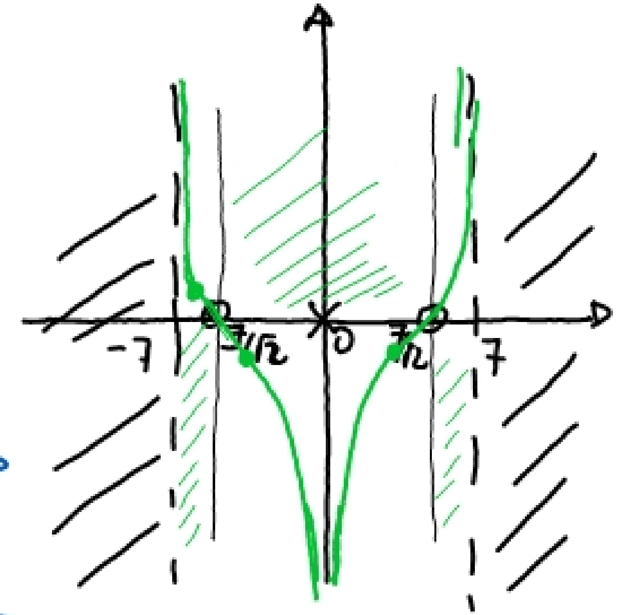
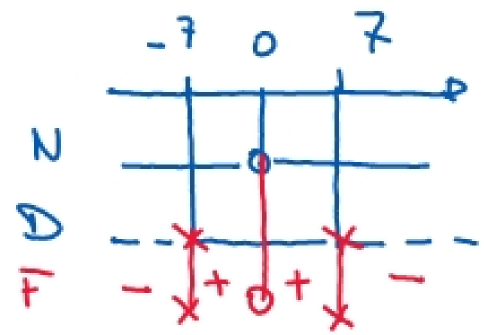
① T.E.

STUDIO COMPLETO DI $y = f(x)$

$$y = \log \frac{x^2}{49-x^2}$$



1- dominio $\quad \dfrac{x^2}{49-x^2} > 0$



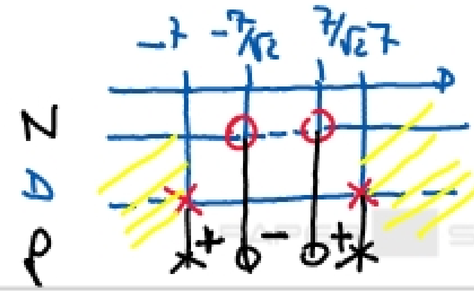
$\Rightarrow$ dom f.: $]-7,0[\, \cup\,]0,7[$

2- dominio simmetrico rispetto all'origine

$$f(-x) = \log \frac{(-x)^2}{49-(-x)^2} = \log \frac{x^2}{49-x^2} = f(x) \Rightarrow \text{pari}$$

3- $f(x) \geq 0 \Rightarrow \log \dfrac{x^2}{49-x^2} \geq 0 \qquad \dfrac{x^2}{49-x^2} \geq 1$

$$\frac{x^2-49+x^2}{49-x^2} \geq 0 \qquad \frac{2x^2-49}{49-x^2} \geq 0$$



4- Limiti

$$\lim_{x\to -7^+} \log \frac{x^2}{49-x^2} = \log \frac{49}{0} = +\infty$$

$$\lim_{x\to 7^-} \log \frac{x^2}{49-x^2} = \log \frac{49}{0} = +\infty$$

$$\lim_{x\to 0^\pm} \log \frac{x^2}{49-x^2} = \log 0 = -\infty$$

$f(x)$ ammette 3 asintoti verticali: $x=-7$
$x=0$
$x=7$

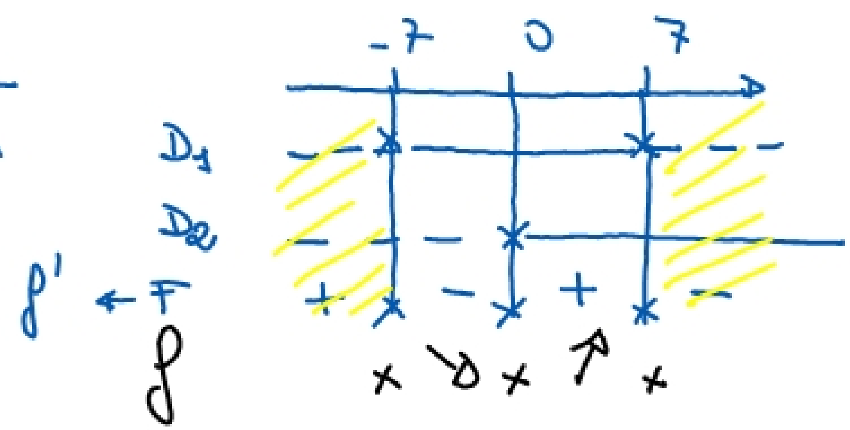
Dominio limitato $\Rightarrow$ no asintoti obliqui o orizzontali

