Lezione 4

LOGARITMI

$$\boxed{y = a^x \iff x = \log_a y}$$

$\forall x \in \mathbb{R}$

$\forall a \in \mathbb{R}^+_0 \setminus \{1\}$

$\forall y \in \mathbb{R}^+_0$

$$\boxed{\begin{array}{l} y > 0 \\ \text{Arg}(\log) > 0 \end{array}}$$

$\Uparrow$ condizione per determinare il dominio di funzioni trascendenti logaritmiche

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$$y = \sqrt{\log(x^2+1)}$$

cond. per il dominio

Arg>0 $\begin{cases} x^2+1 > 0 & \forall x \in \mathbb{R} \\ \log(x^2+1) \geq 0 \end{cases}$

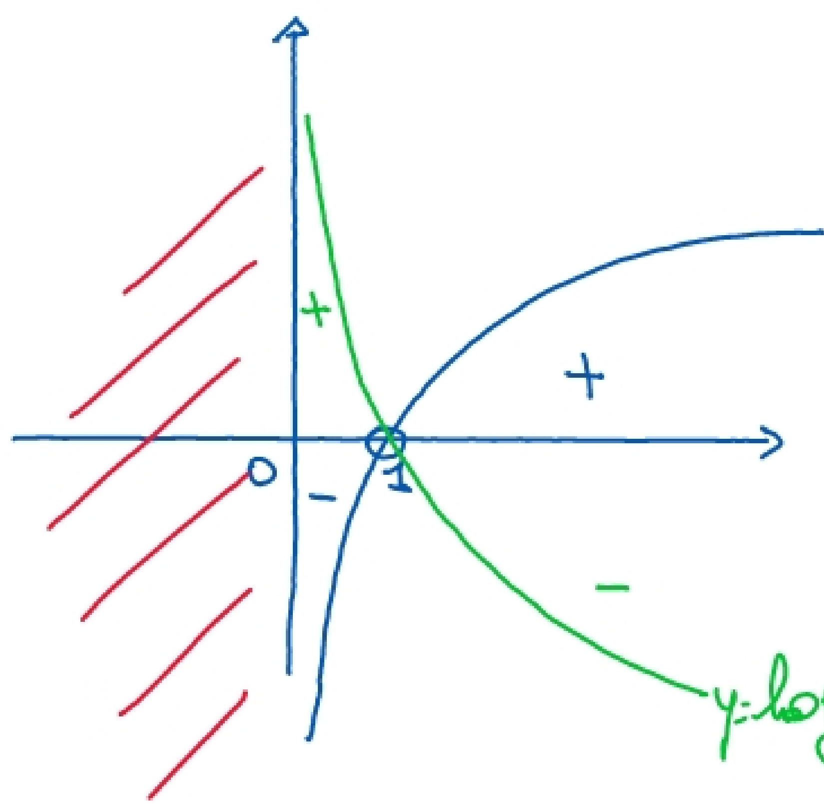
$\sqrt{}$

$\Downarrow$

$x^2+1 \geq 1 \qquad x^2 \geq 0$

$\boxed{\forall x \in \mathbb{R}}$

N.B.



$y = \log_a x \qquad a > 1$

crescente

$f(x) \geq 0 \iff Arg \geq 1$

$f(x) < 0 \iff 0 < Arg < 1$

$y = \log_a x \qquad 0 < a < 1$ decrescente

2) Sono equivalenti le seguenti funzioni

(stesso dominio)

