Lezione 16 : funzioni e limiti

$$y = \begin{cases} -x-\pi & x < -2\pi \\ \operatorname{sen} x & -2\pi \leq x \leq -\pi \\ \operatorname{senh} x & -\pi < x < -1 \\ \operatorname{arcsen} x & -1 \leq x \leq 1 \\ \operatorname{arctg} x & x > 1 \end{cases}$$

$f(1) = \operatorname{arctg} 1 = \pi/4$

$y = -x-\pi$ retta // $y = -x$ $m = 1$

$$f(-2\pi) = -(-2\pi) - \pi = 2\pi - \pi = \pi$$

$y = \operatorname{sen} x$ parte positiva di $\operatorname{sen} x$

$$y = \operatorname{senh} x = \frac{e^{+x} - e^{-x}}{2}$$

$$f(-\pi) = \frac{e^{-\pi} - e^{\pi}}{2}$$

$$f(-1) = \frac{e^{-1} - e^{1}}{2} = -\frac{e - \frac{1}{e}}{2} \sim -0{,}5$$

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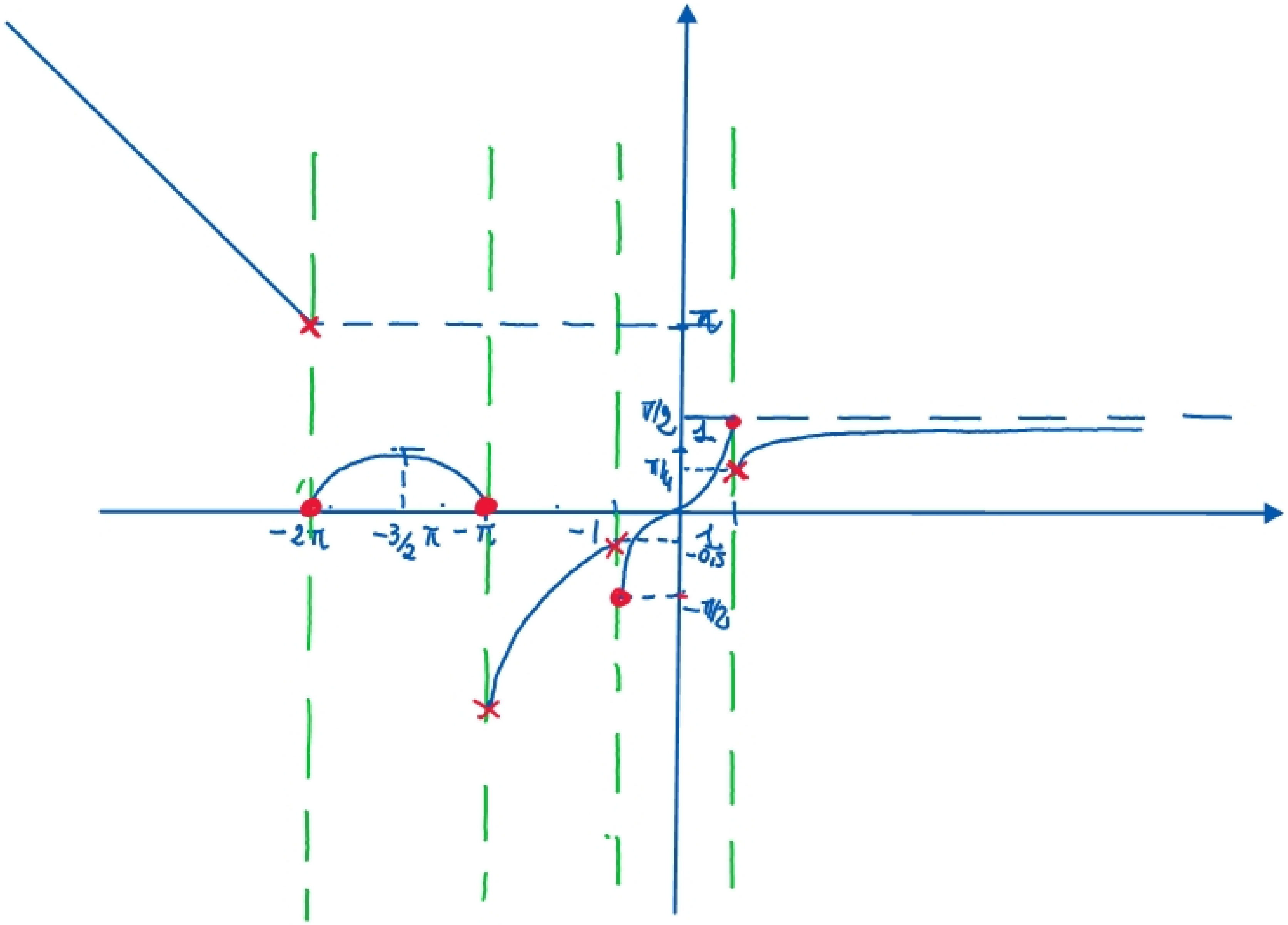

