Lezione 10 : successioni e limiti notevoli

- $$\lim_{n\to+\infty} \frac{2\left((2n)!\right)^{n+1} - 7\left((2(n+1))!\right)^{n}}{2^{n}\cdot n^{n}\cdot\left((2n+1)!\right)^{n}}$$

RICORDARE $2n! =$ il doppio di $n!$

$(2n)! =$ il doppio di n, di cui calcolare il fattoriale

$$2n! = 2\cdot(n)(n-1)\cdot(n-2)\cdot\dots$$

$$(2n)! = 2n\cdot(2n-1)\,(2n-2)\dots\dots\underbrace{(2n-n+1)}_{n+1}\,\underbrace{(2n-n)}_{n}\dots n!$$

A) $$\lim_{n\to+\infty} \frac{2\left((2n)!\right)^{n+1}}{2^n\, n^n \left((2n+1)!\right)^n}$$

$$= \lim_{n\to+\infty} \frac{2\,\left((2n)!\right)^n \cdot (2n)!}{(2n)^n \left((2n+1)!\right)^n} = \lim_{n\to+\infty} \frac{2\cancel{\left((2n)!\right)^n}\,(2n)!}{(2n)^n\,(2n+1)^n \cancel{\left((2n)!\right)^n}} =$$

FRAZ: $$= 2 \underbrace{\overset{\to 1}{\frac{2n}{2n}} \cdot \overset{<1}{\frac{2n-1}{2n}} \cdot \ldots \cdot \overset{<1}{\frac{n+1}{2n}}}_{(2n)\ n\,\text{Termini}} \cdot \frac{n!}{(2n+1)^n} <$$

$$0 < 2\,\frac{n!}{(2n+1)^n} < 2\,\frac{n!}{n^n} \rightsquigarrow 0$$

MA $(2n+1)^n > n^n \Leftarrow 2n+1 > n$

$$\lim_{n\to+\infty} \left[\,-\,-\,-\,\right] = 0$$

$$\lim_{n\to+\infty} \frac{-7\,\left((2(n+1))!\right)^n}{2^n\, n^n \left((2n+1)!\right)^n} =$$

$$\to (2n+2)\,(2n+1)!$$

$$= \lim_{n\to+\infty} \frac{-7\,\left((2n+2)!\right)^n}{(2n)^n \left((2n+1)!\right)^n} =$$

$$= \lim_{n\to+\infty} \frac{-7\,(2n+2)^n \cancel{\left((2n+1)!\right)^n}}{(2n)^n \cancel{\left((2n+1)!\right)^n}} =$$

$$= \lim_{n\to+\infty} -7 \left(\frac{2n+2}{2n}\right)^n = -7 \underbrace{\lim_{n\to+\infty} \left(1+\frac{1}{n}\right)^n}_{e} = -7e$$

LIMITE NOTEVOLE

LIMITI NOTEVOLI

$$\lim_{n \to +\infty} \frac{\operatorname{sen} \frac{1}{n}}{\frac{1}{n}} = \lim_{n \to +\infty} n \operatorname{sen} \frac{1}{n} = 1$$

$$\lim_{n \to +\infty} \frac{\operatorname{sen} a_n}{a_n} = 1 \quad \text{se} \quad a_n \to 0$$

$$\lim_{n \to +\infty} \frac{1 - \cos \frac{1}{n}}{\frac{1}{n}} = \text{F. I.} =$$

$$1 - \cos^2 \frac{1}{n} = \operatorname{sen}^2 \frac{1}{n}$$

$$= \lim_{n \to +\infty} \frac{\overbrace{\left(1 - \cos \frac{1}{n}\right)\left(1 + \cos \frac{1}{n}\right)}^{1 - \cos^2 \frac{1}{n}}}{\frac{1}{n} \underbrace{\left(1 + \underbrace{\cos \frac{1}{n}}_{1}\right)}_{2}} =$$

$$\lim_{n \to +\infty} \frac{\operatorname{sen}^2 \frac{1}{n}}{\frac{1}{n} \cdot 2} =$$

$$= \lim_{n \to +\infty} \underbrace{\boxed{\frac{\operatorname{sen} \frac{1}{n}}{\frac{1}{n}}}}_{1} \cdot \underbrace{\operatorname{sen} \frac{1}{n}}_{0} \cdot \frac{1}{2} = 0$$

$$\Rightarrow \quad \lim_{n \to +\infty} \frac{1 - \cos \frac{1}{n}}{\frac{1}{n^2}} = \lim_{n \to +\infty} \frac{\left(1 - \cos \frac{1}{n}\right)\left(1 + \cos \frac{1}{n}\right)}{\frac{1}{n^2}\left(1 + \cos \frac{1}{n}\right)} =$$

$$= \lim_{n \to +\infty} \boxed{\frac{\operatorname{sen}^2 \frac{1}{n}}{\frac{1}{n^2}}} \cdot \frac{1}{2} = \frac{1}{2}$$

$$\left[\frac{\operatorname{sen} \frac{1}{n}}{\frac{1}{n}}\right]^2 \longrightarrow 1^2 = 1$$

$$\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n}=e$$

$$\lim_{n\to+\infty}\left(1+\frac{1}{a_n}\right)^{a_n}=e \quad \text{se } a_n \to +\infty$$

$$\lim_{n\to+\infty}\left(1+\frac{k}{a_n}\right)^{\frac{a_n}{k}}=e \quad \text{se } a_n \to +\infty \text{ e } k\in\mathbb{R}$$

$$\lim_{n\to+\infty}\left(1+\frac{1}{2n}\right)^{n}=\lim_{n\to+\infty}\left[\left(1+\frac{1}{2n}\right)^{2n}\right]^{1/2}=e^{1/2}$$

$$\frac{2n}{2}=2n\cdot\frac{1}{2}$$

$$\lim_{n\to+\infty}\left(1-\frac{1}{n}\right)^{n}=\lim_{n\to+\infty}\left[\left(1+\frac{1}{-n}\right)^{-n}\right]^{-1}=e^{-1}$$