$$y = \log(x^2-4) \qquad (A)$$

$$y = \log(x-2) + \log(x+2) \qquad (B)$$

(A) $y = \log(x^2-4)$

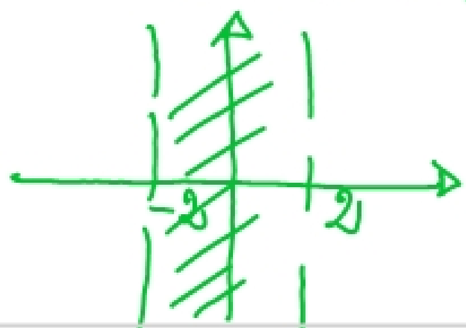
dom: $x^2-4>0$

$x^2-4=0$

$x=\pm 2$

$\Downarrow$

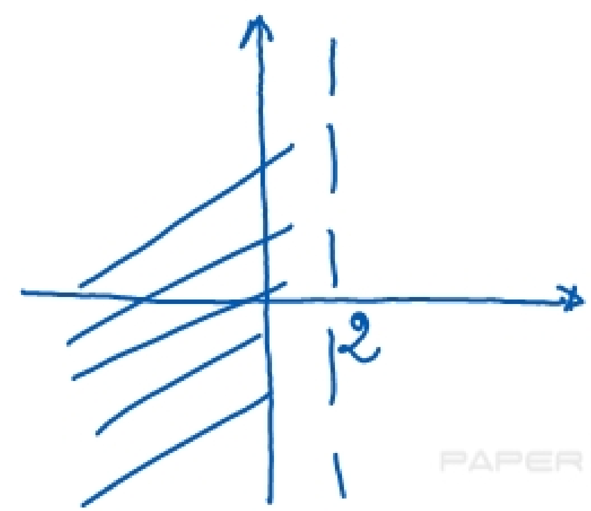
$x<-2 \cup x>2$



(B) $y = \log(x-2) + \log(x+2)$

dom $\begin{cases} x-2>0 \\ x+2>0 \end{cases}$ $\begin{cases} x>2 \\ x>-2 \end{cases}$

$\Rightarrow x>2$



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$$\log(x^2-4) = \log[(x-2)(x+2)] = \log(x-2) + \log(x+2)$$

scomposizione argomento — prop. log.

RIASSUNTO PROPRIETA' LOGARITMI

$$a^{\log_a b} = b \qquad \left[\ f(x)^{g(x)} = e^{\log_e [f(x)]^{g(x)}} = e^{g(x)\log f(x)}\ \right]$$

passaggio all'esponenziale

$$\log_a a^c = c \qquad \log_a 1 = 0$$

$$\log_a a = 1 \qquad [\ 2 = 2 \cdot \log_e e = \log_e e^2\ ]$$

$$\log_a (m \cdot n) = \log_a m + \log_a n$$

$$\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$$

$$\log_a \frac{1}{n} = \log_a n^{-1} = -\log_a n$$

$$= \log_a 1 - \log_a n = -\log_a n$$

N.B. attenzione alle condizioni!

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$$\log_a b^m = m \log_a b$$

$$\log_a \sqrt[n]{b} = \log_a b^{\frac{1}{n}} = \frac{1}{n} \log_a b$$

CAMBIO DI BASE

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$$\log_a b = \frac{1}{\log_b a}$$

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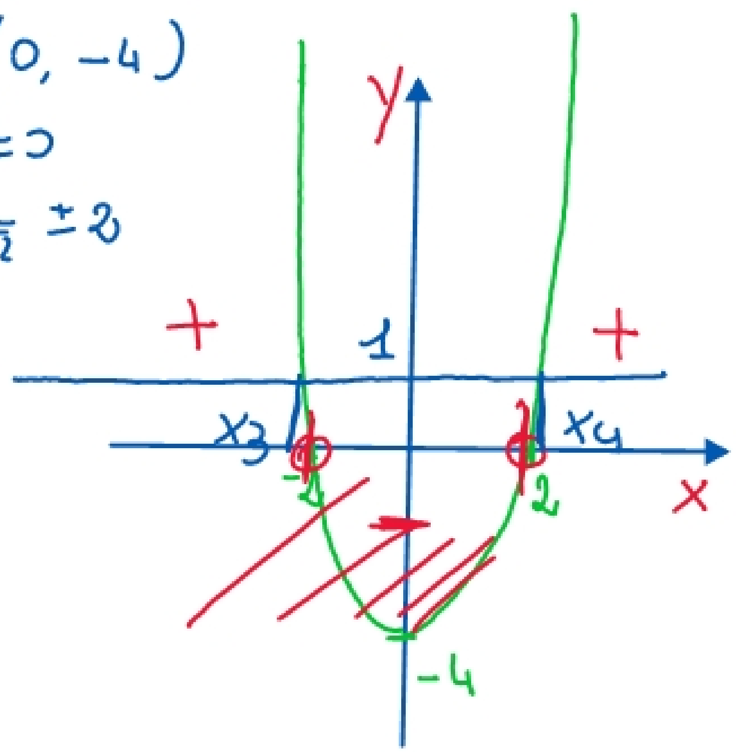
$y = \log(x^2 - 4)$

tracciare un grafico qualitativo a partire da
$y = x^2 - 4$ (parabola)

