LIMITI DI FUNZIONE

LIMITI A VALORE FINITO

$$\lim_{x \to x_0} f(x) = \frac{0}{0}$$

$$\lim_{x \to x_0} f(x) = \frac{\infty}{\infty}$$

$$\lim_{x \to x_0} f(x)^{g(x)} = \infty^{\infty};\ 0^{0};\ \infty^{0};\ 1^{\infty}$$

LIMITE A VALORE INFINITO

$$\lim_{x \to \infty} f(x) = \frac{\infty}{\infty}$$

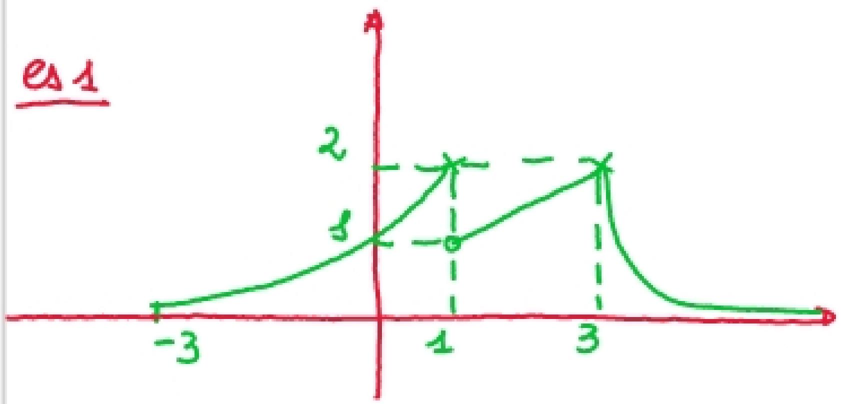
$$\lim_{x \to \infty} f(x) = \frac{0}{0}$$

$$\lim_{x \to \infty} f(x)^{g(x)} = \infty^{\infty};\ 0^{\infty};\ \infty^{0};\ 1^{\infty}$$

1. scomposizione
2. raccoglimento
3. confronto asintotico
4. passaggio all'esponenziale ($e^{\log f(x)}$)
5. limite notevole : sen x
   limite notevole : e
6. sviluppi di Taylor (per $x \to x_0 = 0$)
   Taylor $x \to 0$
   Mc Laurin $x \to x_0$
7. la catena di infiniti (e infinitesimi)

ESISTENZA E NON ESISTENZA

es 1



$$\lim_{x\to 1} f(x) = \nexists$$

$$\lim_{x\to 1^-} f(x) = 2$$

$$\lim_{x\to 1^+} f(x) = 1$$

$$\lim_{x\to 3} f(x) = 2$$

$$\lim_{x\to +\infty} f(x) = 0$$

$$\lim_{x\to -\infty} f(x) = \nexists$$

es 2

$$\lim_{x\to 0} \log x = \nexists \qquad \lim_{x\to 0^+} \log x = -\infty$$

$$\lim_{x\to +\infty} \operatorname{sen} x = \nexists \qquad -1 \le \operatorname{sen} x \le 1$$

$$\lim_{x\to +\infty} x\,(\underbrace{5+\operatorname{sen} x}_{[4,6]}) = +\infty$$

$$\lim_{x\to +\infty} \{x + (\underbrace{5+\operatorname{sen} x}_{[4,6]})\} = +\infty$$

$$\lim_{x\to +\infty} (x + \underbrace{\operatorname{sen} x}_{[-1,1]}) = +\infty$$

es 3

Siano f e g 2 funzioni a valori reali:

$$\lim_{x\to 6} f(x) = 0$$

$$\lim_{x\to 6} f(x)\cdot g(x) = 3$$

$$\Rightarrow \lim_{x\to 6} \frac{1}{g(x)} =$$

es 3 Siano f e g 2 funzioni a valori reali:

$$\lim_{x \to 6} f(x) = 0$$

$$\lim_{x \to 6} f(x) \cdot g(x) = 3$$

$$\Rightarrow \lim_{x \to 6} \frac{1}{g(x)} = ?$$

- A n.d.e.
- ~~B~~ esiste finito
- C non esiste
- D esiste, ma è finito

esempio

$$f(x) = (x-6)$$

$$g(x) = \frac{(x-3)}{(x-6)}$$

$$\lim_{x \to 6} f(x) = \lim_{x \to 6} (x-6) = 0$$

$$\lim_{x \to 6} f(x) \cdot g(x) = \lim_{x \to 6} \cancel{(x-6)} \frac{(x-3)}{\cancel{(x-6)}} = 3$$

$$\Rightarrow \lim_{x \to 6} \frac{1}{g(x)} = \lim_{x \to 6} \frac{x-6}{x-3} = 0$$

es 4

$$\lim_{x \to +\infty} \left\{ x + \underbrace{[1 + \underbrace{\operatorname{sen} x}_{[-1,1]}]}_{[0,+2]} \right\} = +\infty$$

$$\lim_{x \to +\infty} \left\{ x + \underbrace{[-2 + \underbrace{\operatorname{sen} x}_{[-1,1]}]}_{[-3,-1]} \right\} = +\infty$$

