Lezione n. 20:
Studio di funzione

$y = f(x)$

dall' espressione algebrica $\longrightarrow$ grafico (supporto grafico)

1. dominio: valori di x per cui ha significato $f(x)$

- denominatore
- radice a indice pari
- base esponenziale
- base log
- logaritmo
- Tg, ct
- arcsen, arccos

2. eventuali simmetrie

- attenzione alla simmetria del dominio rispetto $x = 0$
- verificare $f(x) = f(-x)$ pari

  $f(x) = -f(-x)$ dispari

3. segno di $f(x)$

$f(x) \geq 0$ [nel semipiano $y \geq 0$ positivo]

4. limiti

asintoti

(*) o valori finiti perché punti di acc. per il dominio

$x \neq x_0$

$$\lim_{x \to x_0} f(x) = \begin{cases} l < \infty & \text{discontinuità eliminabile} \\ l_1 \neq l_2 < \infty & \text{discontinuità a salto} \\ \infty & \text{asintoto verticale } x = x_0 \end{cases}$$

(*) se il dominio è superiormente e/o inferiormente illimitato

$$\lim_{x \to +\infty} f(x) \qquad \lim_{x \to -\infty} f(x) \rightarrow \begin{cases} l < \infty & \text{asintoto orizzontale } y = l \\ \infty & \text{divergente } (*) \end{cases}$$

(*) potrebbe ammettere un asintoto obliquo $y = mx + q$

$$m = \lim_{x \to \infty} \frac{f(x)}{x} (= l) < \infty \qquad q = \lim_{x \to \infty} (f(x) - mx) < \infty$$

5. calcolo della derivata e studio del segno

$f'(x)$

1. dominio di $f'(x)$ per calcolarli punti di non derivabilità

2. studio del segno
   - $f'(x) > 0$ $\quad f(x) \nearrow$
     $f'(x) < 0$ $\quad f(x) \searrow$
   - massimi e minimi (relativi o assoluti)

$f''(x)$

1 studio del segno
   - $f''(x) > 0$ $\quad f(x) \cup$
   - $f'(x) < 0$ $\quad f(x) \cap$
   - flessi

punti di non derivabilità : punti angolosi
punti cuspidali

punti a tg vert/orizz.

T.E. 26-10-09

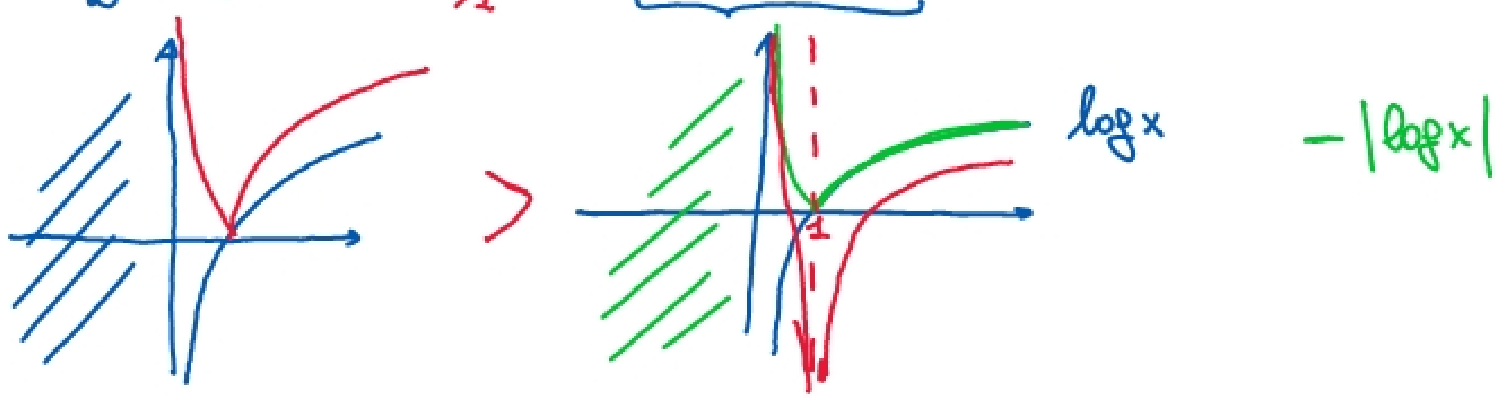
$$f(x) = \frac{1}{2}\log^3 x - (\log^2 3)\log(|\log x|)$$

dominio
$$\begin{cases} x>0 \\ |\log x| > 0 \iff \log x \neq 0 \end{cases} \quad \begin{cases} x>0 \\ x\neq 1 \end{cases} \Rightarrow \,]0,1[\,\cup\,]1,+\infty[$$