$y = x^2 - 4$

$V(0, -4)$

$\begin{cases} y = 0 \\ x_{1,2} = \pm 2 \end{cases}$



$y = \log(x^2 - 4)$

$x_3, x_4 : x^2 - 4 = 1$
$x = \pm\sqrt{5}$

f(x) è pari
- simmetria del dom f
- $f(x) = f(-x)$

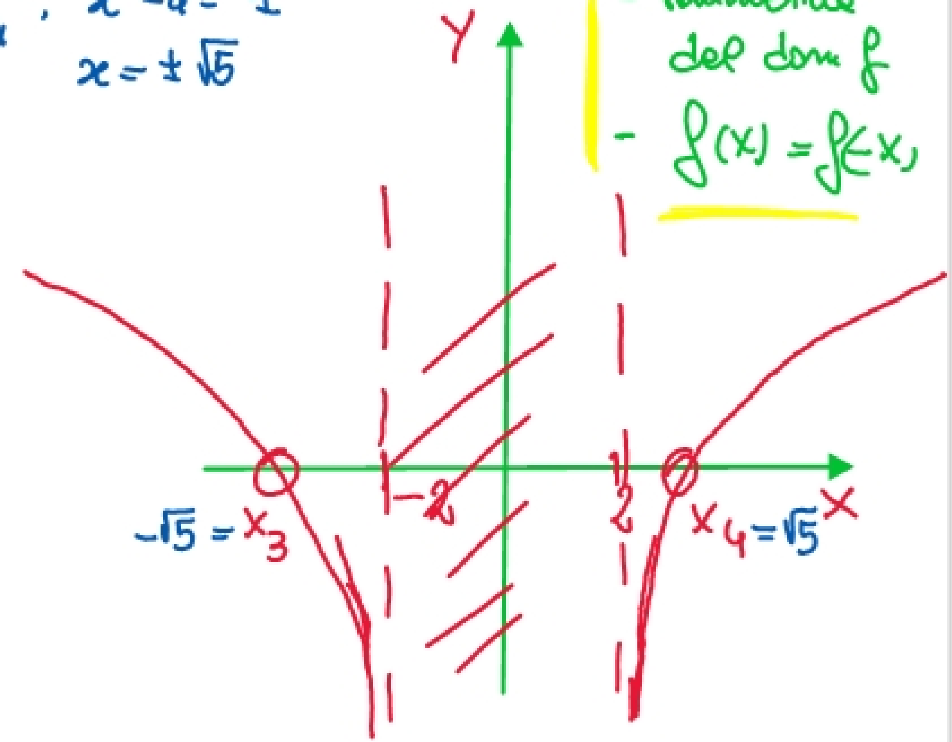


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Grafico qualitativo di

$$y = \log\left[\frac{|x|+1}{|x|-1}\right]$$

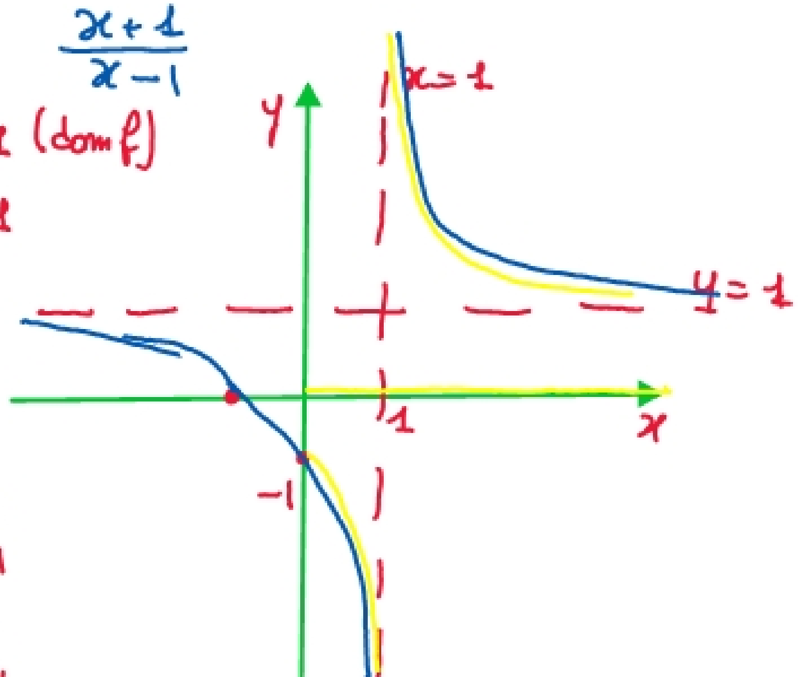
$$y = \frac{|x|+1}{|x|-1} \qquad \begin{cases} \dfrac{x+1}{x-1} & \text{per } x \geq 0 \\ \dfrac{-x+1}{-x-1} = \dfrac{x-1}{x+1} & \text{per } x < 0 \end{cases}$$

