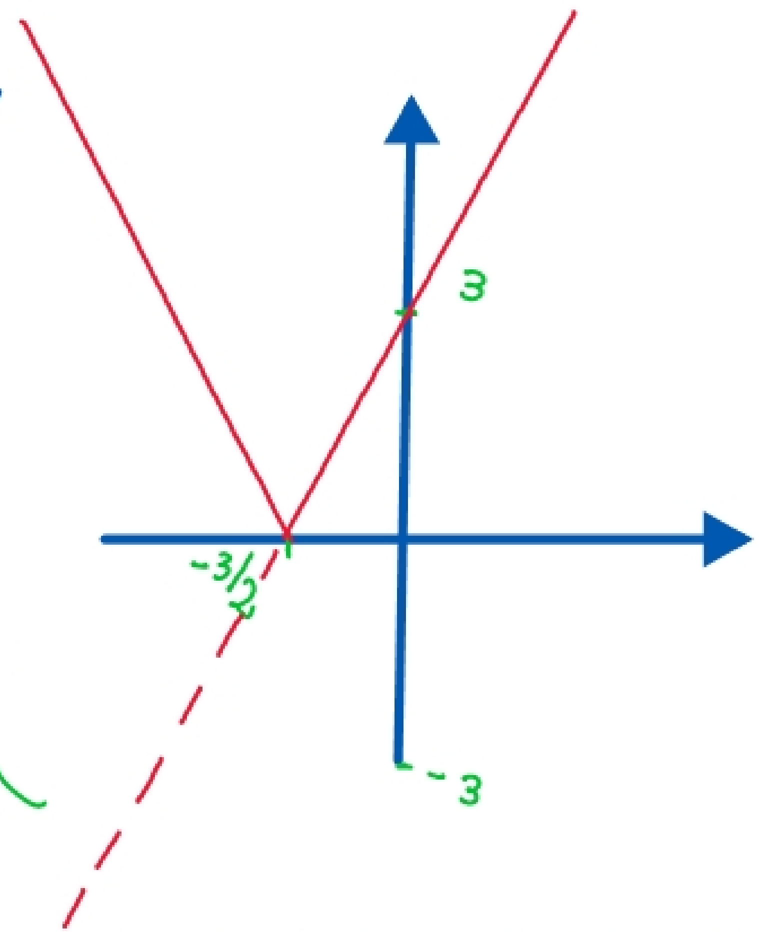
# Lezione 18: derivazione

$$y = |2x+3| = \begin{cases} 2x+3 & x \geq -3/2 \\ -(2x+3) & x < -3/2 \end{cases} \qquad (\text{Arg} \geq 0)$$




$$\begin{cases} x = -3/2 \\ y = 0 \end{cases}$$

$x_0 = -3/2$ la $f(x)$ è continua

derivabilità in $-3/2$

si deve calcolare il limite

del rapporto incrementale a sinistra e a destra di $x_0 = -3/2$

$$\lim_{x \to -\frac{3}{2}^-} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to -3/2^-} \frac{-(2x+3) - 0}{x + 3/2} = \underline{-2}$$

$x_0$

$$\lim_{x \to -\frac{3}{2}^+} \frac{2x+3-0}{x + 3/2} = \underline{2}$$

$f'_-(x_0) \neq f'_+(x_0) \Rightarrow$ la $f(x)$ non è derivabile

in $x_0 = -3/2$

$$y = |2x+3|$$

$$y' = 2 \quad \mathrm{sgn}\,(2x+3)$$

$$y = |2x+3| = \begin{cases} 2x+3 & x \geq -3/2 \\ -2x-3 & x < -3/2 \end{cases}$$

$$y' = \begin{cases} 2 & x \geq -\frac{3}{3} \\ -2 & x < -3/2 \end{cases}$$

$$D(x^n) = n\ x^{n-1}$$

$$D[f(x)]^n = n[f(x)]^{n-1} \cdot D[f(x)]$$

$$D[k] = 0$$

$$D[k f(x)] = k\ D[f(x)]$$

$$y = \sqrt{1-3x^2} = (1-3x^2)^{\frac{1}{2}}$$

$$D(\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

$$D(\sqrt{f(x)}) = \frac{1}{2\sqrt{f(x)}} \cdot D[f(x)]$$

$$y' = \frac{1}{2}(1-3x^2)^{-\frac{1}{2}} \underbrace{(-6x)}_{D(1-3x^2)} =$$

$$= \frac{-6x}{2\sqrt{1-3x^2}}$$

Dom $f$:

$$1-3x^2 \geq 0$$

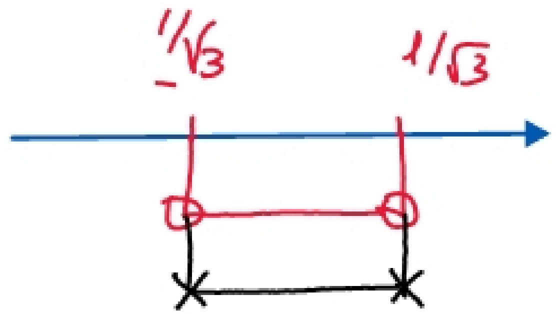
$$-\frac{1}{\sqrt{3}} \leq x \leq \frac{1}{\sqrt{3}}$$

Dom $f'$:

$$1-3x^2 > 0$$

$$-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

domini : $f(x)$ $f'(x)$

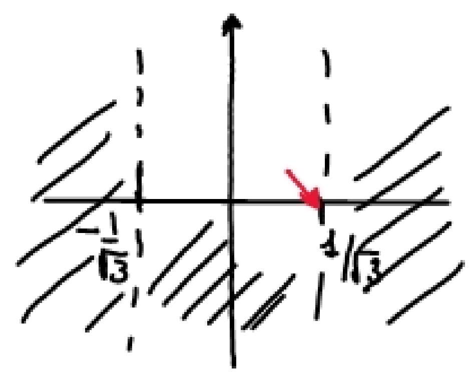


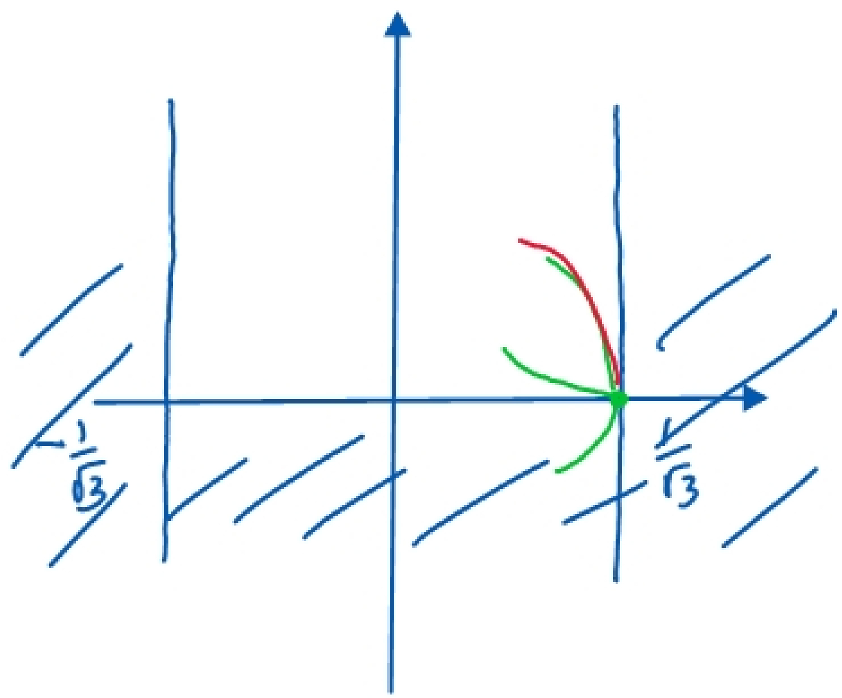

$\text{dom } f' \subset \text{dom } f$ cioè è restrittivo (una restrizione) rispetto al $\text{dom } f$

$\Rightarrow$ vi sono dei punti dove esiste la funzione ma non è derivabile

$$\lim_{x \to \frac{1}{\sqrt{3}}^-} \frac{-6x}{2\sqrt{1-3x^2}} = \frac{-\frac{6}{\sqrt{3}}}{2\sqrt{0}} = +\infty$$