$$\lim_{x \to +\infty} x \, \underbrace{(5 + \underbrace{\operatorname{sen} x}_{[-1,1]})}_{[4,6]} = +\infty$$

$$\lim_{x \to +\infty} x \, \underbrace{(-4 + \underbrace{\operatorname{sen} x}_{[-1,1]})}_{[-5,-3]} = -\infty$$

$$\lim_{x \to +\infty} x \underbrace{\operatorname{sen} x}_{[-1,1]} = \nexists$$

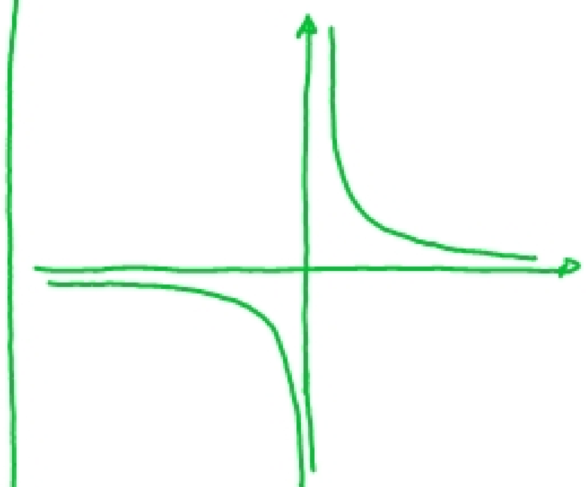
$$\lim_{x \to +\infty} x \, \underbrace{(5 + \underbrace{4 \underbrace{\operatorname{sen} x}_{[-1,1]}}_{[-4,+4]})}_{[1,9]} = +\infty$$

es 5

$$\lim_{x \to 0} \frac{1}{x} = \nexists$$

$$\lim_{x \to 0^+} \frac{1}{x} = +\infty$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$



es 6

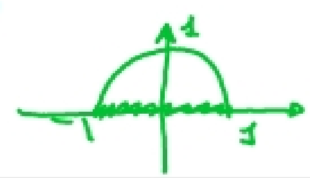
$$\lim_{x \to +\infty} \sqrt{1-x^2} = \nexists \qquad \lim_{x \to -1^-} \sqrt{1-x^2} = \nexists$$

$$\lim_{x \to -1} \sqrt{1-x^2} = \nexists \qquad \lim_{x \to -1^+} \sqrt{1-x^2} = 0$$

$D_f = [-1, 1]$

$1 - x^2 \geq 0$

$-1 \leq x \leq 1$



N.B. attenzione al dominio!

$\frac{0}{0}$

es 1

$$\lim_{x\to 2} \frac{x^2+3x-10}{3x^2-5x-2} = \frac{4+6-10}{12-10-2} = \frac{0}{0} \qquad (x-2) \qquad N(2)=0 \quad D(2)=0$$

$$= \lim_{x\to 2} \frac{(x+5)(x-2)}{(3x+1)(x-2)} = \lim_{x\to 2} \frac{x+5}{3x+1} = \frac{7}{7} = 1$$

es 2

$$\lim_{x\to 0^+} \frac{\sqrt{x+\sqrt{x}}}{x^{1/4}} = \frac{0}{0} \qquad = \lim_{x\to 0^+} \frac{\sqrt{\sqrt{x}\,(\sqrt{x}+1)}}{\sqrt[4]{x}} = \lim_{x\to 0^+} \frac{\sqrt[4]{x}\,\sqrt{\sqrt{x}+1}}{\sqrt[4]{x}} =$$

D_f: $x>0$ $\qquad x=(\sqrt{x})^2$

$$= \lim_{x\to 0^+} \left(\sqrt{\sqrt{x}+1}\right) = 1$$

es 3

$$\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} = \frac{1-1}{0} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{1+x-1+x}{x(\sqrt{1+x}+\sqrt{1-x})} = \lim_{x\to 0} \frac{2x}{x(\sqrt{1+x}+\sqrt{1-x})} =$$

$$= 2/2 = 1$$

$$D \begin{cases} x\neq 0 \\ 1+x \geq 0 \\ 1-x \geq 0 \end{cases} \qquad \begin{cases} x\neq 0 \\ x \geq -1 \\ x \leq 1 \end{cases}$$

es4 $\lim_{x \to \frac{\pi}{6}} \frac{1 + 2\cos\left(\frac{\pi}{2}+x\right)}{1 - 4\sin^2(\pi + x)} = \lim_{x \to \frac{\pi}{6}} \frac{1 - \cancel{2\sin x}}{(1-2\sin x)(1+2\sin x)} = \lim_{x \to \frac{\pi}{6}} \frac{1}{(1+2\sin x)} =$

