DERIVAZIONE « CALCOLO »

es 1

$$D\left(x^3 + \operatorname{sen} x - e^x\right) =$$

$$= 3x^2 + \cos x - e^x$$

es 2 $$D\left(x^2 \cdot \cos x + e^x \cdot \operatorname{sen} x\right) =$$

$$= \underbrace{2x\cos x + x^2(-\operatorname{sen} x)}_{1^\circ \text{ prodotto}} + \underbrace{e^x \operatorname{sen} x + e^x \cos x}_{2^\circ \text{ prodotto}}$$

$$D\left(f(x) \cdot g(x)\right) = D\left(f(x)\right) \cdot g(x) + f(x) \cdot D\left(g(x)\right)$$

$$D\left(x^n\right) = n\, x^{n-1}$$

$$D\left(\left(f(x)\right)^n\right) = n\left(f(x)\right)^{n-1} \cdot D f(x)$$

$$D(\operatorname{sen} x) = \cos x$$

$$D(\cos x) = -\operatorname{sen} x$$

$$D(e^x) = e^x$$

$$D(a^x) = a^x \cdot \log_e a$$

$$D\left(\operatorname{sen} f(x)\right) = \cos\left(f(x)\right) D\left(f(x)\right)$$

es 3

$$D\left(tg\left(\frac{x^2-x}{x+1}\right)\right) =$$

$$= \underbrace{\left(1+tg^2\left(\frac{x^2-x}{x+1}\right)\right)}_{D\,tg(\ )} \cdot \underbrace{\frac{(2x-1)(x+1) - 1\,(x^2-x)}{(x+1)^2}}_{D\,(Arg)} = \dots$$

$$D(Tg(x)) = \frac{1}{\cos^2 x} = 1+tg^2 x$$

$$D(Tg(f(x))) = \frac{1}{\cos^2 f(x)} \cdot D(f(x))$$

es 4

$$D\left(\sqrt[3]{x^3-x^2}\right) = D\left((x^3-x^2)^{1/3}\right) = \underbrace{\frac{1}{3}(x^3-x^2)^{\overset{(n-1)}{-2/3}}}_{D\sqrt[3]{\ }} \underbrace{(3x^2-2x)}_{D(x^3-x^2)} =$$

$$= \frac{3x^2-2x}{3\sqrt[3]{(x^3-x^2)^2}}$$

es 5

$$D\left((\operatorname{sen}x)^{x^2-x}\right) = \text{(vedi dopo)}$$

$$D\left[\frac{N(x)}{D(x)}\right] = \frac{D(N(x))\cdot D(x) - D(D(x))\cdot N(x)}{[D(x)]^2}$$

DERIVATA FUNZIONE DI FUNZIONE

$$D\left[(f(x))^{g(x)}\right] = D\left[e^{\log(f(x))^{g(x)}}\right] = D\left[e^{g(x)\cdot\log(f(x))}\right] =$$

$$= e^{g(x)\cdot\log(f(x))}\underbrace{\left[D(g(x))\cdot\log(f(x)) + g(x)\cdot\frac{1}{f(x)}\cdot D(f(x))\right]}_{\text{derivata dell'esponente}} =$$

$D(e^{f(x)})$

$$= [f(x)]^{g(x)}\left[g'(x)\cdot\log(f(x)) + g(x)\cdot\frac{1}{f(x)}\cdot f'(x)\right]$$

$$D(\log_e x) = \frac{1}{x} \qquad D(\log_a x) = \frac{1}{x}\cdot\log_a e$$

$$D\left[(\operatorname{sen} x)^{x^2-x}\right] = D\left(e^{(x^2-x)\cdot\log(\operatorname{sen} x)}\right) =$$

$$= e^{(x^2-x)\cdot\log(\operatorname{sen} x)}\left((2x-1)\cdot\log(\operatorname{sen} x) + (x^2-x)\cdot\frac{1}{\operatorname{sen} x}\cdot\cos x\right) =$$

$$= (\operatorname{sen} x)^{(x^2-x)}\left((2x-1)\log(\operatorname{sen} x) + (x^2-x)\cdot\operatorname{ctg} x\right)$$

es 5 $\quad D\left(x\, e^{\frac{4}{\log x}}\right) = e^{\frac{4}{\log x}} + \cancel{x} \cdot e^{\frac{4}{\log x}} \cdot \left(-\frac{4}{\cancel{x}} \log^{-2} x\right) = \dots$

$$D\left(\frac{4}{\log x}\right) = 4 \cdot D\left(\log^{-1} x\right) = 4 \cdot \underbrace{\left(-1 \cdot \log^{-2} x\right)}_{t^n} \cdot \underbrace{\frac{1}{x}}_{D(\log x)}$$

es 6 $\quad D\left(x + \frac{\pi}{4} + \operatorname{arctg}\left(1 - 3e^x\right)\right) =$

$$= 1 + \underbrace{\frac{1}{1 + \left(1 - 3e^x\right)^2}}_{D(\operatorname{arctg}(\ ))} \cdot \underbrace{\left(-3e^x\right)}_{D(1-3e^x)}$$

$$D(\operatorname{ctg} x) = -\frac{1}{\operatorname{sen}^2 x} = -1 - \operatorname{ctg}^2 x$$

