Lezione 23: integrazione

$$\int f(x)\,dx = \underbrace{F(x)}_{} + \underbrace{c}_{}$$

$$D\left(F(x)+c\right) = f(x)$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

f definita, continua

$$\int \sqrt{2x+5}\,dx = \int (2x+5)^{\frac{1}{2}}\,dx = \frac{\frac{3}{2}(2x+5)^{3/2}\cdot\frac{2}{2}}{3/2}$$

$$\longrightarrow \quad D(t^n) = n\,t^{n-1}$$

$$\int t^n\,dt = \frac{1}{n+1}\,t^{n+1} + c$$

$$D\left(\frac{1}{n+1}\,t^{n+1} + c\right) = \frac{n+1}{n+1}\,t^n = t^n$$

$$= \frac{1}{3/2}\,\frac{(\boxed{2x}\boxed{+5})^{3/2}}{2} + c$$

$$D\left[(2x+5)^{\frac{3}{2}}\right] = \frac{3}{2}\cdot(2x+5)^{1/2}\cdot 2$$

$$t = 2x+5$$

$$Dt = 2$$

$$= \frac{(2x+5)^{3/2}}{3} + C$$

pongo $2x+5 = t$

$2\,dx = dt \qquad dx = \frac{dt}{2}$

cambio di variabile

cambio del differenziale

$$= \int (t)^{\frac{1}{2}} \frac{dt}{2} = \frac{t^{3/2}}{3/2} \cdot \frac{1}{2} + C = \frac{t^{3/2}}{3} + C =$$

$$= \frac{(2x+5)^{3/2}}{3} + C = \frac{\sqrt{(2x+5)^3}}{3} + C$$

cambio gli estremi di integrazione (sostituzione)

$$\int_1^3 (2x+5)^{\frac{1}{2}} dx = \int_7^{11} t^{\frac{1}{2}} \frac{dt}{2} = \left| \frac{t^{3/2}}{3} \right|_7^{11} = \frac{11^{3/2} - 7^{3/2}}{3}$$

$x_1 = 1 \qquad t_1 = 2x+5 = 7$

$x_2 = 3 \qquad t_2 = 2x+5 = 11$

$$\int \frac{\overbrace{2x \qquad\qquad dx}^{d(x^2+5)}}{2\sqrt{(x^2+5)^3}} = \frac{1}{2}\int 2x\,(x^2+5)^{-3/2}\,dx =$$

$$D(x^2+5) = 2x \qquad\qquad (n+1)$$

$$= \frac{1}{2}\left(\frac{(x^2+5)^{-1/2}}{-1/2}\right) + C = -\frac{1}{\sqrt{x^2+5}} + C$$

$$\int \frac{4}{4}x^3\,(8+x^4)^{-5/3}\,dx = \frac{1}{4}\,\frac{(8+x^4)^{-2/3}}{-2/3} + C = -\frac{3}{8}(8+x^4)^{-2/3}$$

$$D(8+x^4) = 4x^3 \qquad\qquad (n+1)$$

$$\int \frac{2}{2}x\,e^{x^2}\,dx = \frac{1}{2}e^{x^2} + C$$

$$D(e^x) = e^x$$

$$\int \frac{3\,e^{x}}{1+e^{2x}}\,dx = 3\,\operatorname{arctg}(e^x) + c$$

$e^{2x} = (e^x)^2$

$$D(\operatorname{arctg}(t)) = \frac{1}{1+t^2}$$

$$D(3\operatorname{arctg}(e^x)) = 3\,\frac{1}{1+(e^x)^2}\cdot e^x$$

$$\int \frac{1}{x\sqrt{1-\log^2 x}}\,dx = \int \frac{\overbrace{1/x \qquad dx}^{D(\log x)}}{\sqrt{1-(\log x)^2}} = \arcsin(\log x) + c$$

$$D(\log x) = \frac{1}{x}$$

$$D(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{x(\log x)^{2/3}}\,dx = \int \frac{1}{x}(\log x)^{-2/3}\,dx = \frac{(\log x)^{1/3}}{1/3} + c =$$