2 parti di funzioni omografiche

$y = \dfrac{x+1}{x-1}$

$x \neq 1$ (dom f)

$y = 1$

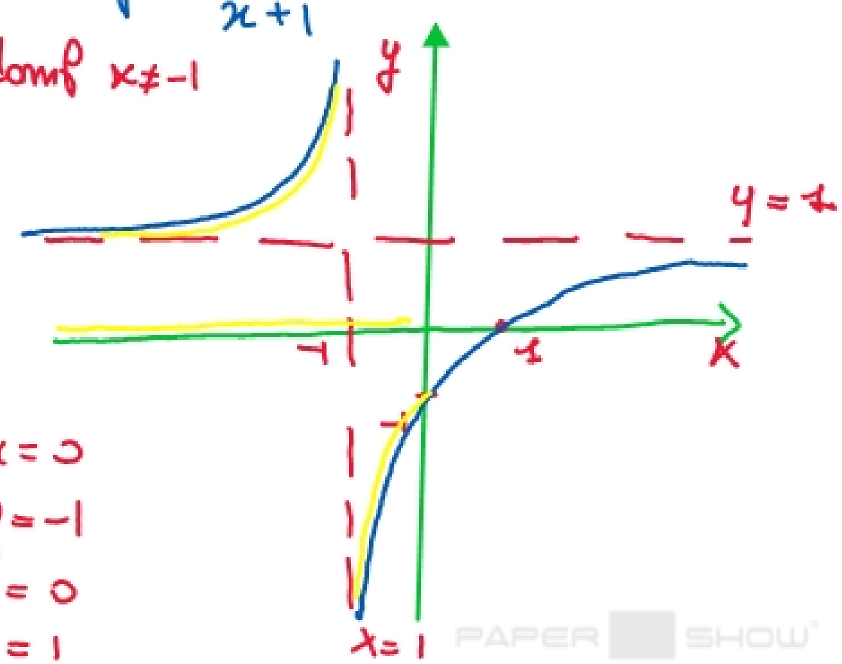


$\begin{cases} x = 0 \\ y = -1 \end{cases}$

$\begin{cases} y = 0 \\ x = -1 \end{cases}$

$y = \dfrac{x-1}{x+1}$

dom f $x \neq -1$