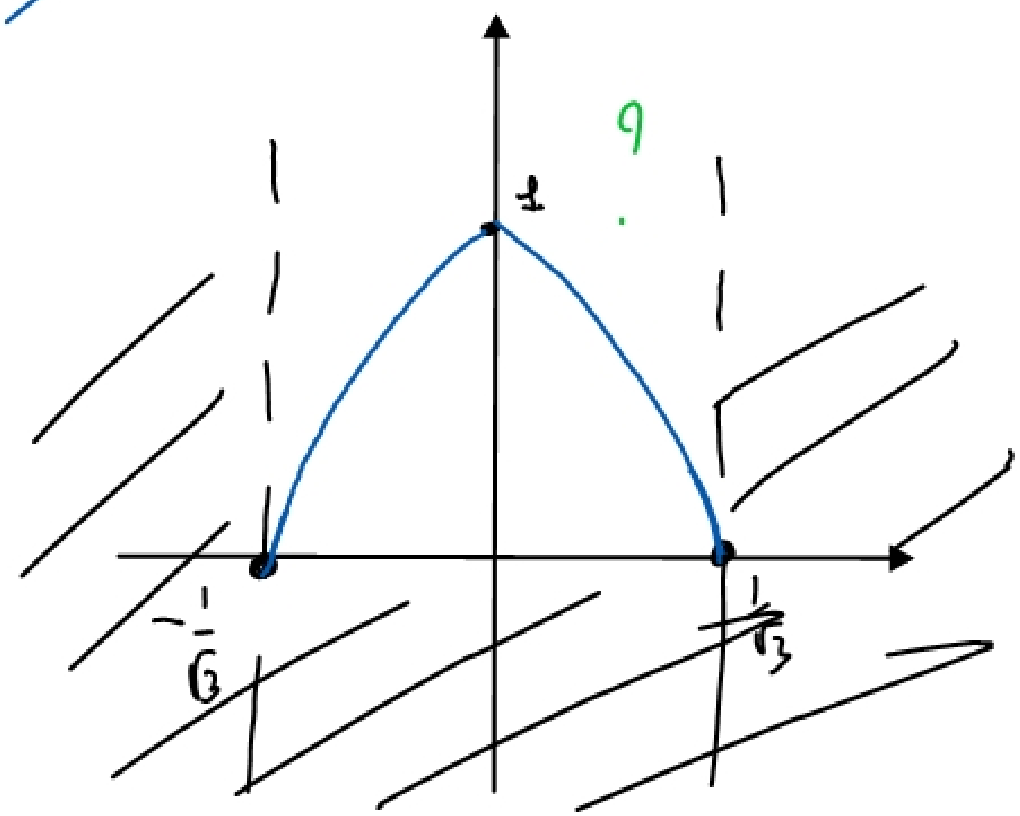
$$\lim_{x \to -\frac{1}{\sqrt{3}}^+} \frac{-6x}{2\sqrt{1-3x^2}} = \frac{+6/\sqrt{3}}{2\sqrt{0}} = +\infty$$






$f(0) = 1$

$x = \pm \frac{1}{\sqrt{3}}$ punti a tangente verticale



• $y = \text{sen}^2(2x) + \cos\left(\frac{x}{\pi}\right) \Rightarrow$

$$y' = \underbrace{2\,\text{sen}(2x)}_{D(t^n)} \cdot \underbrace{\cos(2x)}_{D(\text{sen}(t))} \cdot \underbrace{2}_{D(t)} + \left(-\text{sen}\frac{x}{\pi}\right) \cdot \frac{1}{\pi} =$$

$$= 4\,\text{sen}(2x)\cos(2x) - \frac{1}{\pi}\,\text{sen}\frac{x}{\pi}$$

$D(\cos x) = -\sin x$

$D(\text{sen}\, x) = \cos x$

$D(\text{tg}\, x) =$ (*)

$D(\text{ctg}\, x) =$

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$$D(\text{tg}\, x) = D\left(\frac{\text{sen}\, x}{\cos x}\right) =$$

$$= \frac{\cos x \cdot \cos x - (-\text{sen}\, x) \cdot \text{sen}\, x}{\cos^2 x} =$$

$$= \frac{\cos^2 x + \text{sen}^2 x}{\cos^2 x} = \begin{cases} \dfrac{1}{\cos^2 x} \\ 1 + \text{tg}^2 x \end{cases} \quad (*)$$

$$D\left[f(x) \cdot g(x)\right] =$$

$$= f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$D\left[\frac{f(x)}{g(x)}\right] =$$

