Lezione 17: limiti - continuità

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} = \frac{0}{0} = F.I.$$

$$= \lim_{x\to 0} \frac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{x\,(\sqrt{1+x}+1)} =$$

$$= \lim_{x\to 0} \frac{\cancel{1}+x-\cancel{1}}{x\,(\underset{\downarrow\,1}{\sqrt{1+x}}+1)} = \frac{1}{2}$$

$$\lim_{x\to +\infty} \left(1+\frac{1}{x}\right)^{x+5} = 1^{\infty} = F.I.$$

$$= \lim_{x\to +\infty} \underbrace{\left(1+\frac{1}{x}\right)^{x}}_{e} \underbrace{\left(1+\frac{1}{x}\right)^{5}}_{\downarrow\; 1^5=1} = e$$

$$\lim_{x \to 1} \frac{(\arccos x)^4}{2\,(1-x)^3} = \frac{0}{0} = F.I$$

sostituzione di variabile

$$y = \arccos x \qquad y \to \arccos 1 = 0$$

$$\Downarrow$$

$$x = \cos y \qquad x \to 1$$

$$= \lim_{y \to 0} \frac{y^4}{2\,(1-\cos y)^3} = \lim_{y \to 0} \frac{1}{2}\; \underbrace{\frac{y^2}{1-\cos y}}_{2} \cdot \underbrace{\frac{y^2}{1-\cos y}}_{2} \cdot \underbrace{\frac{1}{1-\cos y}}_{\to \infty} = \infty$$

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \lim_{x \to 0} \frac{(1-\cos x)\,(1+\cos x)}{x^2\,(1+\cos x)} =$$

$$\left(\frac{\operatorname{sen} x}{x}\right)^2 \to 1$$

$$= \lim_{x \to 0} \frac{1-\cos^2 x}{x^2\,(1+\cos x)} = \lim_{x \to 0} \frac{\operatorname{sen}^2 x}{x^2} \cdot \frac{1}{1+\cos x} = \frac{1}{2}$$

• $\lim_{x\to 0^+} \frac{\overbrace{(1+3x)^{\log x}}^{1^{-\infty}} - 1}{\underbrace{\operatorname{sen} x}_{\to 0} \quad \underbrace{\log x^3}_{\to -\infty}} =$ F. I

passaggio all'esponenziale :

$$= \lim_{x\to 0^+} \frac{e^{\log(1+2x)^{\log x}} - 1}{\operatorname{sen} x \cdot 3 \log x} =$$

$$= \lim_{x\to 0^+} \frac{1}{3} \frac{e^{\log x \cdot \log(1+3x)} - 1}{\operatorname{sen} x \quad \log x} =$$

$$\lim_{t\to 0} \frac{e^t - 1}{t} = 1$$

$$= \frac{1}{3} \lim_{x\to 0^+} \underbrace{\boxed{\frac{e^{\log x \cdot \log(1+2x)} - 1}{\log x \cdot \log(1+2x)}}}_{\to 1} \cdot \log(1+2x) \, \frac{1}{\underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1} \cdot x} =$$

$$= \frac{1}{3} \lim_{x \to 0^+} \underbrace{\frac{\log(1+2x)}{2x}}_{\to 1} \cdot 2 = \frac{2}{3}$$

(*)

$$\lim_{t \to +\infty} \left(1 + \frac{1}{t}\right)^t = e$$

$$\lim_{t \to +\infty} \log\left[\left(1 + \frac{1}{t}\right)^t\right] = \log e = 1$$

(*) $\quad t = \frac{1}{x} \rightsquigarrow 0$

∞

$$1 = \lim_{x \to 0} \log(1+x)^{\frac{1}{x}} = \lim_{x \to 0} \frac{1}{x} \log(1+x) = \lim_{x \to 0} \frac{\log(1+x)}{x}$$

Tema d'esame 3-02-09

$$\lim_{x \to \frac{\pi}{2}} \left[1+ \sin\left(\frac{\pi}{2}-x\right)\right]^{\frac{2}{\sin\left(\frac{\pi}{2}-x\right)}} \cdot \log\left[3\,\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)^2}\right] =$$

$$= \lim_{t \to +\infty} \left(1+\frac{1}{t}\right)^{2t} \cdot \lim_{q \to 0} \log\left[3\,\underbrace{\frac{1-\cos q}{q^2}}_{1/2}\right]$$

$$t = \frac{1}{\underbrace{\sin(\underbrace{\tfrac{\pi}{2}-x}_{\to 0})}_{\to 0}} \to \infty \qquad \qquad \underbrace{\underbrace{x}_{\to \pi/2}-\frac{\pi}{2}}_{\to 0} = q \to 0$$

$$= e^2 \cdot \log\frac{3}{2}$$

- $$\lim_{x\to+\infty} \frac{x\left(x^{\frac{1}{x}}-1\right)}{\log x} = \text{F.I.}$$