$\begin{cases} x = 0 \\ y = -1 \end{cases}$

$\begin{cases} y = 0 \\ x = 1 \end{cases}$

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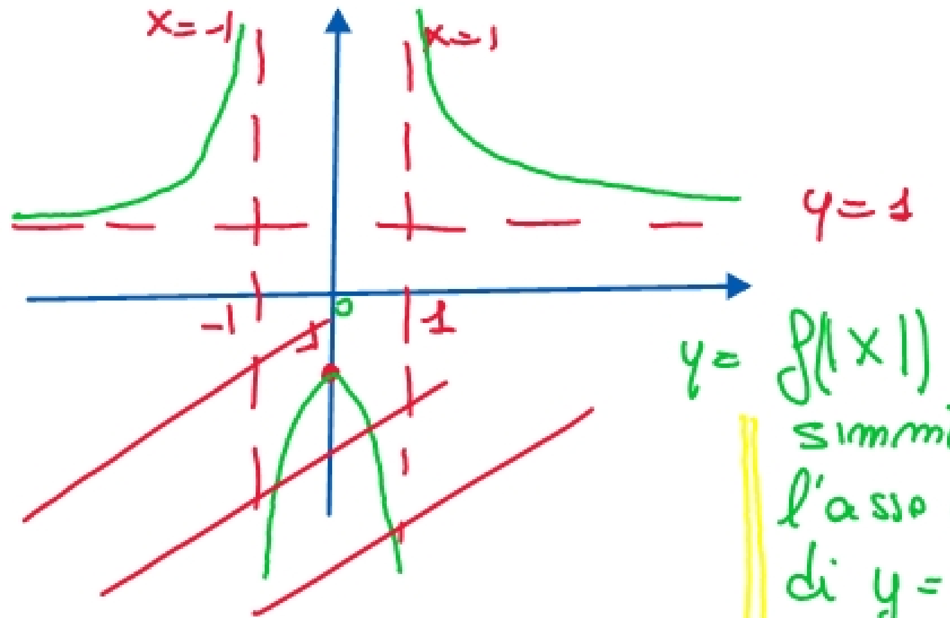
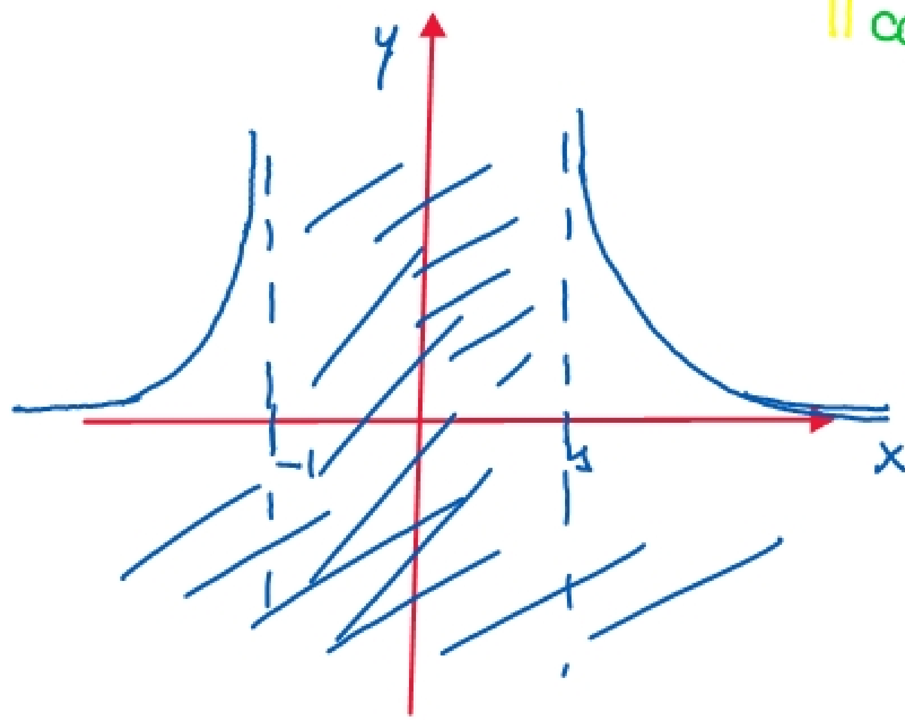
$$y = \frac{|x|+1}{|x|-1}$$