no simmetrie

segno
$$\frac{1}{2}\log^3 x - \log^2 3 \;\log|\log x| \geq 0$$

$$\frac{1}{2}\log^3 x \geq \underset{>1}{\log^2 3}\;\underbrace{\log|\log x|} \qquad \forall x \in \text{dom.}$$



$$\lim_{x \to 0^+} f(x) = +\infty \qquad (\text{raccolgo } \log^2 x)$$

$$\lim_{x \to 1^\pm} f(x) = 0 - \underbrace{\log|0|}_{-\infty} = +\infty$$

$$\lim_{x \to +\infty} f(x) = +\infty \qquad (\text{prevale } \log^2 x)$$

$\Rightarrow$ ✓ asintoto verticale destro $x = 0$

✓ asintoto verticale completo $x = 1$

✓ avendo un comportamento logaritmico, non ammette asintoto obliquo derivata prima

$$f(x) = \begin{cases} \frac{1}{2}\log^2 x - \log^2 3 \, \log(\log x) & \log x > 0 \Rightarrow x > 1 \\ \frac{1}{2}\log^2 x - \log^2 3 \, \log(-\log x) & 0 < x < 1 \end{cases}$$

$$f'(x) = \begin{cases} \frac{1}{2}\, 2 \log x \cdot \frac{1}{x} - \log^2 3 \, \frac{1}{\log x} \cdot \frac{1}{x} \\ \frac{1}{2}\, 2 \log x \cdot \frac{1}{x} - \log^2 3 \, \frac{1}{-\log x} \left(-\frac{1}{x}\right) \end{cases} = \frac{1}{x}\left(\frac{\log^2 x - \log^2 3}{\log x}\right)$$

$\mathrm{dom}\, f' = \mathrm{dom}\, f$

$$f'(x) = \frac{1}{x} \, \frac{(\log x - \log 3)(\log x + \log 3)}{\log x}$$

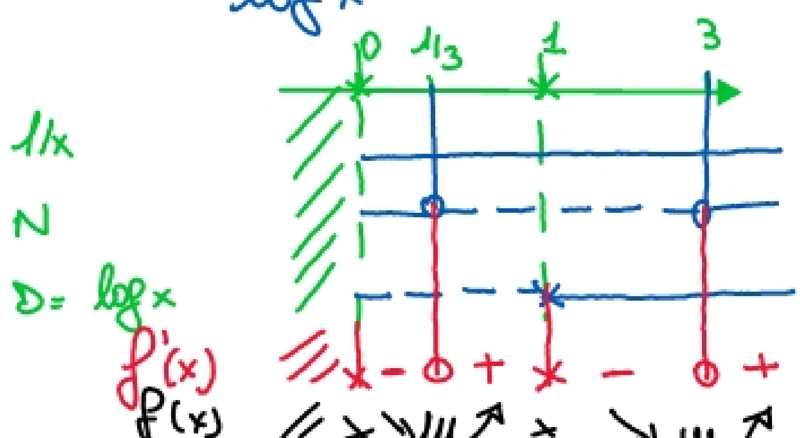
$N \geq 0 \quad \log x < -\log 3 = \log 3^{-1}$

