Lezione 21: sviluppi di Taylor e limiti

(N.B $x_0 = 0$)

$$\sin^2\left(\frac{2}{\sqrt{2}}x\right) = \left[\frac{2}{\sqrt{2}}x - \frac{1}{\cancel{6}_3}\cdot\frac{\cancel{8}^{\cancel{4}\,2}}{\cancel{2\sqrt{2}}}x^3 + o(x^4)\right]^2 = 2x^2 - \frac{4}{3}x^4 + o(x^5)$$

$$= 2x^2 + \frac{2}{9}x^6 + o(x^8) - \frac{4}{3}x^4 + \underline{o(x^5)} + o(x^7)$$

$$\sin t = t - \frac{t^3}{3!} + o(t^4) =$$

$$= \frac{2}{\sqrt{2}}x - \frac{1}{3!}\left(\frac{2}{\sqrt{2}}x\right)^3 + o\left(\left(\frac{2}{\sqrt{2}}x\right)^4\right)$$

$$\sin x = \sum_{n\geq 0} (-1)^n \frac{x^{2n+1}}{(1+2n)!} + o\left(x^{2n+2}\right)$$

$$\sin x = \underset{n=0}{x} - \underset{n=1}{\frac{x^3}{3!}} + o(x^4)$$

$$f(x) = \sum_{k=0}^{n} \frac{f^{k}(x_0)}{k!}(x-x_0)^k + o(x-x_0)^n$$

t.c. $\displaystyle\lim_{x\to x_0} \frac{o(x-x_0)^n}{(x-x_0)^n} = 0$

PAPER SHOW

TE. 26.06.10

$$\lim_{x\to 0^-} \frac{\sin x - \cos x + 1}{2x + x^2 + 1 - \underbrace{\frac{e^x - 1}{x}}_{\to 1}} = \frac{0}{0} \quad \text{F. I.}$$

$$\sin x = x - \frac{x^3}{3!} + o(x^4)$$

$$\cos x = 1 - \frac{x^2}{2!} + o(x^3)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + o(x^3)$$

$$= \lim_{x\to 0^-} \frac{\left[x - \frac{x^3}{3!} + o(x^4)\right] - \left[\cancel{1} + \frac{x^2}{2!} + o(x^3)\right] \cancel{+1}}{2x + x^2 + 1 - \frac{\cancel{1} + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)}{x}} =$$

$$= \lim_{x\to 0} \frac{x + \frac{x^2}{2} - \cancel{\frac{x^3}{3!}} + o(x^3)}{2x + x^2 \cancel{+1} - 1 - \frac{x}{2} - \frac{x^2}{6} + o(x^2)} = \lim_{x\to 0} \frac{x + \frac{x^2}{2} \left[- \frac{x^3}{3!}\right] + o(x^3)}{\frac{3}{2}x + \frac{5}{6}x^2 + o(x^2)} = \frac{0}{0} ?$$

Cambio lo sviluppo perché non funzionale

$$= \lim_{x \to 0^-} \frac{\overbrace{x + o(x^2)}^{\operatorname{sen} x} - \overbrace{\cancel{1} + \frac{x^2}{2!} + o(x^3)}^{\cos x} + \cancel{1}}{2x + x^2 + 1 - \dfrac{\cancel{1} + x + \frac{x^2}{2} + o(x^2) - \cancel{1}}{x}} =$$

$$= \lim_{x \to 0^-} \frac{x + \frac{x^2}{2} + o(x^2)}{2x + x^2 + \cancel{1} - \cancel{1} - \frac{x}{2} - o(x)} =$$

trascurabile

$$= \lim_{x \to 0^-} \frac{x + o(x)}{\frac{3}{2}x + o(x)} = 1 : \frac{3}{2} = \frac{2}{3}$$