$\cos\left(\frac{2}{3}\pi\right) = -\frac{1}{2}$

$\sin \frac{7}{6}\pi = -\frac{1}{2}$

$= \frac{1}{\left(1+2\cdot\frac{1}{2}\right)} = \frac{1}{2}$

$\cos\left(\frac{\pi}{2}+x\right) = -\sin x$

$\sin(\pi + x) = -\sin x$

$\sin^2(\pi + x) = \sin^2 x$

es5 $\lim_{x \to 1} \frac{\sqrt{x} - 1}{x^2 - 1} = \lim_{x \to 1} \frac{(\cancel{\sqrt{x}-1})}{(\cancel{\sqrt{x}-1})(\sqrt{x}+1)(x+1)} = \lim_{x \to 1} \frac{1}{(\sqrt{x}+1)(x+1)} = \frac{1}{2\cdot 2} = \frac{1}{4}$

$x^2 - 1 = (x-1)(x+1) = (\sqrt{x}-1)(\sqrt{x}+1)(x+1)$

es6 $\lim_{x \to 0} \left(\underbrace{\sin^2 \frac{1}{x} + \cos^2 \frac{1}{x}}\right)$ = apparentemente non esiste

$\sin^2 \alpha + \cos^2 \alpha = 1$

$= 1 \quad \forall x \neq 0$

es 7 $\displaystyle\lim_{x\to 0} x^2 \underbrace{\sin^2\frac{1}{x}}_{[0,1]} = 0$

$-1 \le \sin\frac{1}{x} \le 1$ $\quad x \neq 0$

$0 \le \sin^2\frac{1}{x} \le 1$

$\underset{\downarrow 0}{0} \le \underset{\downarrow 0}{x^2 \sin^2\frac{1}{x}} \le \underset{\downarrow 0}{x^2}$ Teo Carabinieri

es 8 $\displaystyle\lim_{x\to 0^+} \sqrt{x}\, \underbrace{\sin 3^{\frac{1}{x}}}_{[-1-1]} = 0$ $\quad x \to 0$ $\quad \to +\infty$

$-1 \le \sin 3^{\frac{1}{x}} \le 1$

$\underset{\downarrow 0}{-\sqrt{x}} \le \underset{\downarrow 0}{\sqrt{x}\sin 3^{\frac{1}{x}}} \le \underset{\downarrow 0}{\sqrt{x}}$

es 9 $\displaystyle\lim_{x\to 0} \frac{1}{x} \underbrace{\sin\left(a\frac{1}{x}\right)}_{[-1,1]} = \nexists$ $\quad (a \in \mathbb{R})$

$\displaystyle\lim_{x\to +\infty} \frac{\overbrace{\sin ax}^{[-1,1]}}{x} = 0$

$\frac{\infty}{\infty}$

- raccoglimento forzato
- comportamento asintotico
- confronto di infiniti

es 1 $\lim_{x\to+\infty} \frac{5x^4-3x+2}{7x^4-5x^3+2x^2-4} = \simeq \lim_{x\to+\infty} \frac{5x^4}{7x^4} = 5/7$

es 2 $\lim_{x\to+\infty} \frac{x^2-4x+1}{x^3-2x} \simeq \lim_{x\to+\infty} \frac{x^2}{x^3} = \lim_{x\to+\infty} \frac{1}{x} = 0$

es 3 $\lim_{x\to+\infty} \frac{x^5-5x+2}{x^3-2x} \simeq \lim_{x\to+\infty} \frac{x^5}{x^3} = \lim_{x\to+\infty} x^2 = +\infty$

$\lim_{x\to-\infty} \frac{x^5-5x+2}{x^3-2x} \simeq \lim_{x\to-\infty} \frac{x^5}{x^3} = \lim_{x\to-\infty} x^2 = +\infty$

es 4 $\lim_{x\to+\infty} \frac{x^3-4x}{x^3+4} \simeq \lim_{x\to+\infty} \frac{x^3}{x^3} = 1$

es5

$$\lim_{x\to+\infty}\left(\underbrace{x}_{+\infty}-\underbrace{\ln(1+4e^x)}_{-\infty}\right)=$$

$+\infty-\infty$

($4e^x$ è maggiore in termini di ordine di infinito)

$$=\lim_{x\to+\infty}\left(x-\ln\left[4e^x\left(1+\frac{1}{4e^x}\right)\right]\right)=$$

$$=\lim_{x\to+\infty}\left(x-\ln 4e^x-\ln\left(1+\frac{1}{4e^x}\right)\right)=$$

$$=\lim_{x\to+\infty}\left(x-\ln 4-\ln e^x-\ln\left(1+\frac{1}{4e^x}\right)\right)=$$

$$=\lim_{x\to+\infty}\left(\cancel{x}-\ln 4-\cancel{x\underbrace{\ln e}_{1}}-\underbrace{\ln\underbrace{\left(1+\frac{1}{4e^x}\right)}_{1}}_{0}\right)=-\ln 4$$

$+\infty - \infty$

es 1 $\lim_{x\to+\infty}\left(\sqrt{\underbrace{x^2-4x+3}_{+\infty-\infty}}+x\right)=\lim_{x\to+\infty}\left(\sqrt{x^2\left(1-\frac{4}{x}+\frac{3}{x^2}\right)}+x\right)=$