$$= \lim_{x\to+\infty} \frac{x}{\log x}\left(e^{\log x^{\frac{1}{x}}}-1\right) =$$

$$= \lim_{x\to+\infty} \frac{x}{\log x}\left(e^{\frac{\log x}{x}}-1\right) = \lim_{t\to 0} \frac{e^{\boxed{-t\log t}}-1}{\underbrace{\boxed{-t\log t}}_{\to 0}} = 1$$

cambio: $t = \frac{1}{x} \underset{x\to+\infty}{\Rightarrow} 0$

$$\log x = \log\left(\frac{1}{x}\right)^{-1} = -\log\frac{1}{x} \quad : \quad \log x = -\log t$$

$$\bullet \lim_{x\to 0} \frac{e^{x} - 1 + \operatorname{sen} x}{(1+x)^2 - 1 + \operatorname{Tg} x} = \frac{0}{0} = \text{F.I.}$$

$$= \lim_{x\to 0} \frac{\cancel{x}\left(\overbrace{\frac{e^x - 1}{x}}^{\to 1} + \overbrace{\frac{\operatorname{sen} x}{x}}^{\to 1}\right)}{\cancel{x}\left(\underbrace{\frac{(1+x)^2 - 1}{x}}_{\to 2} + \underbrace{\frac{\operatorname{Tg} x}{x}}_{\to 1}\right)} = \frac{2}{3}$$

$$\lim_{x\to 0} \frac{(1+x)^2 - 1}{x} = \lim_{x\to 0} \frac{(1+x+1)(1+x-1)}{x} =$$

$$\boxed{1}$$

$$\frac{\operatorname{Tg} x}{x} = \frac{\operatorname{sen} x}{\cos x \cdot x} = \frac{\operatorname{sen} x}{x} \cdot \frac{1}{\cos x}$$

$$= \lim_{x\to 0} \frac{x\,(x+2)}{x} = 2$$

$$\frac{(1+x)^2 - 1}{x} = \frac{\cancel{1} + x^2 + 2x - \cancel{1}}{x} = \frac{x\,(x+2)}{x}$$

$$\bullet \lim_{x \to 0^+} \frac{1 - \operatorname{sen} x - \overset{1-\operatorname{sen}^2 x}{\cos^2 x}}{e^{x^2} - 1} =$$

$$1 - \cos^2 x = \operatorname{sen}^2 x$$

$$= \lim_{x \to 0^+} \frac{\operatorname{sen}^2 x - \operatorname{sen} x}{x^2} \cdot \frac{x^2}{e^{x^2} - 1} =$$

$$= \lim_{x \to 0^+} \left(\underbrace{\frac{\operatorname{sen}^2 x}{x^2}}_{1} - \underbrace{\frac{\operatorname{sen} x}{x}}_{\downarrow 1} \cdot \underbrace{\frac{1}{x}}_{+\infty} \right) \cdot \underbrace{\left(\frac{e^{x^2} - 1}{x^2} \right)^{-1}}_{\downarrow 1}$$

$$= -\infty$$

$$\bullet \lim_{x \to 0^+} \frac{\log(1 - \overset{\to 0}{\cos x})}{\log x} = \text{F.I.}$$

$$= \lim_{x \to 0^+} \frac{\log\left[\frac{1-\cos x}{x^2} \cdot x^2\right]}{\log x} =$$

$$= \lim_{x \to 0^+} \left\{ \frac{\log\left(\overset{\to 1/2}{\frac{1-\cos x}{x^2}}\right)}{\underset{\to -\infty}{\log x}} + \frac{\log x^2}{\log x} \right\} =$$

$$\frac{\log x^2}{\log x} = 2\,\frac{\cancel{\log x}}{\cancel{\log x}}$$

$$= 0 + 2 = 2$$

$$\lim_{x \to \frac{\pi}{2}} \underbrace{\operatorname{tg}^2 x}_{\to +\infty} \cdot \underbrace{\log\left(\operatorname{sen}^2 x\right)}_{\to 0} = \text{F.I.}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\operatorname{sen}^2 x}{\cos^2 x} \log\left(1 - \cos^2 x\right) =$$

$$= \lim_{x \to \frac{\pi}{2}} \underbrace{\operatorname{sen}^2 x}_{\to 1} \left(- \frac{\log\left(1 - \cos^2 x\right)}{- \cos^2 x} \right) =$$

$$= -1$$

TEMA D'ESAME (23.03.11)

$$\lim_{x\to+\infty} \frac{\log(2x)}{e^{2x}\left(\sqrt{9e^{4x}+\log x}-3e^{2x}\right)} =$$

$$= \lim_{x\to+\infty} \frac{\log(2x)\left(\sqrt{9e^{4x}+\log x}+3e^{2x}\right)}{e^{2x}\left(\cancel{9e^{4x}}+\log x-\cancel{9e^{4x}}\right)} =$$

$$= \lim_{x\to+\infty} \underbrace{\frac{\log 2+\log x}{\log x}}_{\to 1} \quad \frac{3\cancel{e^{2x}}\overbrace{\left(\sqrt{1+\underbrace{\frac{\log x}{9e^{4x}}}_{\to 0}}+1\right)}^{1+1=2}}{\cancel{e^{2x}}} =$$

$$= 6$$

CONTINUITA'