5- derivate

$$y = \log \frac{x^2}{49 - x^2}$$

$$y' = f'(x) = \underbrace{\cancel{\frac{49 - x^2}{x^2}}}_{D \log} \cdot \underbrace{\frac{2x(49-x^2) - x^2 \cdot (-2x)}{(49-x^2)^{\cancel{2}}}}_{D \text{ Arg.}} = \frac{2\cancel{x}(49 - \cancel{x^2} + \cancel{x^2})}{x^{\cancel{2}}(49-x^2)} =$$

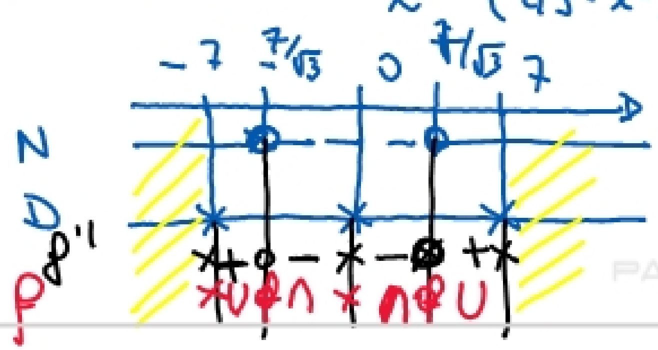
$$= \frac{98}{(49-x^2)X}$$



$Df' \supset Df$

$$f''(x) = 98 \frac{-[-2x \cdot x + 49 - x^2]}{(49-x^2)^2 \, x^2} = 98 \cdot \frac{-(-2x^2 + 49 - x^2)}{x^2 \, (49-x^2)^2} =$$