$=\lim_{x\to+\infty}\left(x\sqrt{1-\underbrace{\frac{4}{x}}_{\to 0}+\underbrace{\frac{3}{x^2}}_{\to 0}}+x\right)=+\infty+\infty=+\infty$

es 2 $\lim_{x\to-\infty}\left(\underbrace{\sqrt{x^2+4x+3}}_{+\infty}\underbrace{+x}_{-\infty}\right)=$

$=\lim_{x\to-\infty}\frac{x^2+4x+3-x^2}{\sqrt{x^2+4x+3}-x}=\lim_{x\to-\infty}\frac{4x+3}{\underbrace{|x|}_{-x}\sqrt{1+\frac{4}{x}+\frac{3}{x^2}}-x}=$

$=\lim_{x\to-\infty}\frac{x\left(4+\frac{3}{x}\right)}{-x\left(\sqrt{1+\frac{4}{x}+\frac{3}{x^2}}+1\right)}=$

$=\frac{4}{-2}=-2$

es 3 $\lim_{x\to+\infty}\left(\log_4\left(1+2\cdot4^x\right)-x\right)=$

$=\lim_{x\to+\infty}\left(\log_4\left(1+2\cdot4^x\right)-x\log_4 4\right)=$

$=\lim_{x\to+\infty}\left(\log_4\left(1+2\cdot4^x\right)-\log_4 4^x\right)=$

$=\lim_{x\to+\infty}\log_4\frac{1+2\cdot4^x}{4^x}=\lim_{x\to+\infty}\log_4\left[\frac{4^x\left(2+\frac{1}{4^x}\right)}{4^x}\right]=\log_4 2=\log_4 4^{\frac{1}{2}}=\frac{1}{2}$

LIMITE NOTEVOLE

$$\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1 \qquad \left(\lim_{n\to+\infty} \frac{\operatorname{sen}\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to+\infty} n \operatorname{sen}\frac{1}{n} \right)$$

$$\lim_{x\to 0} \frac{\sin^2 x}{x} = \lim_{x\to 0} \overbrace{\frac{\operatorname{sen} x}{x}}^{\to 1} \cdot \overbrace{\operatorname{sen} x}^{\to 0} = 0$$

$$\lim_{x\to 0} \frac{\sin \alpha x}{\sin \beta x} = \lim_{x\to 0} \overbrace{\frac{\operatorname{sen} \alpha x}{\alpha x}}^{\to 1} \cdot \alpha x \; \overbrace{\frac{\beta x}{\operatorname{sen} \beta x}}^{\to 1} \cdot \frac{1}{\beta x} = \frac{\alpha}{\beta} \qquad \alpha,\beta \in \mathbb{R}_0$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} = \lim_{x\to 0} \frac{\overbrace{1-\cos^2 x}^{\operatorname{sen}^2 x}}{x^2(1+\cos x)}$$

$$= \lim_{x\to 0} \underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1} \cdot \underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1} \cdot \frac{1}{(1+\cos x)} = \frac{1}{2}$$

$$\lim_{x\to \pi} \frac{1+\cos x}{(\pi - x)^2} = \lim_{y\to 0} \frac{1+\cos(\pi - y)}{y^2} = \lim_{x\to 0} \frac{1-\cos y}{y^2} = \frac{1}{2}$$

$$y = \pi - x \quad \underset{x\to\pi}{\to 0}$$

$$\lim_{x\to 0}\left(\frac{\sin 5x}{x}\right)^{\overset{\to 5}{x+5}} = \lim_{x\to 0}\left(5\,\underbrace{\frac{\operatorname{sen} 5x}{5x}}_{1}\right)^{\overset{\to 0}{x+5}} = 5^5$$

$$\lim_{x\to \frac{\pi}{2}}\frac{\cos x}{x-\frac{\pi}{2}} = \lim_{t\to 0}\frac{\cos\left(t+\frac{\pi}{2}\right)}{t} = \lim_{t\to 0} -\underbrace{\frac{\operatorname{sen} t}{t}}_{1} = -1$$

$t = x-\pi/2 \to 0$, $x \to \pi/2$ $\qquad \cos\left(t+\frac{\pi}{2}\right) = -\operatorname{sen} t$

$$\lim_{x\to 0}\frac{\operatorname{sen} ax}{x} = \lim_{x\to 0}\underbrace{\frac{\operatorname{sen} ax}{ax}}_{1}\cdot a = a \quad \text{con } a\neq 0$$

$$\lim_{x\to 1}\frac{(\arccos x)^4}{2(1-x)^3} = \lim_{y\to 0}\frac{y^4}{2(1-\cos y)^3} =$$

$y = \arccos x \to 0$, $x = \cos y$

$$= \lim_{y\to 0}\frac{1}{2}\left[\underbrace{\frac{y^2}{(1-\cos y)}}_{2}\cdot\underbrace{\frac{y^2}{(1-\cos y)}}_{2}\cdot\frac{1}{\underbrace{(1-\cos y)}_{0}}\right] = \infty$$