$D(\log x)$

Dom
$$\begin{cases} x > 0 \\ x \neq 1 \end{cases}$$

$$n+1 = \left(-\frac{2}{3}+1\right) = \frac{1}{3}$$

$$= 3\sqrt[3]{\log x} + c$$

$$\int_2^5 \frac{1}{x(\log x)^{\frac{2}{3}}} + c = \Big|3\sqrt[3]{\log x}\Big|_2^5 =$$

$$= 3\left(\sqrt[3]{\log 5} - \sqrt[3]{\log 2}\right)$$

$$\int_1^5 \frac{1}{x(\log x)^{2/3}}\,dx = ?$$

dom f : $\begin{cases} x > 0 \\ x \neq 1 \end{cases}$

non è integrabile "propriamente" perché f non è definita in 1

$$\int tg\,x\,dx = -\int -\frac{sen\,x}{cos\,x}\,dx = -\log|cos\,x| + c$$

pongo il modulo per l'esistenza del logaritmo

$cos\,x \neq 0$

$x \neq \frac{\pi}{2} + k\pi$

$$D(cos\,x) = -sen\,x$$

$$D(\log x) = \frac{1}{x}$$

$$D(cos\,x) = \frac{1}{cos\,x}(-sen\,x)$$

$$\int \frac{1}{sin\,2x}\,dx = \int \frac{1}{2\,sin\,x\,cos\,x}\,dx = \int \frac{cos\,x}{2\,sen\,x\,cos^2 x}\,dx =$$

$sin\,2x \neq 0$

$$= \frac{1}{2}\int \underbrace{\frac{1}{cos^2 x}}_{D(Tg\,x)} \cdot \frac{1}{Tg\,x}\,dx = \frac{1}{2}\log|Tg\,x| + c$$

(attenzione al dominio!)

$$\int 4x \cos(3x^2-5)\,dx = \frac{4}{6}\int 6x \cos(3x^2-5)\,dx =$$

$$D(\operatorname{sen} t) = \cos t$$

$$= \frac{4}{6}\operatorname{sen}(3x^2-5) + c$$

$$D(3x^2-5) = 6x$$

$$\int \cos x \sqrt{\sin x}\,dx = \int \cos x\,(\sin x)^{\frac{1}{2}}\,dx =$$

$$D(\operatorname{sen} x) = \cos x$$

$$\int \cos x\,dx = \operatorname{sen} x + c$$

$$t = \operatorname{sen} x$$

$$dt = \cos x\,dx$$

$$= \int t^{\frac{1}{2}}\,dt = \frac{2}{3}\,t^{3/2} + c = \frac{2}{3}(\operatorname{sen} x)^{\frac{3}{2}} + c =$$

$$= \frac{2}{3}\sqrt{(\operatorname{sen} x)^3} + c$$

$$\frac{1}{6}\int \frac{6x}{\cos^2(3x^2+5)}\,dx = \frac{1}{6}\,Tg\,(3x^2+5)+c$$

$$D(Tg\,x) = \frac{1}{\cos^2 x}$$

$$D(3x^2+5) = 6x$$

Integrazioni per parti

$$\int f'(x)\, g(x)\, dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x)\, dx$$

$$f(x) \cdot g(x) = \int f'(x) \cdot g(x)\, dx + \int f(x) \cdot g'(x)\, dx$$

$$D\left(f(x) \cdot g(x) \right) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\int x \cdot \sin x \, dx = x \cdot (-\cos x) - \int -\cos x \, dx =$$