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$$\lim_{x \to -\infty} f(x) = +\infty \qquad \lim_{x \to +\infty} f(x) = \pi/2$$

$$\lim_{x \to -2\pi^-} f(x) = \pi$$

$$\lim_{x \to -2\pi^+} f(x) = 0 = f(-2\pi)$$

$$\lim_{x \to -\pi^-} f(x) = 0 = f(-\pi)$$

$$\lim_{x \to -\pi^+} f(x) = \frac{e^{-\pi} - e^{\pi}}{2}$$

$$\lim_{x \to -1^-} f(x) = -\frac{e - 1/e}{2}$$

$$\lim_{x \to -1^+} f(x) = -\pi/2 = f(-1)$$

$$\lim_{x \to 1^-} f(x) = \pi/2 = f(1)$$

$$\lim_{x \to 1^+} f(x) = \pi/4$$

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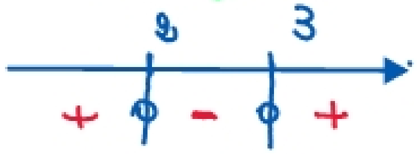
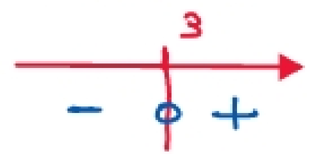
$y = |x^2 - 5x + 6| + |x - 3| =$ dom. $\forall x \in R$

$(x-3), (x-2)$

modulo 1



modulo 2



$$y = \begin{cases} x^2 - 5x + 6 - x + 3 = x^2 - 6x + 9 = (x-3)^2 & x < 2 \\ -x^2 + 5x - 6 - x + 3 = -x^2 + 4x - 3 = \\ = -(x^2 - 4x + 3) & 2 \leq x \leq 3 \\ x^2 - 5x + 6 + x - 3 = x^2 - 4x + 3 & x > 3 \end{cases}$$

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Limiti

LIMITI A VALORE FINITO

$$\lim_{\substack{x \to x_0 \\ x \to \infty}} f(x) = \begin{cases} \infty \\ \ell \\ \text{F. I.} \quad \frac{0}{0}\,;\ \frac{\infty}{\infty}\,;\ \infty^{\infty};\ 0^{0};\ \infty^{0};\ 1^{\infty} \ldots \end{cases}$$

1. raccoglimenti totali (ciò che è in comune o fratto)
2. scomposizioni
3. comportamento asintotico (catena di infiniti)
4. cambi di variabile
5. passaggio all'esponenziale $\left(f(x) = e^{\log f(x)} \right)$
6. limiti notevoli
7. De l'Hospital
8. sviluppi di Taylor $(x \to x_0)$ e Mc Laurin $(x \to x_0)$

Teoremi: 1. Carabinieri, confronto
2. esistenza ed unicità

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$$\lim_{x \to +\infty} \operatorname{sen} x = \nexists \qquad -1 \le \operatorname{sen} x \le 1$$

$$\lim_{x \to +\infty} (\operatorname{sen} x + 5) = \nexists \qquad 4 \le \operatorname{sen} x + 5 \le 6$$

$$\lim_{x \to +\infty} x (\operatorname{sen} x + 5) = +\infty$$

$$\lim_{x \to +\infty} x \operatorname{sen} x = \nexists$$

$$\lim_{x \to +\infty} \frac{\operatorname{sen} x}{x} = 0$$

NON E' UN LIM. NOT.

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$$

$$x = \frac{1}{n}$$

$$\lim_{x \to +\infty} \{x + 2 \operatorname{sen} x\} = +\infty$$

$$\lim_{n \to +\infty} n \operatorname{sen} \frac{1}{n} = \lim_{n \to +\infty} \frac{\operatorname{sen} \frac{1}{n}}{\frac{1}{n}} = 1$$

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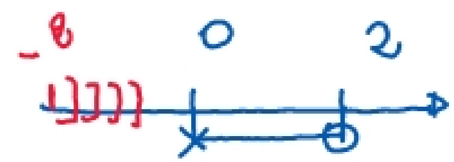
ATTENZIONE: il calcolo del limite avviene per valori appartenenti al dominio o di accumulazione.
Se la $f(x)$ non è definita nell'intorno, non si calcola il limite

$$\lim_{x \to +\infty} \left(\sqrt{4-x^2} + \underbrace{\log x}_{x>0} \right) = \cancel{\nexists}$$

$4-x^2 \geq 0$

$-2 \leq x \leq 2$

dominio è $0 < x \leq 2$



$$\lim_{x \to 0^+} \left(\sqrt{4-x^2} + \log x \right) = 4 - \infty = -\infty$$

$$\ell_{x \to 2^-} \left(\sqrt{4-x^2} + \log x \right) = \log 2$$

$$\lim_{x\to 2} \frac{x^3-2x^2-x+2}{x-2} = \frac{8-8-2+2}{2-2} = \frac{0}{0}$$ F.I.