$\downarrow +\infty$

$$\lim_{x\to 0}\frac{1-\cos x}{4x^2} = \lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\frac{1+\cos x}{1+\cos x}\cdot\frac{1}{4} = \frac{1}{4}\lim_{x\to 0}\frac{1-\cos^2 x}{x^2}\cdot\frac{1}{1+\cos x} =$$

$$= \lim_{x\to 0}\frac{\operatorname{sen}^2 x}{x^2}\cdot\frac{1}{1+\underset{\to 1}{\cos x}} = 1/2$$

$$\lim_{x\to 0}\frac{x^2+x\,\mathrm{sen}\,x}{1-\cos x}=\lim_{x\to 0}\frac{x^2\left(1+\frac{\mathrm{sen}\,x}{x}\right)}{1-\cos x}=\lim_{x\to 0}\frac{1+\overbrace{\frac{\mathrm{sen}\,x}{x}}^{\to 1}}{\underbrace{\frac{1-\cos x}{x^2}}_{\to 1/2}}=\frac{2}{1/2}=4$$

$$\lim_{x\to 0}\frac{\mathrm{tg}\,x-\mathrm{sen}\,x}{x^3}=\lim_{x\to 0}\frac{\frac{\mathrm{sen}\,x}{\cos x}-\mathrm{sen}\,x}{x^3}=\lim_{x\to 0}\frac{\mathrm{sen}\,x\,(1-\cos x)}{\cos x\cdot x^3}=$$

$$=\lim_{x\to 0}\underbrace{\frac{\mathrm{sen}\,x}{x}}_{\to 1}\cdot\underbrace{\frac{1-\cos x}{x^2}}_{\to 1/2}\cdot\underbrace{\frac{1}{\cos x}}_{\to 1}=1/2$$

$$\lim_{x\to\pm\infty}\frac{x-\sin x}{x+\cos x}=\lim_{x\to\pm\infty}\frac{x\,(1-\overbrace{\frac{\mathrm{sen}\,x}{x}}^{\to 0})}{x\,(1+\underbrace{\frac{\cos x}{x}}_{\to 0})}=1$$

$$\lim_{x\to 0}\frac{x^5\cos 1/x}{x^4+x^6}=\lim_{x\to 0}\frac{x^5\cos\frac{1}{x}}{x^4(1+x^2)}=\lim_{x\to 0}\frac{x\cos x}{1+x^2}=0$$

$$\underbrace{\frac{-x}{1+x^2}}_{\to 0}\le\frac{x\cos x}{1+x^2}\le\underbrace{\frac{x}{1+x^2}}_{\to 0}$$

$$\lim_{x\to\frac{\pi}{2}^-}\left(\frac{1}{\cos x}-\mathrm{tg}\,x\right)=\lim_{t\to 0}\left(\frac{1}{\mathrm{sen}\,t}-\frac{\cos t}{\mathrm{sen}\,t}\right)=\lim_{t\to 0}\frac{1}{\mathrm{sen}\,t}\left(\frac{1-\cos t}{t^2}\right)\cdot t^2=$$

$$=\lim_{t\to 0}\underbrace{\frac{t}{\mathrm{sen}\,t}}_{\to 1}\cdot\underbrace{\frac{1-\cos t}{t^2}}_{\to 1/2}\cdot\underbrace{t}_{\to 0}=0$$

$x=\frac{\pi}{2}-t$, $t\to 0$

$\cos\left(\frac{\pi}{2}-t\right)=\mathrm{sen}\,t$

$\mathrm{sen}\left(\frac{\pi}{2}-t\right)=\cos t$

$0 \cdot \infty$

$$\lim_{x \to 0} 4x^2 \left(\frac{3}{x^2} + \frac{1}{x} \sin \frac{1}{x} \right) =$$

$$= \lim_{x \to 0} \left(12 + \underbrace{4x \sin \frac{1}{x}}_{0 \cdot [-1,1]} \right) = \lim_{x \to 0} \left(12 + 4 \frac{\sin \frac{1}{x}}{\underbrace{1/x}_{\to \infty}} \right) = 12$$

$$\lim_{x \to +\infty} \frac{x + \sin x}{x} =$$

$$= \lim_{x \to +\infty} \left(\underbrace{\frac{x}{x}}_{\downarrow\, 1} + \underbrace{\frac{\sin x}{x}}_{\to 0} \right) = 1$$

LIMITE NOTEVOLE

$$\lim_{x \to +\infty} \left(1+\frac{1}{x}\right)^x = e$$

$$\lim_{x \to \infty} \left(1+\frac{a}{x}\right)^x = e^a$$

$$\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$$

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \to +\infty} \frac{\log(1+a^x)}{x} = \log a \quad a \in \mathbb{R}_0^+ \setminus \{1\}$$