N.B. $\displaystyle\lim_{\substack{\cancel{n\to\infty} \\ n\to 0}} \left(1+\frac{1}{\lg n}\right)^{\frac{1}{\log n}} = 1^0 = 1$!!

$$\lim_{n\to+\infty} \frac{\log\left(1+\frac{1}{n}\right)}{\frac{1}{n}} = \lim_{n\to+\infty} n \log\left(1+\frac{1}{n}\right) =$$

$$= \lim_{n\to+\infty} \log \underbrace{\left(1+\frac{1}{n}\right)^n}_{e} = \log e = 1$$

$$\lim_{n\to+\infty} \frac{e^{\frac{1}{n}}-1}{1/n} = 1$$

$$\lim_{n\to+\infty} \frac{(n!)^{n-1} - ((n-1)!)^{n}}{[(n-1)! \cdot (n-10)]^{n-1}} = \qquad n! = n \cdot (n-1)!$$

$$= \lim_{n\to+\infty} \frac{\cancel{((n-1)!)^{n-1}}\,[n^{n-1} - (n-1)!]}{\cancel{((n-1)!)^{n-1}}\,(n-10)^{n-1}} =$$

$$= \lim_{n\to+\infty} \frac{n^{n-1}\left[1 - \underbrace{\frac{(n-1)!}{n^{n-1}}}_{\to 0}\right]}{(n-10)^{n-1}} \sim$$

$$a_n = \frac{(n-1)!}{n^{n-1}} \cdot \frac{n}{n} = \frac{n!}{n^n} \to 0$$

$$0 < \frac{n!}{n^n} \to 0 \qquad 0 < \frac{\overbrace{(n-1)}^{m}!}{n^{n-1}} < \frac{m!}{(m+1)^m} < \frac{m!}{m^m} \to 0$$

$m = n-1$

$$= \lim_{n\to+\infty} \frac{n^{n-1}}{(n-10)^{n-1}} = \lim_{n\to+\infty} \frac{1}{\left(\frac{n-10}{n}\right)^{n-1}} =$$

$$\left(\frac{n}{n-10}\right)^{n-1} = \left(\frac{n-10}{n}\right)^{1-n}$$

$$= \lim_{n\to+\infty} \left[\frac{1}{\left(\frac{n-10}{n}\right)^{n}} \cdot \frac{1}{\left(\frac{n-10}{n}\right)^{-1}} \right] = \frac{1}{e^{-10}} \cdot 1 = e^{10}$$

$$\left[1+\frac{1}{t}\right]^{t}:$$

$$\left(\frac{n-10}{n}\right)^{n} = \left[\left(1-\frac{10}{n}\right)^{-\frac{n}{10}}\right]^{-10} \to e^{-10}$$

$$\left(\frac{n-10}{n}\right)^{-1} = \left(1-\frac{10}{n}\right)^{-1} \to (1-0)^{-1} = 1$$

Teme d' esame 6-01-09

$$\lim_{n\to+\infty} \frac{\overbrace{\alpha^{n}}^{\text{esp.}} + \operatorname{arctg} n! + 1 - \cos\frac{1}{n^2}}{7^{\log n} + \underbrace{2^{n}}_{\text{esp.}} + \log n^3} = \qquad \alpha \in \mathbb{R}^+$$

$\operatorname{arctg} n! \to \pi/2$, $\cos\frac{1}{n^2} \to 1$

$$= \lim_{n\to+\infty} \frac{\alpha^n}{2^n} \frac{1 + \frac{\operatorname{arctg} n!}{\alpha^n} + \left\{\frac{1}{\alpha^n} - \frac{\cos\frac{1}{n^2}}{\alpha^n}\right\}}{\frac{7^{\log n}}{2^n} + 1 + 3\frac{\log n}{2^n}}$$

$$0 < \frac{7^{\log n}}{2^n} = \frac{7^{\frac{\log_7 n}{\log_7 e}}}{2^n} = \frac{\left[7^{\log_7 n}\right]^{\frac{1}{\log_7 e}}}{2^n} = \frac{n^{\frac{1}{\log_7 e}}}{2^n} \to 0$$

$\frac{1}{\log_7 e} > 0$

$$\left(\text{cambiamento di base} \quad \log_e n = \frac{\log_7 n}{\log_7 e}\right)$$

$$= \lim_{n\to+\infty} \frac{\alpha^n + \arctan n! \overset{\to\ \pi/2}{}}{2^n} =$$

$$= \lim_{n\to+\infty} \frac{\alpha^n + \pi/2}{2^n} = \lim_{n\to+\infty} \frac{\alpha^n}{2^n} = \lim_{n\to+\infty} \left(\frac{\alpha}{2}\right)^n$$

se $\frac{\alpha}{2} > 1 \Rightarrow \alpha > 2$ esponenziale con base > 1 $\Rightarrow \lim_{n\to+\infty} \left(\frac{\alpha}{2}\right)^n = +\infty$

se $0 < \frac{\alpha}{2} < 1 \Rightarrow 0 < \alpha < 2$: esponenziale con base $< 1 \Rightarrow$

$$\Rightarrow \lim_{n\to+\infty} \left(\frac{\alpha}{2}\right)^n = 0$$