pari grado

$$\lim_{x\to 0} \frac{x^4}{x} = \lim_{x\to 0} x^3 = 0 \qquad x^4 \sim o(x)$$

$$\lim_{x\to 0} \frac{x^4}{x^2} = \lim_{x\to 0} x^2 = 0 \qquad x^4 \sim o(x^2)$$

$$\lim_{x\to 0} \frac{x^4}{x^3} = \lim_{x\to 0} x = 0 \qquad x^4 \sim o(x^3)$$

$$\lim_{x\to 0} \frac{x^4}{x^4} = 1$$

$$\lim_{x\to 0} \frac{x^4}{x^5} = \lim_{x\to 0} \frac{1}{x} = \infty$$

$$\lim_{x\to 0} \frac{\frac{x^2}{3} + o(x^2)}{o(x)} \sim \lim_{x\to 0} \frac{x^2}{2} \Big/ x = 0$$

PAPER SHOW

$$\lim_{x \to 0} \frac{\frac{x^2}{2}}{\frac{3}{2}x} = 0$$

$$\lim_{x \to 0} \frac{x + \frac{x^2}{2}}{\frac{3}{2}x} = \lim_{x \to 0} \left(\frac{2}{3} + \frac{3}{4}x \right) = 2/3$$

confronto fra i coefficienti dello
stesso grado e minori

PAPER SHOW

TE 2/09/03

$$\lim_{x\to 0}(\cos x)^{\frac{x}{\operatorname{sen} 2x - 2x\cos x}} = 1$$

%

$$= \lim_{x\to 0} e^{\log(\cos x)^{\frac{x}{\operatorname{sen}2x - 2x\cos x}}} = \lim_{x\to 0} e^{\frac{x}{\operatorname{sen}2x - 2x\cos x}\cdot\log(\cos x)} =$$

$$= e^{3/2}$$

ESPONENTE

$$\lim_{x\to 0}\frac{x\cdot\log(\cos x)}{\operatorname{sen}2x - 2x\cos x} = \lim_{x\to 0}\frac{x\log\left(1-\frac{x^2}{2}+o(x^2)\right)}{2x-\frac{(2x)^3}{6}-2x\left(1-\frac{x^2}{2}\right)+o(x^3)} =$$

$$\log(1+x) = x - \frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+o(x^4)$$

$$\operatorname{sen}(x) = x - \frac{x^3}{3!} + o(x^4)$$

$$\cos(x) = 1 - \frac{x^2}{2}+\frac{x^4}{4!}+o(x^5)$$

$$= \lim_{x\to 0}\frac{x\left(-\frac{x^2}{2}+o(x^2)\right)}{\cancel{2x}-\frac{8x^3}{6}-\cancel{2x}+x^3+o(x^3)} =$$

$$= \lim_{x\to 0}\frac{-\frac{x^3}{2}+o(x^3)}{-\frac{1}{3}x^3+o(x^3)} =$$

$$= -\frac{1}{2}\cdot(-3) = +3/2$$ esponente

PAPER SHOW

TE 11.02.11

$$0/0$$

$$\lim_{x\to 0} \frac{4\log\left(1+\dfrac{\sin(x^2)-x^2}{x^2}\right)}{1-2x^2-\cos(2x)} =$$

$$= \lim_{x\to 0} \frac{4\log\left(1+\dfrac{\cancel{x^2}-\frac{(x^2)^3}{6}+o(x^7)-\cancel{x^2}}{x^2}\right)}{1-2x^2-\left(1-\frac{(2x)^2}{2}+\frac{(2x)^4}{4!}+o((2x)^5)\right)} =$$

$$= \lim_{x\to 0} \frac{4\log\left(1+\left(-\frac{x^4}{6}+o(x^5)\right)\right)}{\cancel{1}-\cancel{2x^2}-\cancel{1}+\cancel{\frac{4x^2}{2}}-\frac{16x^4}{24}+o(x^5)} =$$

$$= \lim_{x\to 0} \frac{4\left[-\frac{x^4}{6}+o(x^5)\right]}{-\frac{2}{3}x^4+o(x^5)} = -\frac{4}{6}\cdot\left(-\frac{3}{2}\right) = 1$$