$a>$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \log a \quad (a>0, \neq 1)$$

$$\lim_{x \to 0^+} x^\alpha \left(\log_a x\right)^\beta = 0$$

con: $\alpha \in \mathbb{R}^+$

$\beta \in \mathbb{R}$

$a>0,\ a \neq 1$

$$\lim_{x\to+\infty} \frac{\log(1+2^x)}{x} = \qquad \infty/\infty$$

$$= \lim_{x\to+\infty} \frac{\log\left[2^x \cdot \left(\frac{1}{2^x}+1\right)\right]}{x} = \lim_{x\to+\infty} \frac{\log 2^x + \log\left(\frac{1}{2^x}+1\right)}{x} =$$

$$= \lim_{x\to+\infty} \left(\frac{x}{x}\log 2 + \frac{\log\left(\frac{1}{2^x}+1\right)\cdot 2^x}{x \cdot 2^x}\right) = \log 2 + \lim_{x\to+\infty} \frac{\overbrace{\log\left(1+\frac{1}{2^x}\right)^{2^x}}^{\to \log e}}{\underbrace{2^x \cdot x}_{+\infty}} =$$

$$= \log 2$$

$$\lim_{x\to 1} x^{\frac{x}{1-x}} = \lim_{x\to 1} e^{\log x^{\frac{x}{1-x}}} = \qquad 1^{\infty}$$

$$= \lim_{x\to 1} e^{\frac{x}{1-x}\log x} = e^{\lim_{t\to 0} \overbrace{(1-t)}^{\to 1} \cdot \underbrace{\frac{\log(1-t)}{-t}}_{1} (-1)} =$$

$$\left[\; t = 1-x \underset{\to 0}{} \;\; (x\to 1) \qquad x = 1-t \;\right] \qquad = e^{-1} = 1/e$$

$$\lim_{x\to 0^+} \frac{(1+2x)^{\log x}-1}{\sin x \cdot \log x^3} =$$

$$= \lim_{x\to 0^+} \frac{e^{\log (1+2x)^{\log x}}-1}{\sin x \cdot 3 \cdot \log x} = \lim_{x\to 0^+} \frac{e^{\log x \cdot \log(1+2x)}-1}{\underbrace{\frac{\sin x}{x}}_{\to 1} \cdot x \cdot 3 \cdot \log x} =$$

$$= \lim_{x\to 0^+} \underbrace{\frac{e^{\log x \cdot \log(1+2x)}-1}{\log x \cdot \log(1+2x)}}_{\left[\frac{e^t-1}{t}\right]} \cdot \underbrace{\frac{\log(1+2x)}{2x}}_{\left[\frac{\log(1+t)}{t}\right]} \; \frac{1}{3} \cdot 2 = 2/3$$

$$\lim_{x\to 0^+} \left[\log(e+x)\right]^{\frac{1}{\sqrt{x}}} =$$

$$= \lim_{x\to 0^+} e^{\log\left[\log(e+x)\right]^{\frac{1}{\sqrt{x}}}} = e^{\lim_{x\to 0^+} \frac{1}{\sqrt{x}} \cdot \log(\log(e+x))} = e^0 = 1$$

$$\Rightarrow \lim_{x\to 0^+} \frac{\log(\log(e+x))}{\sqrt{x}} = \lim_{x\to 0^+} \frac{\log\left[\log\left(e\left(1+\frac{x}{e}\right)\right)\right]}{\sqrt{x}} =$$

$$\log e = 1$$

$$= \lim_{x\to 0^+} \underbrace{\frac{\log\left(1+\log\left(1+\frac{x}{e}\right)\right)}{\log\left(1+\frac{x}{e}\right)}}_{\to 1} \cdot \underbrace{\frac{\log\left(1+\frac{x}{e}\right)}{\frac{x}{e}}}_{\to 1} \cdot \frac{\overset{\sqrt{x}}{\cancel{x}}}{e} \cdot \frac{1}{\cancel{\sqrt{x}}} = 0$$

$$\lim_{x\to 0} \frac{\overbrace{\operatorname{sen}(\log(1-x))}^{x}}{\operatorname{sen} x} = \lim_{x\to 0} \underbrace{\frac{\operatorname{sen}(\log(1-x))}{\log(1-x)}}_{\downarrow 1} \cdot \underbrace{\frac{\log(1-x)}{-x}}_{1} \cdot \underbrace{\frac{-x}{\operatorname{sen} x}}_{1} =$$

$\log 1 = 0 \qquad \operatorname{sen}(\log 1) = 0$

$$= -1$$

$$\lim_{x\to \frac{\pi}{2}} \left[1 + \sin\left(\tfrac{\pi}{2} - x\right)\right]^{\frac{2}{\operatorname{sen}\left(\frac{\pi}{2}-x\right)}} \cdot \log\left[3 \cdot \frac{1-\cos\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)^2}\right] =$$

$t = \frac{\pi}{2} - x$

$$= \lim_{t\to 0} \left[1 + \operatorname{sen} t\right]^{\frac{2}{\operatorname{sen} t}} \cdot \underbrace{\log\left[3 \; \underbrace{\frac{1-\cos t}{t^2}}_{1/2}\right]}_{\log 3/2} = \log\frac{3}{2} \cdot \lim_{y\to\infty} \underbrace{\left(1 + \frac{1}{y}\right)^{y\cdot 2}}_{e^2} =$$

