# Lezione 8 : successioni

## Promemoria

$$(-1)^n = \begin{cases} (-1)^{2m} = 1 & n = 2m \Rightarrow \text{pari} \\ (-1)^{2m+1} = -1 & n = 2m+1 \Rightarrow \text{dispari} \end{cases}$$

$$a_n = (-1)^n \cos n = \qquad -1 \le \cos n \le 1 \quad \text{limitato}$$

$$= \begin{cases} \cos n & n \text{ pari} \\ -\cos n & n \text{ dispari} \end{cases}$$

$$a_n = (-1)^n e^n = \qquad e^n > 0 \,, \ \underline{e^n \geq 1}$$

$$= \begin{cases} e^n & n \text{ pari} \\ -e^n & n \text{ dispari} \end{cases}$$

$$a_n = (-1)^n e^n = \begin{cases} e^n & n \text{ pari} & \geq 1 \\ -e^n & n \text{ dispari} & \leq -1 \end{cases}$$

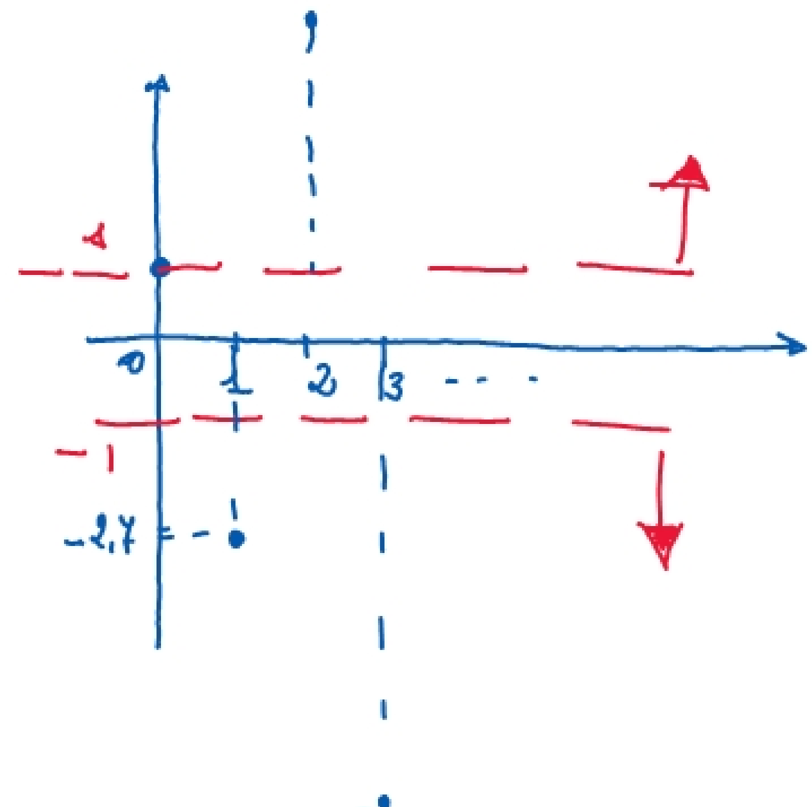
$a_0 = 1$

$a_1 = -e^1 = -e \sim -2{,}7$

$a_2 = e^2$

$a_3 = -e^3$

$\vdots$




$$\lim_{n \to +\infty} (-1)^n e^n = \nexists$$

succ. a segni alterni
non esiste il limite

$a_{2n} = e^{2n} = e^{n}$ sottosuccessione di $a_n$, con l'indice pari

$$\lim_{2n \to +\infty} a_{2n} = \lim_{n \to +\infty} e^{n} = +\infty$$

sottosuccessione diverge a più infinito

$a_{2n+1} = -e^{2n+1} = -e^{n}$ sottosuccessione di $a_n$, con l'indice dispari

$$\lim_{2n+1 \to +\infty} a_{2n+1} = \lim_{n \to +\infty} \left(-e^{n}\right) = -\infty$$