$y = f(|x|)$

simmetria rispetto l'asse delle ordinate di $y = f(x)$ con $x \geq 0$

$$y = \log\left[\frac{|x|+1}{|x|-1}\right]$$



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funzioni trigonometriche

1) $y = \operatorname{sen} x$

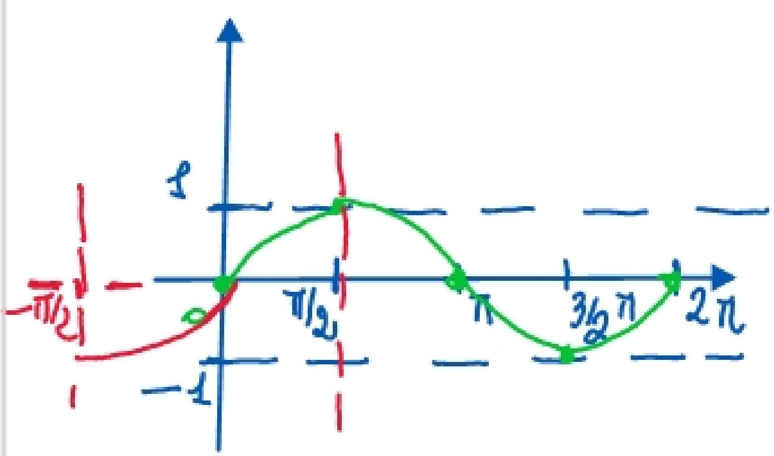
<u>periodico</u> $2\pi \to [0, 2\pi]$

$x = \text{Arg}$ è un angolo

$y = \operatorname{sen} x$ è una misura lineare

limitato: $-1 \leq \operatorname{sen} x \leq 1 \quad \forall x \in \mathbb{R}$

$\operatorname{sen} x \in [-1, 1]$



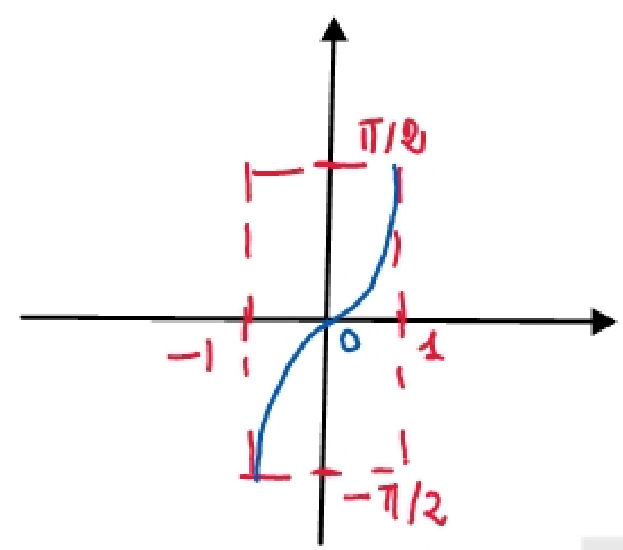
$y = \arcsin x$