$$D(\operatorname{arcsen} x) = \frac{1}{\sqrt{1-x^2}}$$

$$D(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$$

$$D(k) = 0$$

$$D(\operatorname{arctg} x) = \frac{1}{1+x^2}$$

es 8 $D\left(\sqrt[3]{x}\cdot e^{\frac{1}{x}}\right) = D\left(x^{1/3}\cdot e^{x^{-1}}\right) = \frac{1}{3\sqrt[3]{x^2}}\cdot e^{\frac{1}{x}} - \frac{1}{x^2}\cdot e^{\frac{1}{x}}\cdot\sqrt[3]{x} = \ldots$

$$D\left(x^{\frac{1}{3}}\right) = \frac{1}{3}\,x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$$

$$D\left(e^{x^{-1}}\right) = e^{x^{-1}}\cdot\left(-\frac{1}{x^2}\right)$$

$$D\left(x^{-1}\right) = -1\,x^{-2} = -\frac{1}{x^2}$$

es 9 $D\left(\frac{x^2-x}{\log(x-1)}\right) =$

$$= \frac{(2x-1)\cdot\log(x-1) - (x^2-x)\cdot\frac{1}{\cancel{x-1}}}{\log^2(x-1)} =$$

$x\,\cancel{(x-1)}$

$$= \frac{(2x-1)\,\log(x-1) - x}{\log^2(x-1)}$$

es 1:

$$f(x) = \frac{1}{\sqrt{|\operatorname{sen}^3 x|}} = \begin{cases} \frac{1}{\sqrt{-\operatorname{sen}^3 x}} & \operatorname{sen} x < 0 \quad \text{---} \quad x \in [\quad] \\ \frac{1}{\sqrt{\operatorname{sen}^3 x}} & \operatorname{sen} x > 0 \quad \text{----} \quad x \in [\quad] \end{cases}$$

$$= \begin{cases} (-\operatorname{sen} x)^{-3/2} & \operatorname{sen} x < 0 \\ (\operatorname{sen} x)^{-3/2} & \operatorname{sen} x > 0 \end{cases}$$

$$f'(x) = \begin{cases} -\frac{3}{2}(-\operatorname{sen} x)^{-5/2}\underbrace{(-\cos x)}_{D(-\operatorname{sen} x)} = -\frac{3}{2}(\operatorname{sen} x)^{-5/2}\cos x & \operatorname{sen} x < 0 \\ -\frac{3}{2}\underbrace{(\operatorname{sen} x)^{-5/2}}_{D(\operatorname{sen}^{-3/2})}\underbrace{(\cos x)}_{D(\operatorname{sen} x)} & \operatorname{sen} x > 0 \end{cases}$$

es 11

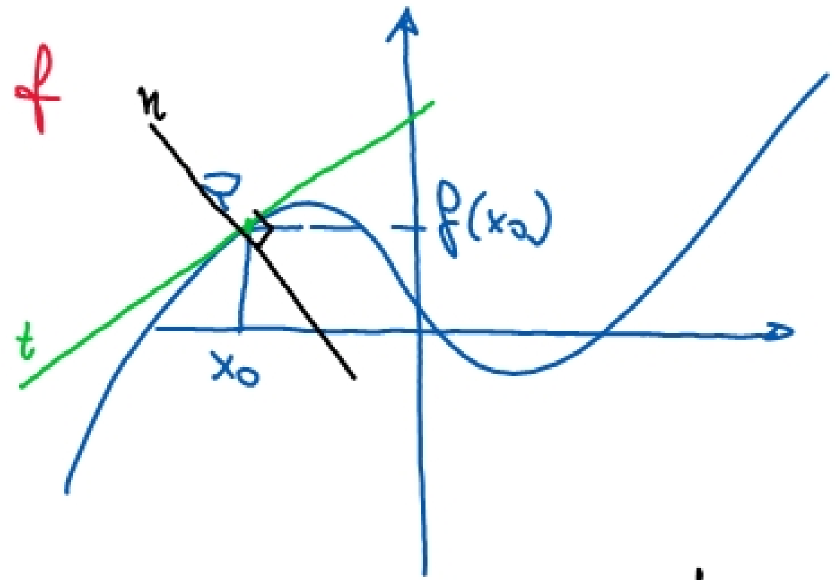
$$f(x) = \begin{cases} \sqrt{\dfrac{|1-e^x|}{\sqrt[5]{x}}} & x \neq 0 \\ 0 & x = 0 \end{cases} = \begin{cases} \dfrac{\sqrt{1-e^x}}{\sqrt[5]{x}} & x < 0 \\ 0 & x = 0 \\ \dfrac{\sqrt{e^x-1}}{\sqrt[5]{x}} & x > 0 \end{cases} \Rightarrow \begin{matrix} \dfrac{(1-e^x)^{1/2}}{x^{1/5}} \\ \\ \dfrac{(e^x-1)^{1/2}}{x^{1/5}} \end{matrix}$$

$$\left[\; 1-e^x > 0 \qquad e^x < 1 = e^0 \qquad x < 0 \;\right]$$

$$f'(x) = \begin{cases} \dfrac{\frac{1}{2}(e^x-1)^{-\frac{1}{2}}(e^x)\cdot x^{\frac{1}{5}} - (e^x-1)^{1/2}\,\frac{1}{5}\,x^{-\frac{4}{5}}}{x^{2/5}} & x > 0 \\ 0 & x = 0 \\ \dfrac{\frac{1}{2}(1-e^x)^{-\frac{1}{2}}(-e^x)\cdot x^{\frac{1}{5}} - (1-e^x)^{\frac{1}{2}}\cdot\frac{1}{5}x^{-4/5}}{x^{2/5}} & x < 0 \end{cases}$$