$\Rightarrow$ $(x-2)$ è divisore del N e D

$$N(x) = x^3-2x^2-x+2 = x^2(x-2)-(x-2) = (x-2)(x^2-1)$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2-1)}{\cancel{x-2}} = 4-1 = 3$$

$$\lim_{x\to 2} \frac{1}{x-2} = \frac{1}{0} = ?\,\infty$$

$x-2$: $-$ per $x<2$, 0 in $x=2$, $+$ per $x>2$

$$f(1{,}9) = \frac{1}{-0{,}1} = -10 \;\rightarrow\; \lim_{x\to 2^-} \frac{1}{x-2} = -\infty$$

$$f(2{,}1) = \frac{1}{0{,}1} = +10 \;\rightarrow\; \lim_{x\to 2^+} \frac{1}{x-2} = +\infty$$

N.B: attenzione ai divisori ed ai segni dei fattori!

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$$\lim_{x\to+\infty} \frac{\ln(2x+e^x+\cos x)}{x} = \frac{\infty}{\infty} =$$

$$= \lim_{x\to+\infty} \frac{\ln\left[e^x\left(\frac{2x}{e^x}^{\to 0}+1+\frac{\cos x}{e^x}^{\to 0}\right)\right]}{x} =$$

$$= \lim_{x\to+\infty} \frac{\ln e^x + \overbrace{\ln\left(\frac{2x}{e^x}+1+\frac{\cos x}{e^x}\right)}^{0}}{x} =$$

$$= \lim_{x\to+\infty} \frac{x \log e}{x} = 1$$

LIMITI PARTICOLARI

$$\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1$$

$$\lim_{x\to 0} \frac{e^x-1}{x} = 1$$

$$\lim_{x\to 0} \frac{e^x-1}{2\operatorname{sen} x} = \frac{0}{0} \quad \text{F.I.}$$

$$= \lim_{x\to 0} \frac{1}{2} \underbrace{\frac{x}{\operatorname{sen} x}}_{\to 1} \cdot \underbrace{\frac{e^x-1}{x}}_{\to 1} = \frac{1}{2}$$

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$$\lim_{x\to 0} \frac{\sqrt[3]{1+x}-1}{x} = \frac{0}{0} = \lim_{t\to 1} \frac{t-1}{t^3-1} = \frac{0}{0}$$

$$t = \sqrt[3]{1+x} \qquad t \to 1$$

$$x = t^3-1 \qquad x \to 0$$

$$= \lim_{t\to 1} \frac{\cancel{t-1}}{\cancel{(t-1)}(t^2+t+1)} = \frac{1}{3}$$

$$\lim_{x\to 0} \frac{(1-\cos x)^2}{tg^3 x - sen^3 x} = \frac{0}{0} = \lim_{x\to 0} \frac{(1-\cos x)^2}{\frac{sen^3 x}{\cos^3 x} - sen^3 x} =$$

$$= \lim_{x\to 0} \frac{(1-\cos x)^2 \cdot \cos^3 x}{sen^3 x\,(1-\cos^3 x)} =$$

$$= \lim_{x\to 0} \frac{(1-\cos x)^{\cancel{2}} \cdot \overbrace{\cos^3 x}^{1}}{\cancel{(1-\cos x)}\,\underbrace{(1+\cos x+\cos^2 x)}_{3} \cdot sen^3 x} =$$

$$= \lim_{x \to 0} \frac{(1-\cos x)}{3 \operatorname{sen} x \cdot \operatorname{sen}^2 x} =$$

$$= \lim_{x \to 0} \frac{1-\cos x}{3 \operatorname{sen} x \, (1-\cos^2 x)} =$$

$$= \lim_{x \to 0} \frac{\cancel{1-\cos x}}{3 \underbrace{\operatorname{sen} x}_{\downarrow 0} \, \cancel{(1-\cos x)} \, \underbrace{(1+\cos x)}_{\downarrow 2}} = \infty$$

$$\lim_{x \to 0^-} (\quad) = -\infty$$

$$\lim_{x \to 0^+} (\quad) = +\infty$$

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