Lezione 1 — Le funzione

def. $\left[(A, B, f) \quad A, B \subseteq \mathbb{R} \quad \forall x \in A \ \exists!\, y \in B : \underline{\underline{y = f(x)}} \right]$

oss. $y = f(x) \iff y - f(x) = 0 \iff F(x, y) = 0$

dominio : A $\qquad$ dom f

codominio : B

immagine $\quad y \in B \quad y = f(x) \qquad \operatorname{im} f \subseteq B$

Rappresentazione Tabellare

x	y
−3	−7
−2	+2
−1	−5
0	−3
1	0
2	1
3	4
4	7

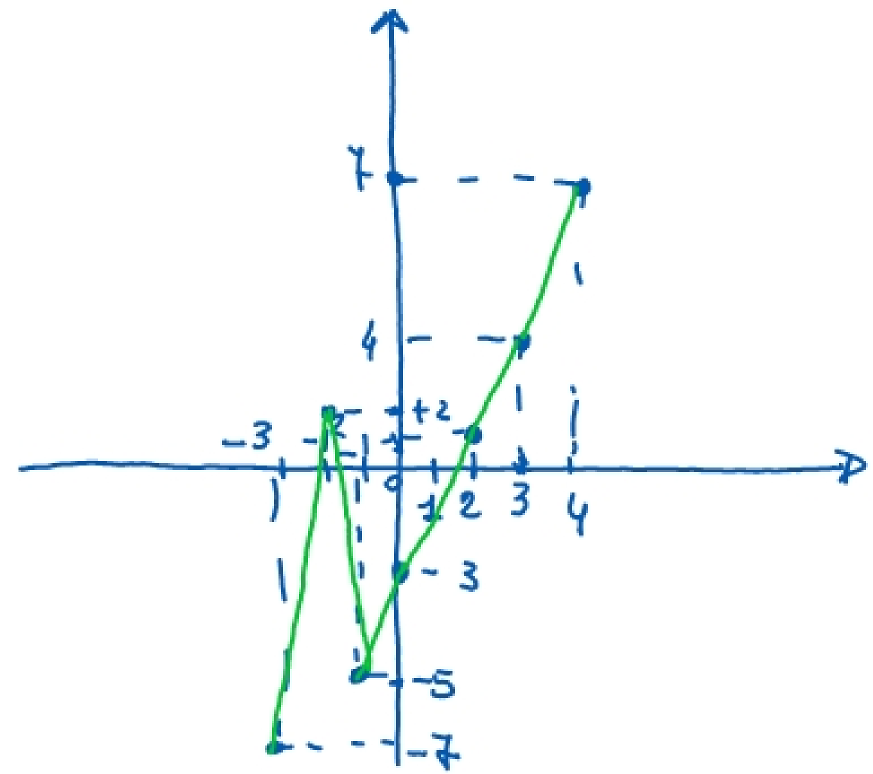


grafico (x, y)

dominio $A = \{ x \in \mathbb{Z} \;\; -3 \leq x \leq 4 \}$

im f $= \{ -7, -5, -3, 1, 4, 7 \}$

Rappresentazione analitica/algebrica

$y = f(x)$

$y = \sqrt{5-x^2}$

$y = \frac{\log(x^2-4)}{\operatorname{sen}(2x+1)} - \operatorname{arctg}(x+4)$

CLASSIFICAZIONE

FUNZIONI REALI A VARIABILE REALE

- ALGEBRICHE
  - RAZIONALI
    - POLINOMIALI (1)
    - FRATTE (2)
  - IRRAZIONALI (3)
- TRASCENDENTI
  - TRIGONOMETRICHE (4)
  - LOGARTMICHE (5)
  - ESPONENZIALI (6)
  - IPERBOLICHE (7)

ESEMPI

(1) $y = a x^n + b x^{n-1} + \dots + c$ $\quad n \in \mathbb{N}$

$a, b, \dots, c \in \mathbb{R}$

$$y = 5x^7 - 4x^3 + 2x^2 - \frac{7}{3}x + \pi$$

$f(0) = \pi$ $\quad$ sostituisco $x = 0$

x: $\quad y = 7$

$$7 = 5x^7 - 4x^3 + 2x^2 - \frac{7}{3}x + \pi$$

dominio $\quad \forall x \in \mathbb{R}$

(2)

$$y = \frac{A(x)}{B(x)}$$

2 polinomi (numeratore/denominatore)