inverso considerando l'intervallo in cui $y = \operatorname{sen} x$ è crescente

$[-\pi/2, \pi/2]$ $\boxed{\begin{array}{c}[-1, 1] \\ x\end{array}}$

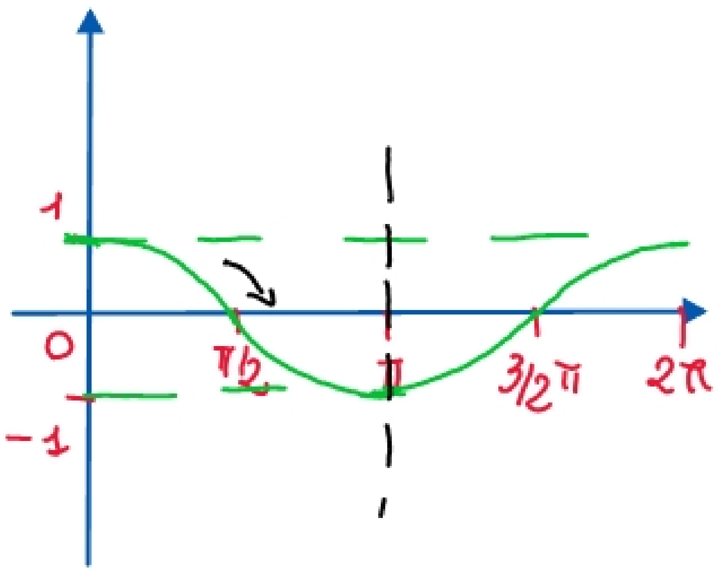
y

cond. come dominio



2) valgono tutte le informazioni/notazioni date per y = sen x

$y = \cos x$

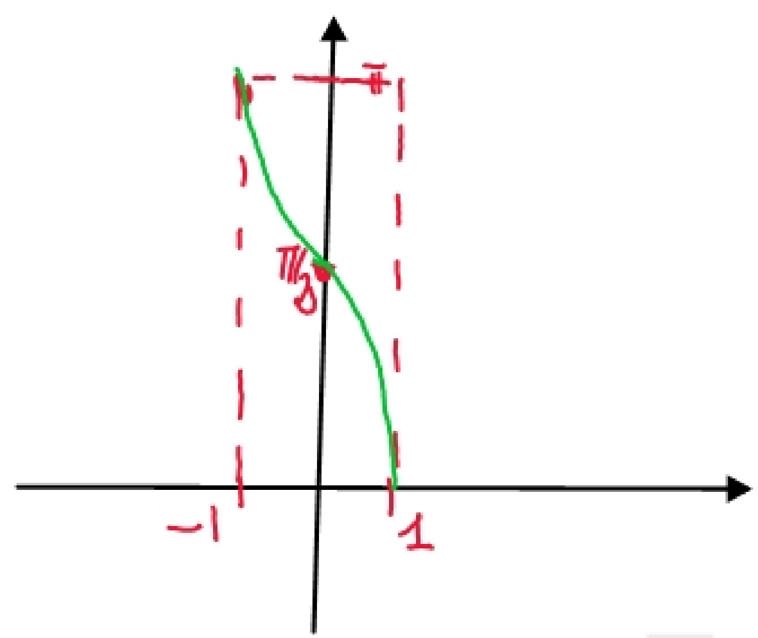


Tratto da invertire

$y \in [0, \pi]$ im f

$x \in [-1, 1]$ dom

$y = \arccos x$



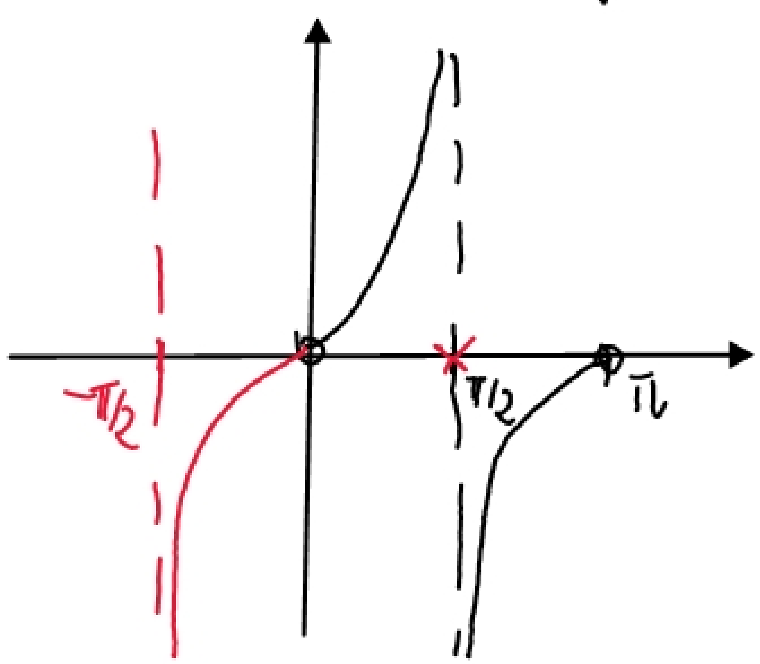
3) $y = \text{tg}\, x = \dfrac{\text{sen}\, x}{\cos x}$

✓ periodicità $[0, \pi] \leftarrow \pi$

✓ illimitata

✓ $\cos x \neq 0$ $\quad \boxed{x \neq \frac{\pi}{2} + k\pi}$ ↗ dom($\text{Tg}\, x$)