$\cup$

$\log x > \log 3$

$\Downarrow$

$x < \frac{1}{3} \quad \cup \quad x > 3$



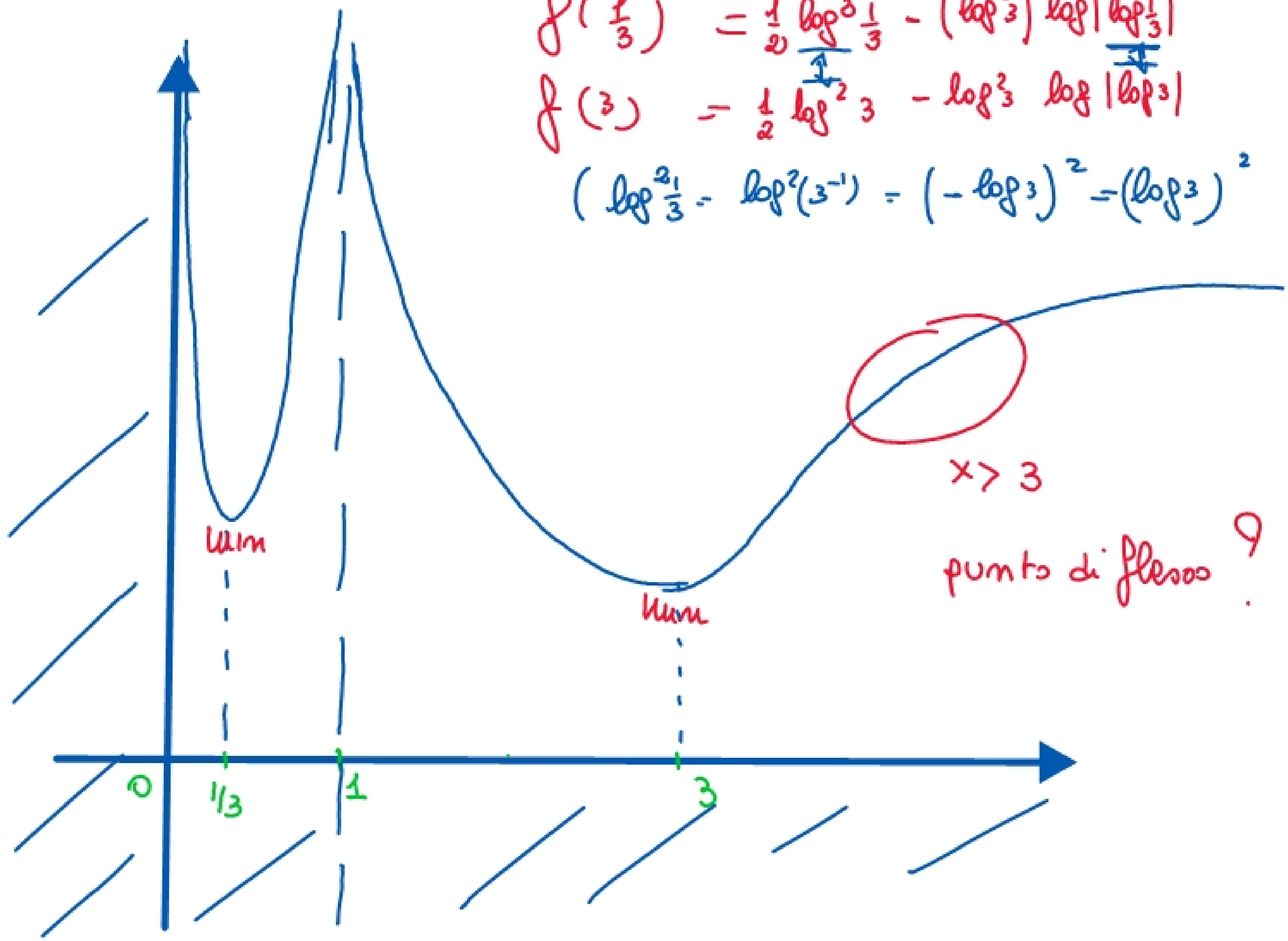
punti di minimo

$x = \frac{1}{3} \qquad x = 3$

$$f\left(\tfrac{1}{3}\right) = \frac{1}{2}\log^2\frac{1}{3} - \left(\log^2 3\right)\log\left|\log\frac{1}{3}\right|$$

$$f(3) = \frac{1}{2}\log^2 3 - \log^2 3\,\log\left|\log 3\right|$$

$$\left(\log^2\frac{1}{3} = \log^2(3^{-1}) = (-\log 3)^2 = (\log 3)^2\right.$$



$$f'(x) = \frac{1}{x}\left(\frac{\log^2 x - \log^2 3}{\log x}\right)$$

$$f''(x) = -\frac{1}{x^2}\left(\frac{\log^2 x - \log^2 3}{\log x}\right) + \frac{1}{x}\left(\frac{2\log x \frac{1}{x}\cdot \log x - \frac{1}{x}\left(\log^2 x - \log^2 3\right)}{\log^2 x}\right) =$$

$$= -\frac{1}{x^2}\,\frac{\log^2 x - \log^2 3}{\log x} + \frac{1}{x^2}\,\frac{2\log^2 x - \log^2 x + \log^2 3}{\log^2 x} =$$

$$= \frac{1}{x^2}\left[\frac{-\log^2 x + \log^2 3}{\log x} + \frac{\log^2 x + \log^2 3}{\log^2 x}\right] =$$

$$= \frac{1}{x^2}\cdot\frac{1}{\log^2 x}\left(-\log^3 x + \log x \cdot \log^2 3 + \log^2 x + \log^2 3\right) =$$

$$+ \qquad +$$

$$\left(-\log^3 x + \log^2 x + \log x\,\log^2 3 + \log^2 3\right)$$

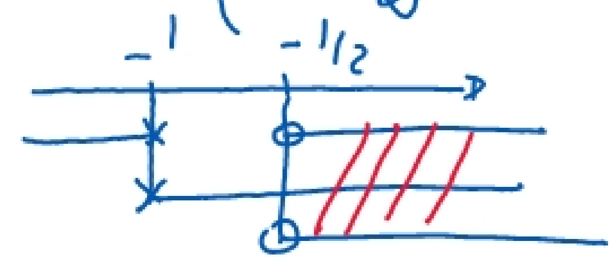
$$\operatorname{dom} f''(x) = \operatorname{dom} f'(x) = \operatorname{dom} f(x) \qquad x \neq 0,\; x \neq 1$$