Teme d'esame 13-05-04

$$f(x) = \begin{cases} \dfrac{x+1}{|x+1|}\left(\dfrac{x+8}{x-7}\right)^{5/3} & x \neq -1, \quad x \neq 7 \\ 1 & x=-1, \quad x=7 \end{cases}$$

$x=-1$

$$\lim_{x \to -1^-} \left\{ \frac{\cancel{x+1}}{-(\cancel{x+1})} \left(\frac{x+8}{x-7}\right)^{\frac{5}{3}} \right\} = -\left(\frac{7}{-8}\right)^{5/3} = -\sqrt[3]{\left(\frac{7}{-8}\right)^5} = +\left(\frac{7}{8}\right)^{5/3}$$

$$\lim_{x \to -1^+} \left\{ \frac{\cancel{x+1}}{\cancel{x+1}} \left(\frac{x+8}{x-7}\right)^{\frac{5}{3}} \right\} = \left(-\frac{7}{8}\right)^{5/3} = -\left(\frac{7}{8}\right)^{5/3}$$

I LIMITI DX E SIN SONO FINITI MA DIVERSI

$\Downarrow$

DISCONTINUITA' A SALTO IN $X=-1$

$x = 7$

$$\lim_{x \to 7^-} \cancel{\frac{x+1}{|x+1|}} \left(\frac{x+8}{x-7} \right)^{5/3} = \left(\frac{15}{0^-} \right)^{5/3} = -\infty$$

$$\frac{x+1}{x+1} = 1$$

$$\lim_{x \to 7^+} \left(\frac{x+8}{x-7} \right)^{5/3} = \left(\frac{15}{0^+} \right)^{5/3} = +\infty$$

TEMA D'ESAME 5-7-04

$$f(x) = \begin{cases} 2\left(x-\frac{\pi}{2}\right) e^{\frac{1}{x-\frac{\pi}{2}}} + 7\,\frac{\operatorname{sen} x}{|x-\pi|} & x \neq \frac{\pi}{2},\ x \neq \pi \\ 2 & x = \frac{\pi}{2},\ x = \pi \end{cases}$$

$x = \frac{\pi}{2}$

$$\lim_{x \to \frac{\pi}{2}} \left\{ 2\left(x-\frac{\pi}{2}\right) e^{\frac{1}{x-\frac{\pi}{2}}} + \underbrace{7\,\frac{\overset{\nearrow +}{\operatorname{sen} x}}{\underset{\searrow \left|-\frac{\pi}{2}\right| = \pi/2}{|x-\pi|}}}_{14/\pi} \right\} =$$

$x - \frac{\pi}{2} = t \to 0$

$x = t + \frac{\pi}{2}$

$$= \lim_{t \to 0} \left\{ \underbrace{2t\, e^{\frac{1}{t}}} + \frac{14}{\pi} \right\}_{+} = +\infty$$

PUNTO DI INFINITO

$\frac{1}{t} = y \to +\infty$ $\quad = \lim_{y \to +\infty} 2\,\frac{e^{y}}{y} = +\infty$

$x = \pi$

$$\lim_{x\to\pi} \left\{ \underbrace{2\left(x-\frac{\pi}{2}\right) e^{\frac{1}{x-\pi/2}}}_{2\left(\pi-\frac{\pi}{2}\right) e^{\frac{1}{\pi-\pi/2}} = \pi\, e^{2/\pi}} + \frac{7 \operatorname{sen} x^{\to 0}}{|x-\pi|_{\to 0}} \right\}$$

$$= \pi \, e^{2/\pi} \quad \pm 1$$

$\to -\operatorname{sen} t$

$$\lim_{x\to\pi} 7 \frac{\operatorname{sen} x}{|x-\pi|} = 7\lim_{t\to 0} \frac{\operatorname{sen}(t+\pi)}{|t|} =$$

$x-\pi = t \to 0$

$$= 7 \lim_{t\to 0} \left(-\frac{\operatorname{sen} t}{|t|}\right) =$$

$$7\lim_{t\to 0^+} \frac{-\operatorname{sen} t}{t} = -1$$

$$7\lim_{t\to 0^-} \frac{-\operatorname{sen} t}{-t} = 1$$

DISCONTINUITA' A SALTO