Retta tangente

$y = f(x)$ $\qquad$ $x_0 \in \text{dom}\, f$



$P(x_0, f(x_0))$

$y' = f'(x)$

$m_{Tg} = f'(x_0)$

$m_n = -\dfrac{1}{m_{Tg}}$

eq. retta tangente

$$y - y_P = m_{Tg}(x - x_P)$$

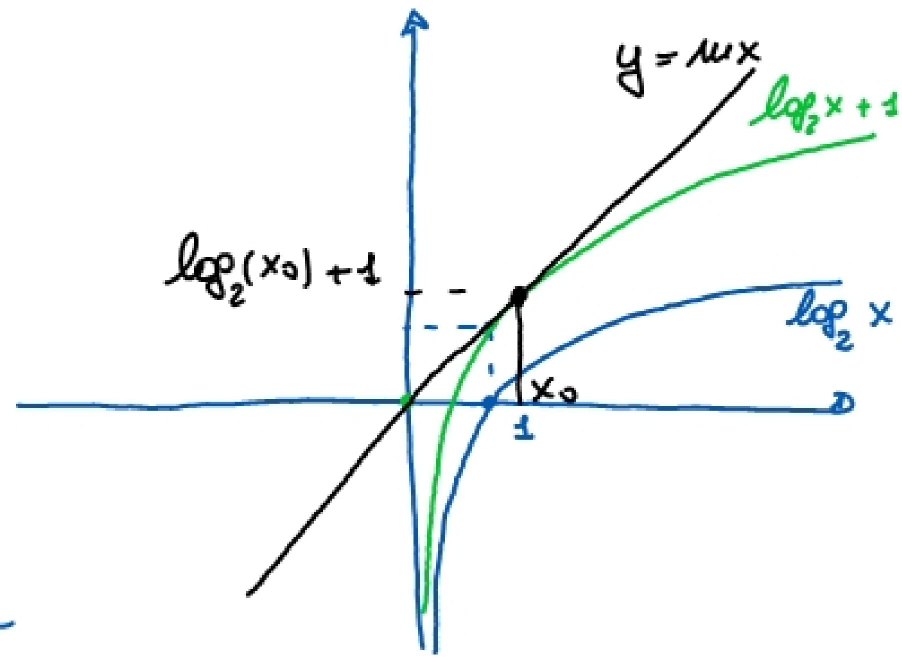
$$y - f(x_0) = f'(x_0)(x - x_0)$$

$$y = f'(x_0)(x - x_0) + f(x_0)$$

es: data la funzione $f(x) = \log_2 x + 1$

Trovare le tangenti alla curva passanti per $(0,0)$

$y = f(x) = \log_2 x + 1$ $(-)$



x_0: $y_0 = f(x_0) = \log_2 x_0 + 1$

$f'(x_0) = \frac{1}{x} \cdot \log_2 e$

$m_{Tg} = f'(x_0) = \frac{1}{x_0} \log_2 e$

retta Tangente: $y - y_0 = m_{Tg}(x - x_0) \rightarrow y - (\log_2 x_0 + 1) = \left(\frac{1}{x_0} \log_2 e\right)(x - x_0)$

passaggio per $(0,0)$ $x = 0, y = 0$: $-(\log_2 x_0 + 1) = \frac{1}{\cancel{x_0}} \log_2 e (-\cancel{x_0})$

$+\log_2 x_0 + 1 = +\log_2 e$

$$\log_2 x_0 + 1 = \log_2 e$$

$$\log_2 x_0 = \log_2 e - \underset{\log_2 2}{\overset{\shortparallel}{1}} = \log_2 e - \log_2 2 = \log_2 e/2$$

$$x_0 = e/2$$

eq. retta tangente, passante per $(0,0)$:

$$y - \left(\log_2 e/2 + 1\right) = \frac{1}{e/2} \log_2 e \left(x - e/2\right)$$

...

$$y = \frac{2}{e} \log_2 e \; x$$

RICORDARSI

1. in x_0 la $f(x)$ deve essere continua per poter calcolare l'eventuale derivata.

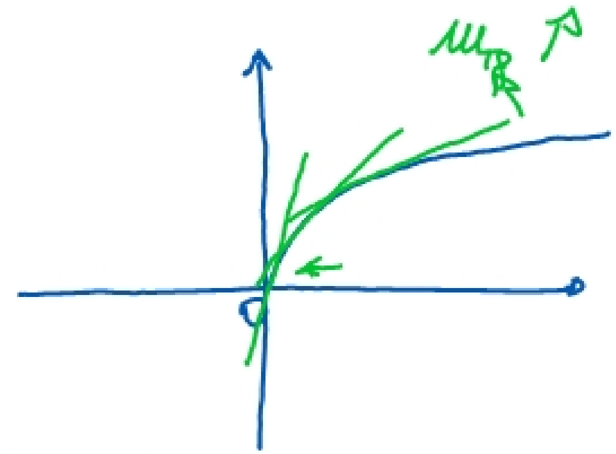
2. CONTINUITA' $\not\Rightarrow$ DERIVABILITA'