$B(x) \neq 0$

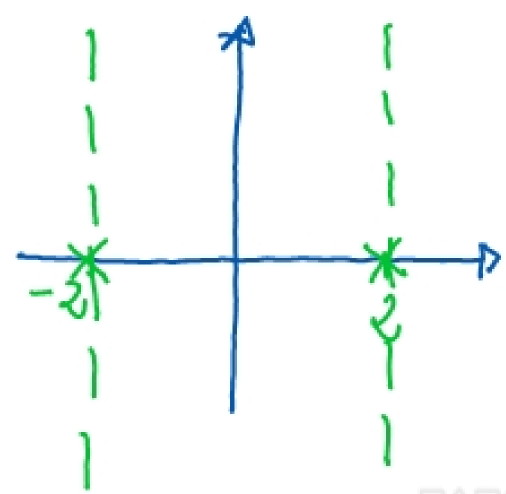
↑ cond. di esistenza per una frazione

$$y = \frac{7x^5 - \pi x^4 + 7{,}2\bar{8}\,x}{x^2 - 4}$$

$x^2 - 4 \neq 0 \qquad x^2 \neq 4 \qquad x \neq \pm\sqrt{4} = \pm 2$

$\mathrm{dom}\, f = \{ \forall x \in \mathbb{R} \setminus \{-2, 2\} \}$

$]-\infty, -2[\; \cup \;]-2, 2[\; \cup \;]2, +\infty[$



③ $y = \sqrt[n]{A(x)}$

n pari $(n = 2m)$

$A(x) \geq 0$

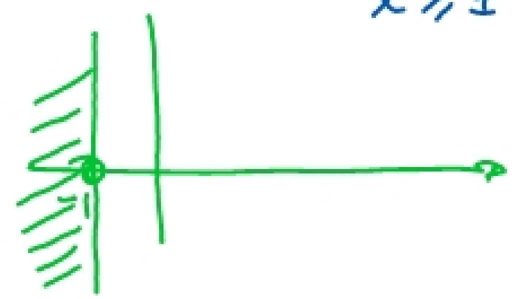
(radicando ≥ 0)

n dispari $(n = 2m+1)$

$y = \sqrt[4]{x+1}$

$x+1 \geq 0$

$x \geq -1$



dom f: $[-1, +\infty[$

$y = \sqrt[7]{4 - x^2 + x}$

$\forall\, x \in \mathbb{R}$

(no condizioni)

es se $f(x) = x e^{x^2}$

allora

$f(\lambda x) = (\lambda x) e^{(\lambda x)^2} = \lambda x e^{\lambda^2 x^2}$

$f(5) = 5 e^{25}$

$f(x+4) = (x+4) e^{(x+4)^2} = (x+4) e^{x^2+16+8x}$

es se $f(x) = x^2 - x$

$f(1-h) = (1-h)^2 - (1-h) =$

$= \cancel{1} + h^2 - 2h - \cancel{1} + h =$

$= h^2 - h$

es

è data la funzione $f(x) = \sqrt{2x^3 - x^2 - 2x + 1}$ e la funzione $g(x) = (x^5 - 16x)^{-0,5}$, e A e B sono i rispettivi domini.

Determinare l'insieme $C = (A \cup B) \setminus (A \cap B)$

A: $f(x) = \sqrt{2x^3 - x^2 - 2x + 1}$

funz. irrazionale
la $\sqrt{\ }$ ha indice pari

$$\underline{2x^3 - x^2} - \underline{2x + 1} \geq 0$$

$$x^2(2x-1) - (2x-1) \geq 0$$

$$(2x-1)(x^2-1) \geq 0$$

$$(2x-1)(x-1)(x+1) \geq 0$$

Studio del segno

1° fatt. $\quad 2x-1 \geq 0 \quad\quad x \geq \frac{1}{2}$

2° fatt. $\quad x-1 \geq 0 \quad\quad x \geq 1$

3° fatt. $\quad x+1 \geq 0 \quad\quad x \geq -1$

		-1		$1/2$		1	
1° fatt.	$-$		$-$	0	$+$		$+$
2° fatt.	$-$		$-$		$-$	0	$+$
3° fatt.	$-$	0	$+$		$+$		$+$
prodotto	$-$	0	$+$	0	$-$	0	$+$