$$= 98 \cdot \frac{3x^2 - 49}{x^2(49-x^2)^2}$$



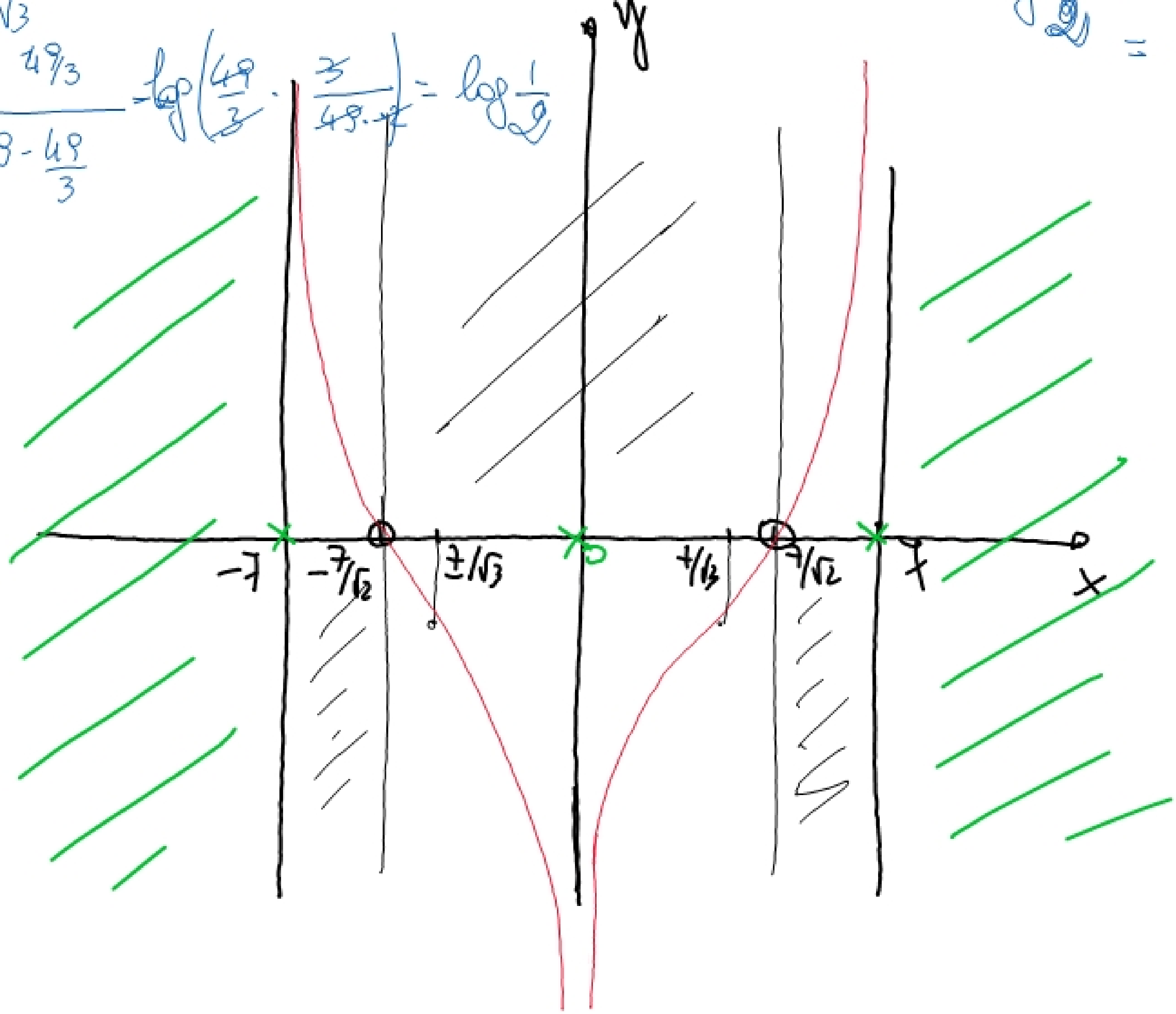
Flessi

F_1 in $x = -7/\sqrt{3}$

F_2 in $x = 7/\sqrt{3}$

$$\begin{cases} x_{1,2} = \pm \dfrac{7}{\sqrt{3}} \\ y = \log \dfrac{49/3}{49 - \frac{49}{3}} = \log\left(\dfrac{49}{3} \cdot \dfrac{3}{49 \cdot 2}\right) = \log \dfrac{1}{2} \end{cases}$$

$$\log \frac{1}{2} < 0$$
$$= -\log 2$$



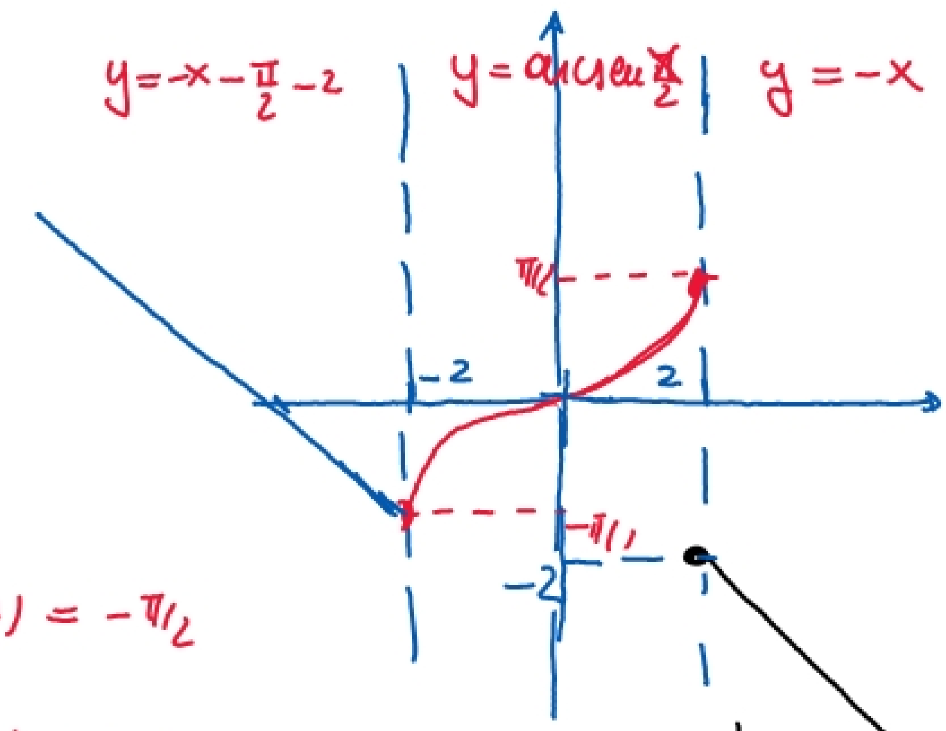
TE 3

$$f(x)=\begin{cases} -x-\frac{\pi}{2}-2 & \text{se } x<-2\\ \arcsin\left(\frac{x}{2}\right) & \text{se } -2\le x\le 2\\ -x & \text{se } x>2\end{cases}$$