3. DERIVABILITA' $\Rightarrow$ CONTINUITA'

DOMINI $f(x)$ e $f'(x)$

1) $y = \sqrt{x}$ dominio di f $x \geq 0$

$y' = \dfrac{1}{2\sqrt{x}}$ dominio di f' $x > 0$



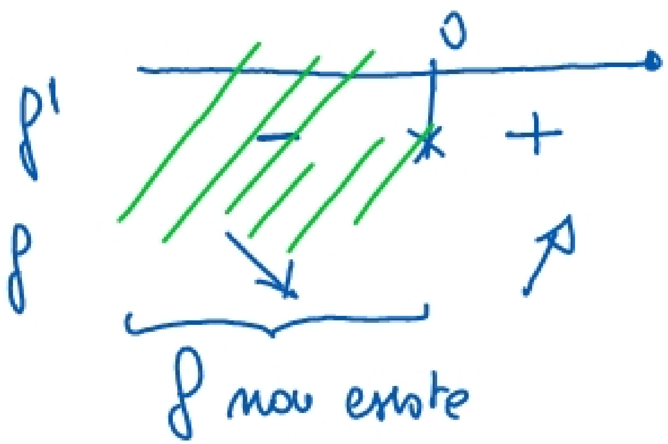
$x_0 = 0$ la f è continua ma non derivabile

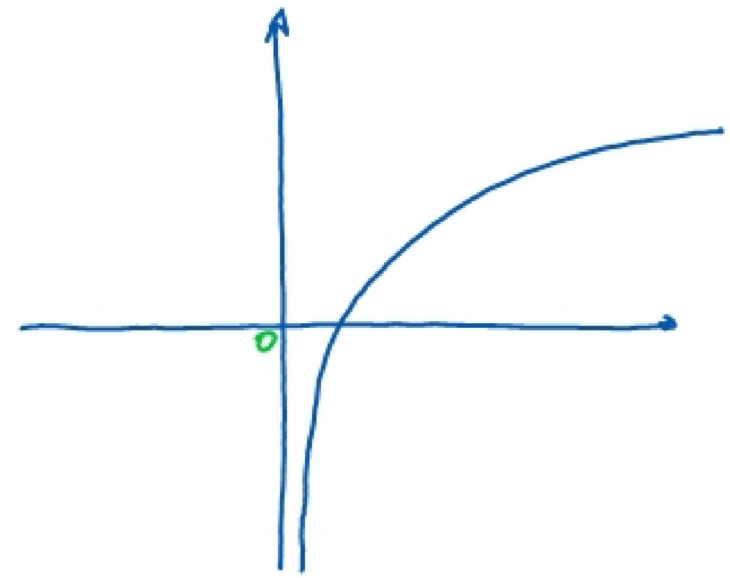
$D f' \subset D f$

$f'(x)$ $\lim_{x \to 0^+} \dfrac{1}{2\sqrt{x}} = +\infty$ pt. a Tangente verticale (Tangenza)

2) $y = \log x$ dom f $x > 0$

$y' = \frac{1}{x}$ dom f' $x \neq 0$



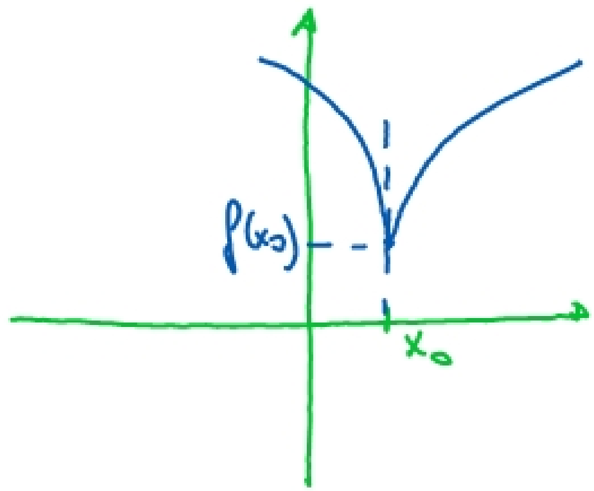


non esiste per $x < 0$ $\Rightarrow$ non devo considerare questo intervallo

PUNTI DI NON DERIVABILITA'

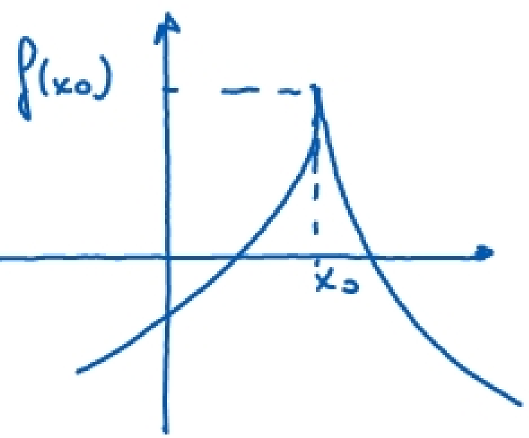
in x_0 $f(x)$ è continua, ma non è derivabile