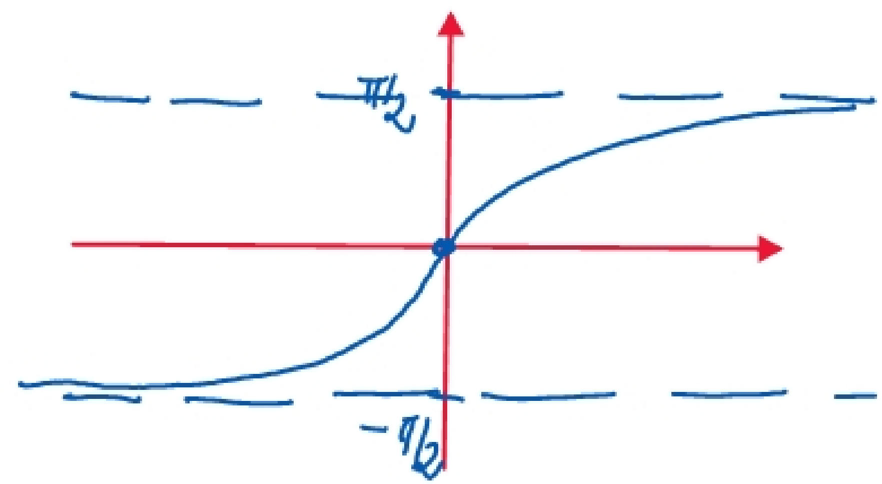
✓ crescente



$y = \text{arctg}\, x$

$y \in \left]-\pi/2, \frac{\pi}{2}\right[$ inf

$\forall x \in \mathbb{R}$ dom f



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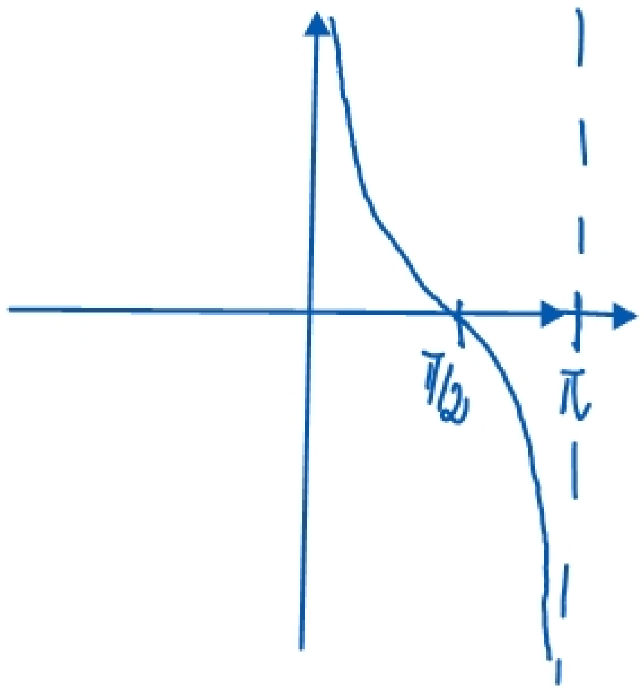
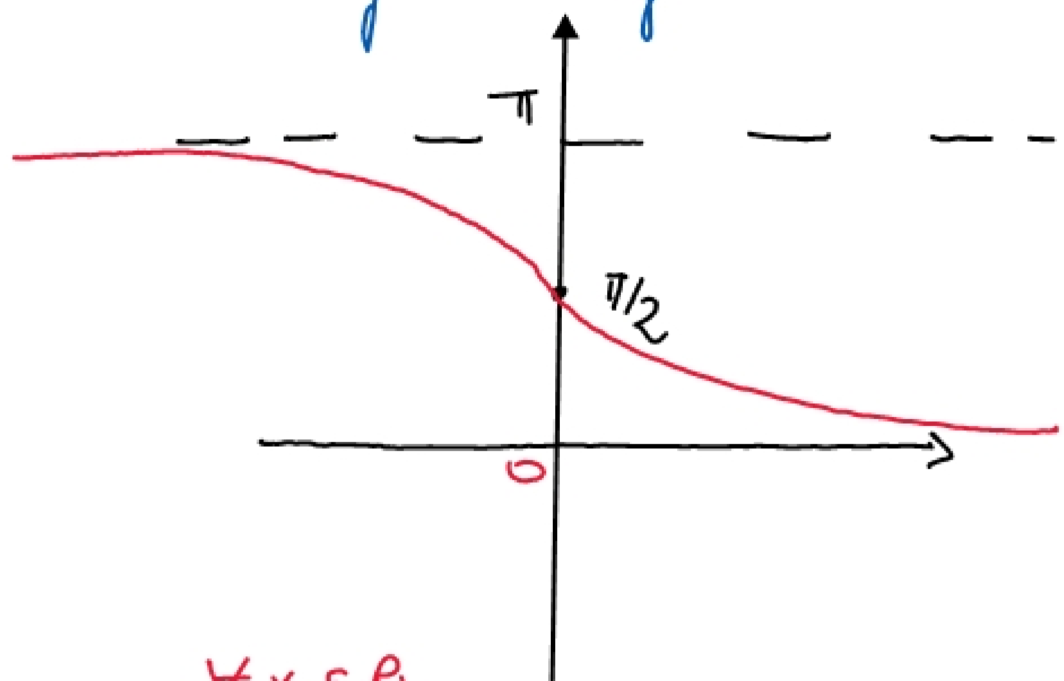
4) $y = ctg\, x = \dfrac{1}{Tg\, x} = (Tg\, x)^{-1} = \dfrac{\cos x}{\operatorname{sen} x}$ $\qquad \operatorname{sen} x \neq 0$

$x \neq k\pi$

vale quanto precisato al punto 3



$y = arcctg\, x$



$\forall x \in \mathbb{R}$

$y \in \,]0, \pi[$

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$$y = \operatorname{arctg} \frac{|x|+1}{|x|-1}$$

$$y = \operatorname{arctg} \frac{|x+3|}{x+1}$$

$$y = \log |\operatorname{sen} x - \sqrt{3} \cos x|$$

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