$$A = \{ x \in \mathbb{R} : \; [-1, 1/2] \cup [1, +\infty[\; \}$$

$$g(x) = (x^5 - 16x)^{-0,5} = (x^5 - 16x)^{-\frac{1}{2}} = \frac{1}{\sqrt{x^5 - 16x}}$$

$$\begin{array}{ll} \text{denominatore} \neq 0 \\ \text{radicando} \geq 0 \end{array} \quad \begin{cases} \sqrt{x^5 - 16x} \neq 0 \\ x^5 - 16x \geq 0 \end{cases}$$

$$\Rightarrow \quad x^5 - 16x > 0$$

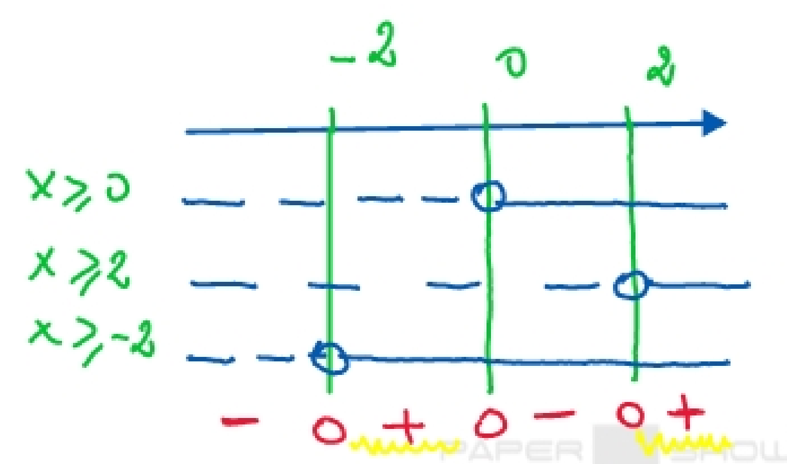
$$x(x^4 - 16) > 0$$

$$x(x^2 - 4)(x^2 + 4) > 0$$

$$x(x-2)(x+2)\underbrace{(x^2+4)}_{+} > 0$$

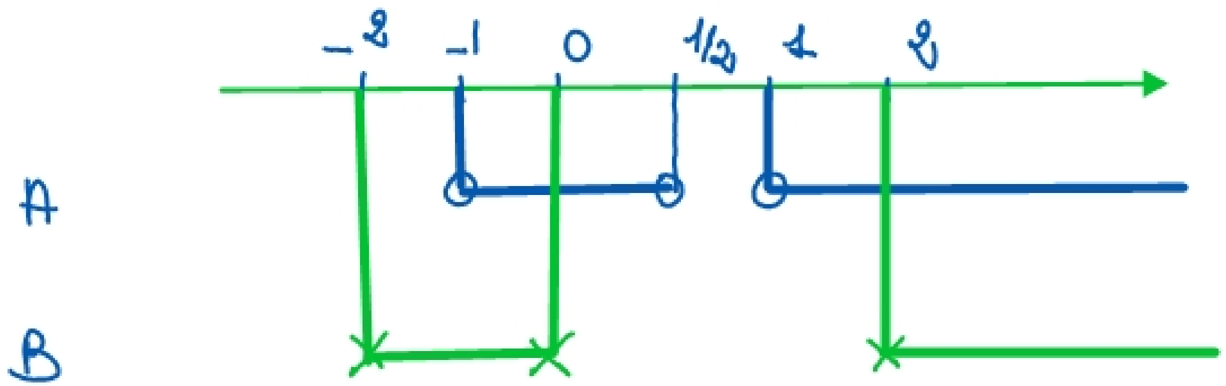
st. segno

$$\begin{array}{ll} x \geq 0 & x \geq 0 \\ x - 2 \geq 0 & x \geq 2 \\ x + 2 \geq 0 & x \geq -2 \end{array}$$



$$A = \{x \in \mathbb{R} : [-1 ; \tfrac{1}{2}] \cup [1 + \infty[\}$$

$$B = \{x \in \mathbb{R} :]-2 ; 0[\cup]2, +\infty[\}$$



$$A \cup B = \{]-2, \tfrac{1}{2}] \cup [1, +\infty[\}$$

$$A \cap B = \{[-1, 0[\cup]2, +\infty[\}$$

$$C = (A \cup B) \setminus (A \cap B) =$$

$$= \{]-2, -1[\cup [0, \tfrac{1}{2}] \cup [1, 2]\}$$