$t \to 0$

$y = \frac{1}{\operatorname{sen} t} \to \infty$

$$= \log\frac{3}{2} \cdot e^2$$

$$\lim_{x\to+\infty}\left(1+\frac{1}{2x-1}\right)^{7x}=\lim_{y\to+\infty}\left(1+\frac{1}{y}\right)^{7\cdot\frac{y+1}{2}}=\lim_{y\to+\infty}\underbrace{\left(1+\frac{1}{y}\right)^{\frac{7}{2}y}}_{e}\underbrace{\left(1+\frac{1}{y}\right)^{\frac{7}{2}}}_{\substack{\downarrow\\ 1}}=e^{7/2}$$

$$y=2x-1\to\infty \quad\Rightarrow\quad x=\frac{y+1}{2}$$

$$e^{\log x^{\frac{1}{x}}}=e^{\frac{1}{x}\log x}$$

$$\lim_{x\to+\infty}\frac{x\left(x^{\frac{1}{x}}-1\right)}{\log x}=\lim_{x\to+\infty}x\underbrace{\frac{\left(e^{\frac{1}{x}\cdot\log x}-1\right)}{\log x}}_{1}=+\infty$$

$$=\lim_{x\to+\infty}\frac{e^{\frac{\log x}{x}}-1}{\frac{\log x}{x}}=1 \qquad \lim_{x\to+\infty}\frac{\log x}{x}=0$$

$$\lim_{x\to1}x^{\frac{x}{1-x}}=\lim_{x\to1}e^{\frac{x}{1-x}\log x}=\lim_{t\to0}e^{\frac{1-t}{t}\log(1-t)}= \qquad t=1-x \to 0$$

$$=\lim_{t\to0}e^{\underbrace{(1-t)}_{1}\cdot\left[\underbrace{-\frac{\log(1-t)}{-t}}_{-1}\right]}=e^{-1}=1/e$$

PASSAGGIO ALL' ESPONENZIALE

$$1^\infty \,;\; 0^0 \,;\; \infty^\infty \,; \qquad [f(x)]^{g(x)} = e^{\log [f(x)]^{g(x)}} = e^{g(x)\log f(x)}$$

$$\lim_{x\to 0^+} (\sin x^2)^{\frac{1}{\log_5 x^2}} = \lim_{t\to 0^+} (\sin t)^{\frac{1}{\log_5 t}} =$$

$t = x^2$

$\downarrow \quad \downarrow$

$0 \quad 0$

$$= \lim_{t\to 0^+} e^{\log (\sin t)^{\frac{1}{\log_5 t}}} = \lim_{t\to 0^+} e^{\frac{1}{\log_5 t}\left(\log(\sin t)\right)} = (*)$$

$$\lim_{t\to 0^+} \frac{\log(\sin t \cdot \frac{t}{t})}{\log_5 t} = \lim_{t\to 0^+} \frac{\log t + \log \frac{\sin t}{t}}{\frac{\log t}{\log 5}} = \log 5$$

$\log_5 t \to \log$

$\frac{\sin t}{t} \to 1 \to \log 1 = 0$

$$= \lim_{t\to 0^+} \log 5 \left(\frac{\log t}{\log t} + \frac{\log \frac{\sin t}{t}}{\log t} \right)$$

$$= e^{\log 5} = 5$$

$$\lim_{x\to 0} (1+\sin x)^{\frac{1}{x}} = \lim_{x\to 0} e^{\log(1+\sin x)^{1/x}} = e^{1} = e$$

$$\lim_{x\to 0} \frac{1}{x}\cdot\log(1+\sin x) = \lim_{x\to 0} \frac{1}{x}\cdot \underbrace{\frac{\log(1+\sin x)}{\sin x}}_{1} \cdot \underset{\downarrow\, 1}{\sin x}$$

$$\lim_{x\to +\infty} x^{\log\left(1+\frac{1}{x}\right)} = \lim_{x\to +\infty} e^{\log x^{\log\left(1+\frac{1}{x}\right)}} =$$

$$\lim_{x\to +\infty} \log\left(1+\frac{1}{x}\right)\cdot \log x \,\frac{x}{x} = \lim_{x\to +\infty} \underset{\downarrow\, 1}{\frac{\log\left(1+\frac{1}{x}\right)}{1/x}} \cdot \underset{\downarrow\, 0}{\frac{\log x}{x}} = 0$$

$$\lim_{x\to\frac{\pi}{2}} (1+\cos x)^{\operatorname{tg} x} = \lim_{x\to\frac{\pi}{2}} e^{\log(1+\cos x)^{\operatorname{tg} x}} = e^{1} = e$$

$$\lim_{x\to\frac{\pi}{2}} \operatorname{tg} x \cdot \log(1+\cos x) = \lim_{x\to\frac{\pi}{2}} \frac{\overset{\to 1}{\operatorname{sen} x}}{\cos x} \cdot \underbrace{\log(1+\cos x)}_{\to 1} =$$

$t = x - \frac{\pi}{2} \to 0$ $x = \left(t + \frac{\pi}{2}\right)$

$\pi/2$

PROPOSTE

$$\lim_{x\to 0^+} x^{x} = \lim_{x\to 0^+} e^{x\cdot\log x} = e^{0} = 1$$

$$\lim_{x\to+\infty} x^{\frac{1}{x}} = \lim_{t\to 0^+} \left(\frac{1}{t}\right)^{t} = \lim_{t\to 0} \frac{1}{t^{t}} = 1 \quad \left[= \lim_{x\to+\infty} e^{\overbrace{\frac{1}{x}\log x}^{\to 0}}\right] = 1$$