1) CUSPIDE



$$\lim_{x \to x_0^-} f'(x) = -\infty$$

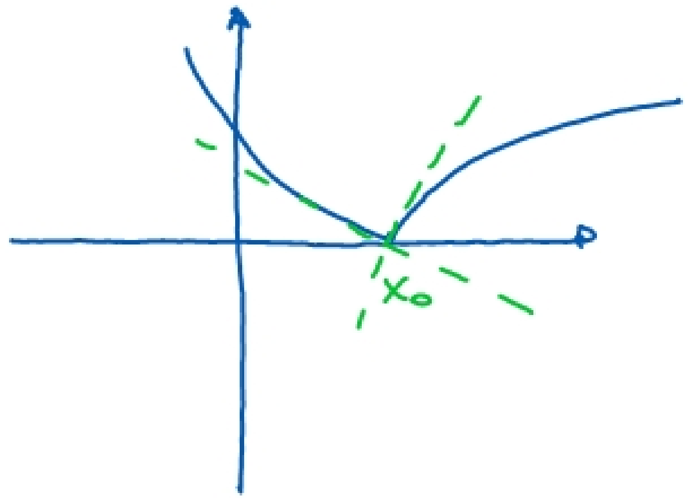
$$\lim_{x \to x_0^+} f'(x) = +\infty$$



$$\lim_{x \to x_0^-} f'(x) = +\infty$$

$$\lim_{x \to x_0^+} f'(x) = -\infty$$

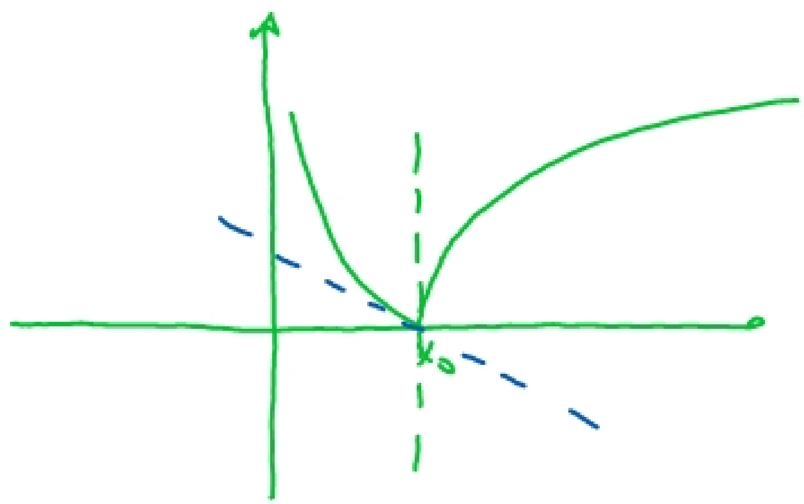
2) ANGOLOSI:



$$\lim_{x \to x_0^-} f(x) = \ell < \infty$$

$$\lim_{x \to x_0^+} f(x) = \ell' < \infty$$

$$\ell \neq \ell'$$

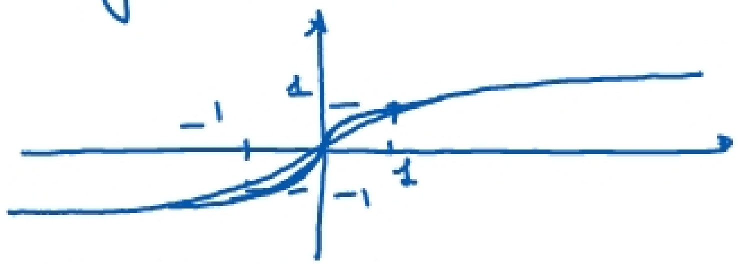


$$\lim_{x \to x_0^-} f(x) = \ell < \infty$$

$$\lim_{x \to x_0^+} f(x) = +\infty$$

3) punti a tangenza verticale

$$y = \sqrt[3]{x} = x^{\frac{1}{3}}$$



$$\lim_{x \to 0^{\pm}} \frac{1}{3\sqrt[3]{x^2}} = +\infty$$

$$y' = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}} \quad \text{dom } f' \; x \neq 0$$

in $x_0 = 0$ f non è derivabile

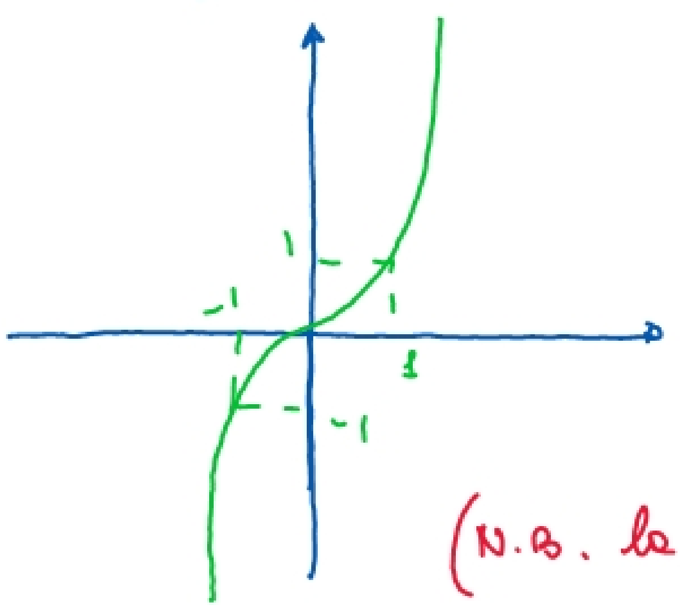
		0	
f'	$+$	$\times$	$+$
f	$\nearrow$		$\nearrow$

$x=0$ c'è la f'