$\downarrow$ $D(\cos x) = -\sin x$ // $D(-\cos x) = \sin x$

$g(x)$ $\quad D(x) = 1$

$$= -x \cos x + \sin x + C$$

$$\int_{\pi/6}^{\pi/3} x \sin x \, dx = x(-\cos x)\Big|_{\pi/6}^{\pi/3} + \int_{\pi/6}^{\pi/3} \cos x \, dx =$$

$$= -x \cos x \Big|_{\pi/6}^{\pi/3} + \sin x \Big|_{\pi/6}^{\pi/3} =$$

$$= -\left(\frac{\pi}{3} \cos \frac{\pi}{3} - \frac{\pi}{6} \cdot \cos \frac{\pi}{6}\right) + \left(\sin \frac{\pi}{3} - \sin \frac{\pi}{6}\right) =$$

$$= -\left(\frac{\pi}{6} - \frac{\sqrt{3}}{12} \pi\right) + \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$$

$$\int 2x\, e^{-x}\, dx = 2 \int x\, e^{-x}\, dx =$$

$f(x) = x$

$f'(x) = 1$

$f'(x) = e^{-x}$ $\quad f(x) = -e^{-x}$

$$= 2 \left\{ -e^{-x} \cdot x - \int -e^{-x}\, dx \right\} =$$

$$= 2 \left\{ -x e^{-x} + (-e^{-x}) + c \right\} =$$

$$= -2x e^{-x} - 2e^{-x} + c =$$

$$= 2e^{-x} (x + 1) + c$$

dom: $1+x>0$

$$\int \log(1+x)\, dx = x \cdot \log(1+x) - \int \frac{x}{1+x}\, dx =$$

$$g(x) = \log(1+x) \qquad g'(x) = \frac{1}{1+x}$$

$$f'(x) = 1 \qquad f(x) = x$$

$$\int \frac{x}{x+1}\, dx = \int \frac{(x+1)-1}{x+1}\, dx = \int \left(1 - \frac{1}{x+1}\right) dx =$$

N, D sono di pari grado

N non è la derivata del den.

$$= x - \log|1+x| + c_1$$

$$= x \log(1+x) - x + \log(1+x) + c$$

non ci serve il val. ass ($|1+x|$) per il dominio della funzione integranda

$x-5>0 \ - - -$

$$\int \underbrace{2x}_{f(x)=x^2} \cdot \underbrace{\log(x-5)}_{f'(x)=\frac{1}{x-5}} \, dx = x^2 \cdot \log(x-5) - \int \frac{x^2}{x-5} \, dx =$$

$$(x^2-25) = (x-5)(x+5)$$

$$\int \frac{x^2}{x-5} \, dx = \int \frac{x^2-25+25}{x-5} \, dx = \int \left(x+5 + \frac{25}{x-5} \right) dx =$$

grad N > grado D

divisione polinomiale $(x^2):(x-5) = \ldots$

$$= \frac{x^2}{2} + 5x + 25 \cdot \log|x-5| + c_1$$

$$= x^2 \log(x-5) - \frac{x^2}{2} - 5x - 25 \log(x-5) + c$$

la funz. int. è definita per $x-5>0 \Rightarrow |x-5| = x-5$

$$\int x \log^2(5x)\, dx =$$

dom $\underline{x>0}$

$$= \frac{x^2}{2} \log^2(5x) - \int \frac{x^{\cancel{2}}}{\cancel{2}} \cdot \overbrace{\frac{\cancel{2} \log(5x)\, \cancel{5}}{\cancel{5}\cancel{x}}}^{D(\log^2(5x))} dx =$$

$$= \frac{x^2}{2} \log^2(5x) - \underbrace{\int \frac{1}{\cancel{5}} \log(5x)\, \cancel{5}x \, dx}_{\text{per parti}} =$$

$$= \frac{x^2}{2} \log^2(5x) - \left(\frac{x^2}{2} \log(5x) - \frac{1}{2} \int \frac{x^2}{2} \cdot \frac{1}{\cancel{5}x} \cdot \cancel{5}\, dx \right) =$$

$$= \frac{x^2}{2} \log^2 5x - \frac{x^2}{2} \log(5x) + \frac{x^2}{4} + c$$