$0^0 = 1$

$$\lim_{x\to+\infty} e^{x} \operatorname{sen}\left(e^{-x}\cos x\right) = \nexists$$

$$\lim_{x\to+\infty}\left[(\log(\log x))^{\log x} - x(\log x)^{\log(\log x)}\right] = +\infty$$

$y=\log x$, $y \to +\infty$, $x\to+\infty$, $x=e^y$

$e^{\log(\log y)^y}$ | $e^{\log(y^{\log y})}$

$$= \lim_{y\to+\infty}\left[(\log y)^y - e^y (y)^{\log y}\right] = \lim_{y\to+\infty}\left[e^{y\cdot\log(\log y)} - e^y e^{\log y\cdot \log y}\right] =$$

$$= \lim_{y\to+\infty}\left[e^{y\log(\log y)} - e^{y+\log^2 y}\right] =$$

$$= \lim_{y\to+\infty} e^{\overset{\to+\infty}{y}\cdot \overset{\to+\infty}{\log(\log y)}}\left[1 - \underbrace{e^{y+\log^2 y - y\log(\log y)}}_{0}\right] =$$

$e^{+\infty} \to +\infty$

$$\underset{+\infty}{y}\left(1 + \underset{\to 0}{\frac{\log^2 y}{y}} - \underset{-\infty}{\log\log y}\right) \circledast \implies -\infty$$

PAPER SHOW

$$\lim_{x\to+\infty}\left(\frac{|\sin x|}{x}\right)^{x}=\lim_{x\to+\infty} e^{x\log\frac{|\sin x|}{x}} \Rightarrow e^{+\infty\cdot-\infty} \Rightarrow e^{-\infty}=0$$

$$\lim_{x\to+\infty}\log\frac{\overbrace{|\sin x|}^{[-1,1]}}{\underbrace{x}_{\to+\infty}}=\log 0=-\infty$$

$$\lim_{x\to\frac{\pi}{2}}\left(\sin^2 x\right)^{tg^2 x}=\lim_{x\to\frac{\pi}{2}} e^{\log(\sin^2 x)^{tg^2 x}}=\lim_{x\to\frac{\pi}{2}} e^{Tg^2 x\cdot\log(\sin^2 x)}=(*)$$

$$\lim_{x\to\frac{\pi}{2}}\frac{\sin^2 x}{\cos^2 x}\cdot\log(\overbrace{\sin^2 x}^{1-\cos^2 x})=\lim_{x\to\frac{\pi}{2}}\overbrace{\sin^2 x}^{\to 1}\cdot\underbrace{\frac{\log(1-\cos^2 x)}{-\cos^2 x}}_{1}(-1)=$$

$$=-1$$

$$(*)=e^{-1}=1/e$$

$$\lim_{x \to \frac{\pi}{2}} (\operatorname{sen}^2 x)^{tg^2 x} =$$

$$\lim_{x \to \frac{\pi}{2}} (1 - \cos^2 x)^{\frac{\operatorname{sen}^2 x}{\cos^2 x}} =$$

$$= \lim_{x \to \frac{\pi}{2}} \left[(1 - \cos^2 x)^{\frac{1}{\cos^2 x}} \right]^{\operatorname{sen}^2 x \to 1} \simeq \lim_{t \to -\infty} \left(1 + \frac{1}{t}\right)^{-t} =$$

$$t = -\frac{1}{\underbrace{\cos^2 x_{\to \pi/2}}_{0}} \to -\infty$$

$$= \lim_{t \to -\infty} \left[\left(1 + \frac{1}{t}\right)^{t} \right]^{-1} = e^{-1}$$

$$\lim_{x\to 0^+} \frac{(1-x)^{\log(8x)}-1}{(\operatorname{sen} x)^{\beta}} = \qquad \beta \in \mathbb{R}$$

$$= \lim_{x\to 0^+} \frac{e^{\log\left[(1-x)^{\log 8x}\right]}-1}{(\operatorname{sen} x)^{\beta}} =$$

$$= \lim_{x\to 0^+} \frac{e^{\log(8x)\cdot\log(1-x)}-1}{(\operatorname{sen} x)^{\beta}} =$$

$$= \lim_{x\to 0^+} \underbrace{\frac{e^{\log 8x\cdot\log(1-x)}-1}{\log(8x)\cdot\log(1-x)}}_{1} (-x)\, \log(8x)\cdot \underbrace{\frac{\log(1-x)}{-x}}_{1}\, \underbrace{\frac{x^{\beta}}{\operatorname{sen}^{\beta} x}}_{1} \cdot \frac{1}{x^{\beta}}$$

$$\lim_{x\to 0^+} \log(8x)\cdot\log(1-x) = 0$$

$$= \lim_{x\to 0^+} -\frac{\log(8x)}{x^{\beta-1}}$$

se $\beta-1>0$ $\quad \beta>1 \quad \ell \to 0$

$\beta=1 \quad \to +\infty$

$\beta<1 \quad \to 0$