---

* punti a tangenza orizzontale

$$y = x^3$$



$$y' = 3x^2$$

		0	
f'	$+$	0	$+$
f	$\nearrow$		$\nearrow$

$$m_{x_0=0} = 0$$

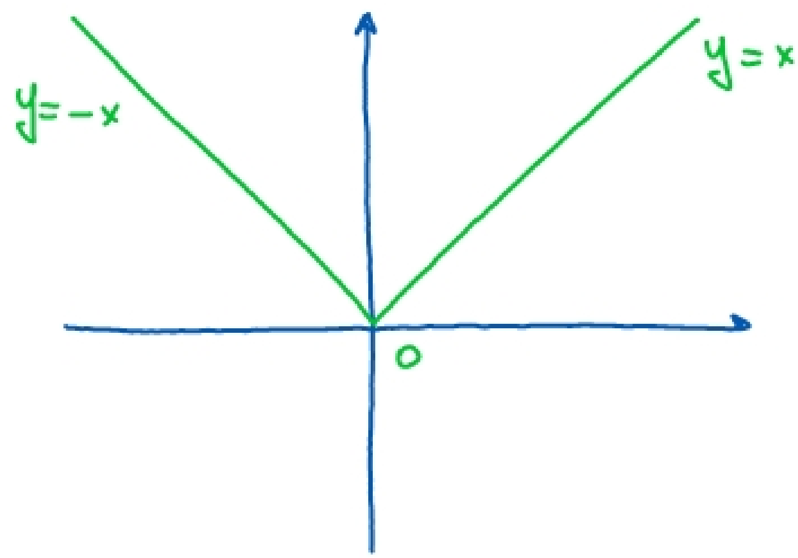
retta tg in $(0,0)$ è orizzontale (asse delle ascisse)

(N.B. la $f'(0) = 0$ ma non è di massimo o di minimo)

es 1 $y = |x|$

- continuità, derivabilità
- grafico

$$f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



la funzione è continua in $x = 0$

la funzione non è derivabile in $x = 0$.

$$f'(x) = \begin{cases} +1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

es 2 $y = \sqrt[3]{x^2}$

- grafico
- continuità
- derivabilità

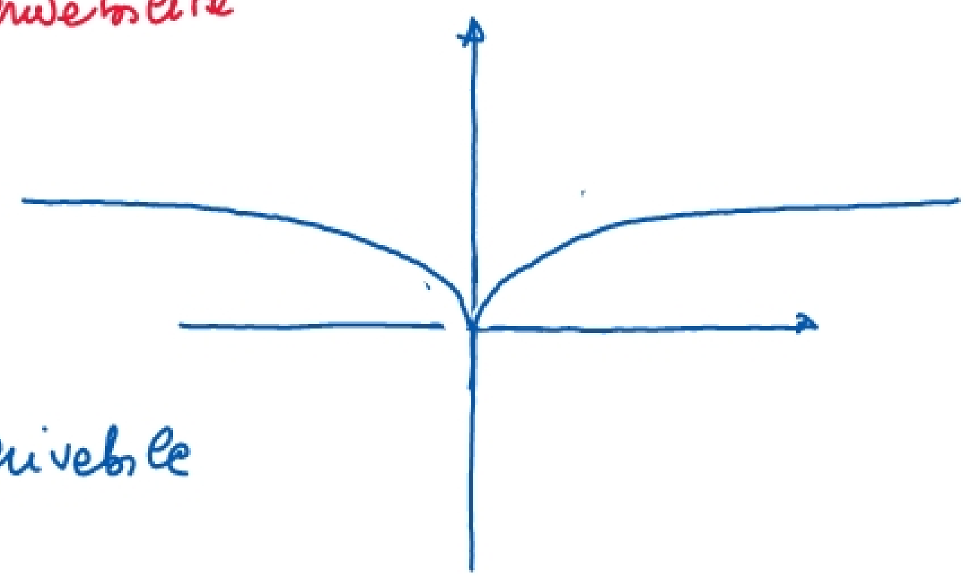
$y = x^{\frac{2}{3}}$

$y' = \frac{2}{3} x^{-1/3} = \frac{2}{3} \frac{1}{\sqrt[3]{x}}$

in $x=0$ la funzione non è derivabile

$\lim_{x \to 0^+} \frac{2}{3} \frac{1}{\sqrt[3]{x}} = +\infty$

$\lim_{x \to 0^-} \frac{2}{3} \frac{1}{\sqrt[3]{x}} = -\infty$



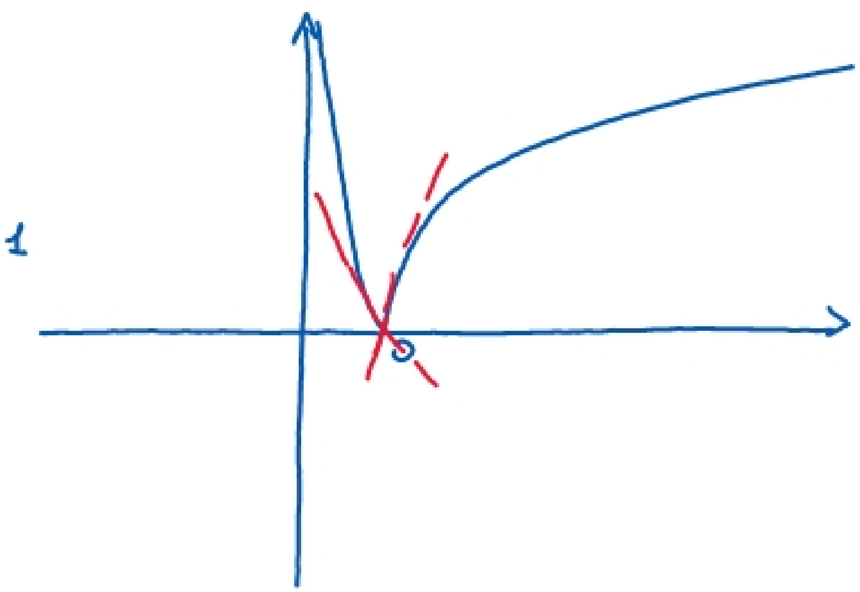
la funzione è:

- continua in $x=0$
- non derivabile in $x=0$
- in $x=0$ punto a tangente verticale

es 3 $\quad y = |\log x|$

- grafico
- continuità
- derivabilità

in $x = 0$ la funzione è continua



$$f(x) = \begin{cases} \log x & \log x \geq 0 \Rightarrow x \geq 1 \\ -\log x & 0 < x < 1 \end{cases}$$

$$f'(x) = \begin{cases} 1/x & x \geq 1 \\ -1/x & 0 < x < 1 \end{cases}$$

la funzione non è derivabile in $x = 1$, punto angoloso

T.E. $f(x) = (x-7)\sqrt[3]{x+7}$

$(x+7)^{1/3}$

$\operatorname{dom} f = \mathbb{R}$

f continua nel suo dominio

$f'(x) = \sqrt[3]{x+7} + (x-7)\dfrac{1}{3\sqrt[3]{(x+7)^2}}$

$\operatorname{dom} f' = \mathbb{R} \setminus \{-7\}$

$x \neq -7$

PASSAGGIO AL LIMITE PER $f'(x)$

$$\lim_{x\to-7^\pm} f'(x) = \lim_{x\to-7^\pm} \left\{ \underbrace{\sqrt[3]{x+7}}_{\to 0} + \underbrace{(x-7)}_{\to -14} \frac{1}{\underbrace{3\sqrt[3]{(x+7)^2}}_{\to 0}} \right\} = -\infty$$

$1/0^+ \to +\infty$

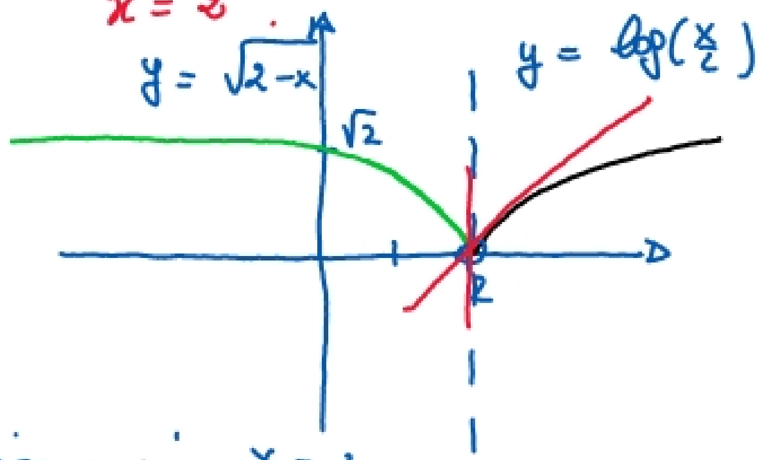
N.B $f(-7) = 0$

RAPPORTO INCREMENTALE !.

$$\lim_{x\to-7^\pm} \frac{\overset{f(x)}{(x-7)\sqrt[3]{x+7}} - \overset{f(x_0)}{0}}{\underset{(x-x_0)}{x+7}} = -\infty$$

T.E.

$$f(x) = \begin{cases} \log\left(\frac{x}{2}\right) & \text{per } x \geq 2 \\ \sqrt{2-x} & \text{per } x < 2 \end{cases}$$



CONTINUITA':

$$f_+(2) = \log\left(\frac{2}{2}\right) = \log 1 = 0$$

$$\lim_{x \to 2^-} \left(\sqrt{2-x}\right) = 0$$

f è continua in $x = 2$

$$f'(x) = \begin{cases} \frac{1}{\frac{x}{2}} \cdot \frac{1}{2} = \frac{1}{x} & \text{per } x \geq 2 \\ \frac{1}{2\sqrt{2-x}} \cdot (-1) = -\frac{1}{2\sqrt{2-x}} & \text{per } x < 2 \end{cases}$$

$$f'_+(2) = \frac{1}{2}$$

$$\lim_{x \to 2^-} f'(x) = \lim_{x \to 2^-} \left(-\frac{1}{2\sqrt{2-x}}\right) = -\infty$$

[per la parabola $x=2$ è tangente verticale]

RAPPORTO INCREMENTALE.

$$\lim_{x \to 2^+} \frac{\log\left(\frac{x}{2}\right) - \log\frac{2}{2}}{x-2} = \lim_{x \to 2^+} \frac{\log\frac{x}{2}}{(x-2)} = \ldots\ldots$$

per il logaritmo $x=2$ è punto con tangente obliqua)

RAPPORTO INCREMENTALE

$$\lim_{x\to 2^+} \frac{\log \frac{x}{2} - \log 1}{x-2} = \lim_{x\to 2^+} \frac{\log \frac{x}{2}}{x-2} \underset{\text{Taylor}}{=}$$

$$x-2 = t$$

$$\downarrow_{2} \qquad \downarrow_{0}$$

$$x = t+2$$

$$= \lim_{t\to 0^+} \frac{\log \frac{t+2}{2}}{t} =$$

$$= \lim_{t\to 0^+} \frac{\log\left(1+\frac{t}{2}\right)}{t} = \lim_{t\to 0} \frac{\frac{t}{2} + o(t)}{t} = \frac{1}{2}$$

T.E. $f(x) = \sqrt{|\operatorname{arctg}(x-7)|} + 2(x-7)$

$$= \begin{cases} \sqrt{\operatorname{arctg}(x-7)} + 2(x-7) & x \geq 7 \\ \sqrt{\operatorname{arctg}(7-x)} + 2(x-7) & x < 7 \end{cases}$$