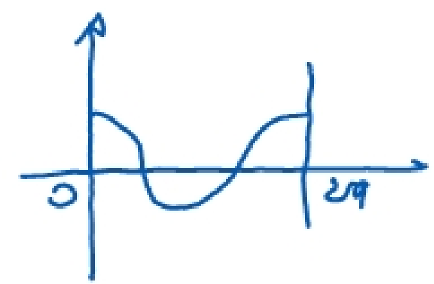
sottosuccessione diverge a meno infinito

$$a_n = (-1)^n \cos n = \begin{cases} \cos 2n \\ -\cos(2n+1) \end{cases}$$

$$\lim_{n\to+\infty} a_n = \nexists$$

$$\lim_{n\to+\infty} \cos n = \nexists$$

$$\lim_{n\to+\infty} -\cos n = \nexists$$




$$-1 \leq \cos x \leq 1$$

una successione e 2 sottosuccessioni che non ammettono limite

(Teo di esistenza e unicità del limite)

altri esempi di successioni:

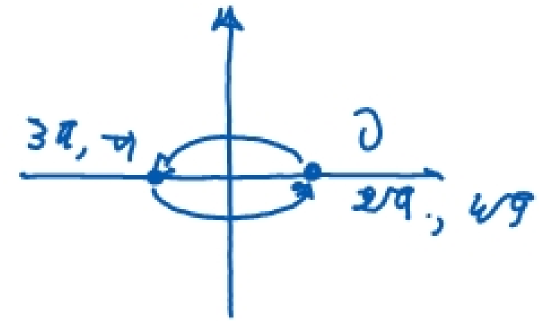
$$a_n = \cos n\pi = \begin{cases} 1 & n = 2m \quad (\vartheta = 0, 2\pi + 2k\pi) \\ -1 & n = 2m+1 \quad (\vartheta = \pi + 2k\pi) \end{cases}$$

$$\lim_{n \to +\infty} \cos n\pi = \nexists$$

$$b_n = \cos n\frac{\pi}{2} = \begin{cases} \pm 1 & n = 2m \\ 0 & n = 2m+1 \end{cases}$$

$$\lim_{n \to +\infty} \cos n\frac{\pi}{2} = \nexists$$

$$c_n = \operatorname{sen} n\pi = 0$$




$$\lim_{n \to +\infty} \operatorname{sen} n\pi = 0$$

succ. identicamente nulla

$$d_n = \operatorname{sen} n\frac{\pi}{2} = \begin{cases} \pm 1 & n = 2m+1 \\ 0 & n = 2m \end{cases}$$

$$\lim_{n \to +\infty} \operatorname{sen} n\frac{\pi}{2} = \nexists$$

$$e_n = \operatorname{sen}^2 n + \cos^2 n = 1$$

$$\lim_{n \to +\infty} \operatorname{sen}^2 n = \nexists \qquad \lim_{n \to +\infty} \cos^2 n = \nexists$$

$$\lim_{n \to +\infty} \left(\operatorname{sen}^2 n + \cos^2 n\right) = 1$$

Ricordarsi che:

$$\lim_{n \to +\infty} a^n = \begin{cases} +\infty & a > 1 \\ 1 & a = 1 \\ 0 & |a| < 1 \\ \nexists & a \leq -1 \end{cases}$$

$$\lim_{n \to +\infty} \left(\frac{1}{2}\right)^n = 0$$

$$\lim_{n \to +\infty} \left(-\frac{1}{2}\right)^n = 0 \quad !!!$$

$$\lim_{n \to +\infty} \frac{a^n}{n!} = 0 \qquad \forall a \in \mathbb{R}$$

$$\lim_{n \to +\infty} \sqrt[n]{a} = 1 \qquad a > 0$$

$$\lim_{n \to +\infty} \sqrt[n]{n!} = +\infty$$

$$\lim_{n \to +\infty} \sqrt[n]{n} = 1$$