TE. 6.4.08

$$f(x) = \arcsin\left(\frac{x}{x+1}\right) - \frac{1}{3}\sqrt{2x+1} + \frac{1}{3}$$

dominio

$$\begin{cases} -1 \le \dfrac{x}{x+1} \le 1 \\ 2x+1 \ge 0 \end{cases} \qquad \begin{cases} \dfrac{x}{x+1} \ge -1 \\ \dfrac{x}{x+1} \le 1 \end{cases} \qquad \begin{cases} \dfrac{2x+1}{x+1} \ge 0 \\ \dfrac{-1}{x+1} \le 0 \end{cases} \qquad \begin{cases} x < -1 \cup x \ge -\dfrac{1}{2} \\ x > -1 \\ x \ge -\dfrac{1}{2} \end{cases}$$



$$x \ge -\frac{1}{2}$$

$$f\left(-\frac{1}{2}\right) = \arcsin\left(\frac{-\frac{1}{2}}{-\frac{1}{2}+1}\right) \underbrace{-\frac{1}{3}\sqrt{2\left(-\frac{1}{2}\right)+1}}_{0} + \frac{1}{3} = -\frac{\pi}{2} + \frac{1}{3} < 0$$

$$\lim_{x \to +\infty} \left[\underbrace{\arcsin\left(\underbrace{\frac{x}{x+1}}_{\nearrow 1}\right)}_{\frac{\pi}{2}} \underbrace{-\frac{1}{3}\sqrt{2x+1}}_{-\infty} + \frac{1}{3} \right] = -\infty$$

segno $f(x) < 0$

Δεγμο :

$$\arcsin \frac{x}{x+1} - \frac{1}{3}\sqrt{2x+1} + \frac{1}{3} \geq 0$$

$$\arcsin \frac{x}{x+1} \geq \frac{1}{3}\sqrt{2x+1} - \frac{1}{3} = \frac{1}{3}\left(\sqrt{2x+1} - 1\right)$$

$$f'(x) = \frac{1}{\sqrt{1-\frac{x^2}{(x+1)^2}}} \cdot \frac{x+1-x}{(x+1)^2} - \frac{1}{3}\,\frac{1}{\cancel{2}\sqrt{2x+1}} \cdot \cancel{2} =$$

$$= \frac{\cancel{x+1}}{\sqrt{\cancel{x^2}+2x+1-\cancel{x^2}}} \cdot \frac{1}{(x+1)^{\cancel{2}}} - \frac{1}{3\sqrt{2x+1}} =$$

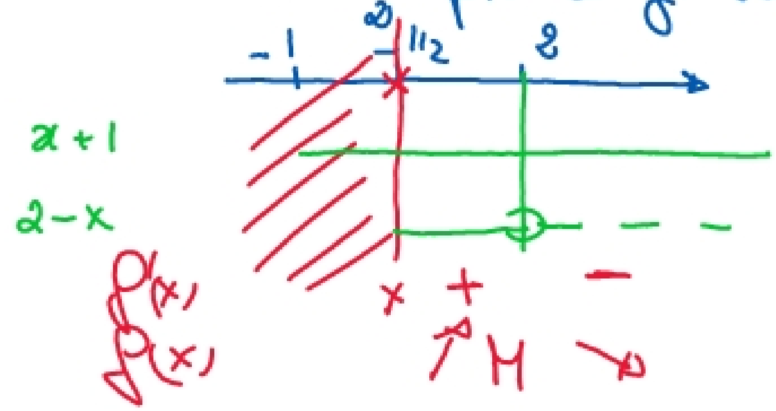
$$= \frac{3 - x - 1}{(x+1)\sqrt{2x+1}\;\;3} = \frac{2-x}{3(x+1)\underbrace{\sqrt{2x+1}}_{+}}$$

dom: $x > -\frac{1}{2}$

dom $f' \subset$ dom f

$$\lim_{x \to -\frac{1}{2}^+} \frac{2-x}{3(x+1)\sqrt{2x+1}} = +\infty$$

$x = -\frac{1}{2}$ pt. a tg. verticale



$x = 2$ max

$$f(0) = \frac{\pi}{2} - \frac{1}{3} + \frac{1}{3}$$

$$f(2) = \arcsin\left(\frac{2}{3}\right) - \frac{1}{3}\sqrt{5} + \frac{1}{3}$$