$x_0 = 7$ continuité

$$f(7) = \underbrace{\sqrt{|\operatorname{arctg}(\underset{0}{7-7})|}}_{0} + 2(\underset{=0}{7-7}) = 0$$

$$f'(x) = \begin{cases} \frac{1}{(x-7)^2+1} \cdot \frac{1}{2\sqrt{\operatorname{arctg}(x-7)}} + 2 & x > 7 \\ \frac{1}{(7-x)^2+1} (-1) \cdot \frac{1}{2\sqrt{\operatorname{arctg}(7-x)}} + 2 & x < 7 \end{cases}$$

dom f' $x \neq 7$

$$\lim_{x \to 7^-} \left\{ \underbrace{\frac{1}{2\sqrt{\operatorname{arctg}(7-x)}}}_{+\infty} \cdot \underbrace{\frac{-1}{(7-x)^2+1}}_{-1} + 2 \right\} = -\infty$$

$x_0 \Rightarrow$ pt. cuspide

$$\lim_{x \to 7^+} \left\{ \underbrace{\frac{1}{2\sqrt{\operatorname{arctg}(x-7)}}}_{+\infty} \cdot \underbrace{\frac{1}{(x-7)^2+1}}_{+1} + 2 \right\} = +\infty$$

CON IL RAPPORTO INCREMENTALE

$$\lim_{x\to 7^{\pm}} \frac{f(x)-f(x_0)}{(x-x_0)} =$$

$$= \lim_{x\to 7^{\pm}} \frac{\sqrt{|\operatorname{arctg}(x-7)|}+2(x-7)-0}{(x-7)} =$$

$$= \lim_{x\to 7^{\pm}} \left\{ \frac{\sqrt{|\operatorname{arctg}(x-7)|}}{(x-7)}+2 \right\} = \begin{cases} +\infty \\ -\infty \end{cases}$$

(anche con Taylor)

(provate il denominatore come infinitesimo)

T.E. $f(x) = \sqrt{e^{7x} - 7x - 1}$ dom $f = \mathbb{R}$

$(x_0 = 0)$

CONTINUITA' $f(0) = \sqrt{e^0 - 7\cdot 0 - 1} = \sqrt{1 - 0 - 1} = 0$

f è continua.

DERIVABILITA' $f'(x) = \frac{1}{2\sqrt{e^{7x} - 7x - 1}} (e^{7x} \cdot 7 - 7) = \frac{7}{2} \frac{e^{7x} - 1}{\sqrt{e^{7x} - 7x - 1}}$

$$\lim_{x \to 0^\pm} \frac{e^{7x} - 1}{\sqrt{e^{7x} - 7x - 1}} \cdot \frac{7}{2} = \lim_{x \to 0^\pm} \frac{1 + 7x - 1 + o(x)}{\sqrt{1 + 7x + \frac{49}{2} x^2 - 7x - 1 + o(x^2)}} \cdot \frac{7}{2}$$

(con Taylor)

$$= \frac{7}{2} \lim_{x \to 0^\pm} \frac{7x + o(x)}{\sqrt{\frac{49}{2} x^2 + o(x^2)}} = \frac{7}{2} \frac{7}{\pm 7/\sqrt{2}} = \pm \frac{\sqrt{2}}{2} \cdot 7$$

punto angoloso.

pari fredo.

RAPPORTO INCREMENTALE

$$\lim_{x \to 0^+} \frac{f(x) - f(x_0)}{x - x_0} =$$

$$= \lim_{x \to 0^+} \frac{\sqrt{e^{7x} - 7x - 1} - 0}{x - 7} =$$

$$= \lim_{x \to 0^+} \frac{\sqrt{e^{7x} - 7x - 1}}{x - 7} \underset{\text{Taylor}}{=} \frac{\sqrt{\frac{49}{2}x^2 + o(x^2)}}{x - 7} =$$

pari grad.

$$= \pm \frac{7}{\sqrt{2}} = \pm \frac{7 \cdot \sqrt{2}}{2}$$

T. E.

$$f(x) = \begin{cases} x^{7\beta} & x \geq 0 \\ 2x^2 \log(-x) & x < 0 \end{cases}$$

per quale/i valore/i di $\beta \in \mathbb{R}^+$ la f è derivabile nel suo dominio?

$(x_0 = 0)$

CONTINUITA'

$f_+(0) = 0$ e $\lim_{x \to 0^-} 2x^2 \log(-x) = 0$ (ordine di infinitesimo)

è continua $\forall \beta \in \mathbb{R}$

DERIVATA:

$$f'(x) = \begin{cases} 7\beta\, \cancel{x}^{7\beta - 1} & x \geq 0 \\ 2\left(2x \log(-x) + x \cdot \frac{1}{-x} \cdot (-1)\right) & x < 0 \end{cases} = \begin{cases} 7\beta\, x^{7\beta-1} & x \geq 0 \\ 2x\left(2\log(-x)+1\right) & x < 0 \end{cases}$$

$f'_+(0) = 0$

$\lim_{x \to 0^-} 2\left(x\left(2\log(-x)+1\right)\right) = 0$?

$\Rightarrow$ l'uguaglianza tra le derivate è vera se $7\beta - 1 > 0$

$\Rightarrow \beta > 1/7$

calcolo del limite: $\left[x \cdot \left(2 \log(-x) + 1\right)\right] \underset{T.}{=} x\left(2 \log\left(1 + (-1+x)\right) + 1\right) = 2\left(2(-x+1)\right) =$
$= -2x^2 + x \to 0$