$$= \frac{f'(x) \cdot g(x) - g'(x)\, f(x)}{g^2(x)}$$

$$y = \log\left\{\operatorname{arctg}\left[9x^3 \cdot \left|\frac{x+2}{x-1}\right|\right]\right\}$$

$$D(\operatorname{arctg} x) = \frac{1}{1+x^2}$$

dom: $\operatorname{arctg}\left(9x^3 \cdot \left|\frac{x+2}{x-1}\right|\right) > 0$

$$D(\log x) = \frac{1}{x}$$

$$\underbrace{9x^3 \left|\frac{x+2}{x-1}\right|}_{\substack{x \neq 1 \\ x \neq -2 \\ +}} > 0 \quad \Rightarrow x > 0, \ x \neq 1$$

$$D(e^x) = e^x$$

$$D(a^x) = a^x \log a$$

$$y' = \underbrace{\frac{1}{\operatorname{arctg}\left[9x^3 \cdot \left|\frac{x+2}{x-1}\right|\right]}}_{D(\log \dots)} \cdot \underbrace{\frac{1}{1+\left(9x^3 \left|\frac{x+2}{x-1}\right|\right)^2}}_{D(\text{argomento } \log)} \cdot \underbrace{9\left\{3x^2 \left|\frac{x+2}{x-1}\right| + x^3 \operatorname{sgn}\left|\frac{x+2}{x-1}\right| \frac{x-1-x-2}{(x-1)^2}\right\}}_{\substack{\text{prodotto} \\ D(\text{argomento del arctg})}}$$

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• $y = [\cos(3x)]^{2x^2-3x} =$

passaggio all'esponenziale

$$= e^{\log(\cos 3x)^{2x^2-3x}} =$$

$$= e^{(2x^2-3x)\cdot\log(\cos 3x)}$$

$$y' = e^{(2x^2-3x)\cdot\log(\cos 3x)} \cdot \left( (4x-3)\log(\cos 3x) + (2x^2-3x)\cdot\frac{1}{\cos 3x}\cdot(-\sin 3x)\cdot 3 \right) =$$

$$= (\cos 3x)^{2x^2-3x}\left( (4x-3)\log(\cos 3x) - 3\,(2x^2-3x)\cdot \operatorname{tg} 3x \right)$$

dominio =

$$\begin{cases} \cos(3x) > 0 \\ \cos(3x) \neq 1 \end{cases}$$

$$\begin{cases} -\frac{\pi}{2} + 2k\pi < 3x < \frac{\pi}{2} + 2k\pi \\ 3x \neq 2k\pi \end{cases}$$

$$\begin{cases} -\frac{\pi}{6} + \frac{2}{3}k\pi < x < \frac{\pi}{6} + \frac{2}{3}k\pi \\ x \neq \frac{2}{3}k\pi \end{cases}$$

$$D(\cosh x) = D\left(\frac{e^x+e^{-x}}{2}\right) = \frac{1}{2}\left(e^x - e^{-x}\right) = \operatorname{senh} x$$

$$D(\operatorname{senh} x) = D\left(\frac{e^x - e^{-x}}{2}\right) = \frac{1}{2}\left(e^x + e^{-x}\right) = \cosh x$$

$$D(\operatorname{ctg} x) = D\left(\frac{\cos x}{\operatorname{sen} x}\right) = \frac{-\operatorname{sen}^2 x - \cos^2 x}{\operatorname{sen}^2 x} =$$

$$= \begin{cases} -\dfrac{1}{\operatorname{sen}^2 x} \\ -1 - \operatorname{ctg}^2 x \end{cases}$$

• $y = \sqrt{1+2x^2}$ calcolare la retta tangente nel punto $x=2$

1. $x=2$ appartiene al dominio di $f$

2. $m_{x=2} = f'(x=2) = \left[\dfrac{1}{2\sqrt{1+2x^2}}(4x)\right]_{x=2} =$

$$= \frac{1}{\cancel{2}_1\sqrt{1+8}}\,\cancel{8}^4 = \frac{4}{3}$$

3. eq. retta : $\quad y - \overset{f(x_0)}{f(x=2)} = \overset{f'(x_0)}{m_{x=2}}\,(x - \overset{x_0}{2})$

$$y - 3 = \frac{4}{3}(x-2) \qquad y = \frac{4}{3}x - \frac{8}{3} + 3 =$$

$$= \frac{4}{3}x + \frac{1}{3}$$