dominio $= \mathbb{R}$

$$\lim_{x\to+\infty}(-x)=-\infty$$

$$\lim_{x\to-\infty}\left(-x-\frac{\pi}{2}-2\right)=+\infty$$



$$f(-2)=\arcsin\left(-\frac{2}{2}\right)=\arcsin(-1)=-\pi/2$$

$$f(2)=\arcsin\left(\frac{2}{2}\right)=\arcsin(1)=\pi/2$$

$y=-x-\pi/2-2$

$$\lim_{x\to-2^-}\left(-x-\pi/2-2\right)=2-\pi/2-2=-\pi/2$$

$$\begin{cases} y=0\\ -x-\pi/2-2=0\end{cases}\qquad x=-\pi/2-2$$

$y=-x$

$$\lim_{x\to 2^+}(-x)=-2$$

la funzione è

1) in $x=-2$ continua: $\lim_{x \to -2^-} f(x) = f(-2)$

in $x = 2$ presenta una discontinuità a salto

2) non presenta asintoti, per la natura degli operatori algebrici che la definiscono

Se calcolo la derivata:

$$f'(x) = \begin{cases} -1 & x < -2 \\ \dfrac{1}{\sqrt{1 - \frac{x^2}{2}}} \cdot \dfrac{1}{2} & -2 < x < 2 \\ -1 & x > 2 \end{cases}$$

N.B. segno f'

		-2		2	
f'	$-$	$\times$	$+$	$\times$	$-$
f	$\searrow$	$\times$	$\nearrow$	$\times$	$\searrow$

attenzione in $x = \pm 2$

discussione sulla derivabilità di f!

N.B. in $x = -2$ discuto la derivabilità, perché f è continua

in $x = 2$ non discuto la derivabilità, perché f non è continua.

$$\lim_{x\to -2^+} \frac{f(x)-f(\overset{=-2}{x_0})}{x-\underset{=-2}{x_0}} = \lim_{x\to -2^+} \frac{\arcsin\left(\frac{x}{2}\right)-\arcsin\left(\frac{-2}{2}\right)}{x+2} =$$

$$= \lim_{x\to -2^+} \frac{\arcsin\left(\frac{x}{2}\right)-\arcsin(-1)}{x+2} = +\infty$$

(prevale il denominatore infinitesimo)

$$\lim_{x\to -2^-} \frac{-x-\pi/2-2-\left(+\cancel{2}-\pi/2-\cancel{2}\right)}{x+2} = \lim_{x\to -2^-} \frac{-x-\cancel{\pi/2}-2+\cancel{\pi/2}}{x+2} =$$

$$= \lim_{x\to -2^-} \left(-\frac{x+2}{x+2}\right) = -1$$

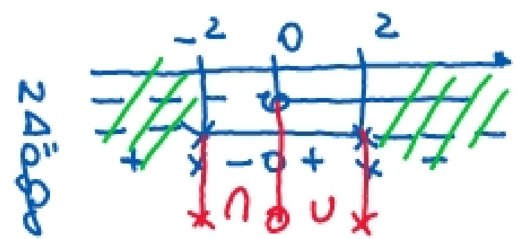
punto angoloso $x=-2$

derivata seconda:

$$f''(x) = \begin{cases} 0 & x<-2 \\ \dfrac{x}{\sqrt{(4-x^2)^3}} & -2<x<2 \\ 0 & x>2 \end{cases}$$

[no concavità/convessità si tratta di una retta]

$$f''(x) = \frac{x}{(4-x^2)\sqrt{4-x^2}}$$



T.E. 3

$$f(x) = 2|x|\, e^{-(x^2+7)} = \begin{cases} -2x\, e^{-(x^2+7)} & x < 0 \\ 2x\, e^{-(x^2+7)} & x \geq 0 \end{cases}$$

1. dominio $= \mathbb{R}$

2. simmetrie

$$f(-x) = 2|-x|\, e^{-((-x)^2+7)} = 2|x|\, e^{-(x^2+7)} = f(x)$$

pari

3. segno

$$f(x) \geq 0 \quad \forall x \in \text{dominio}.$$

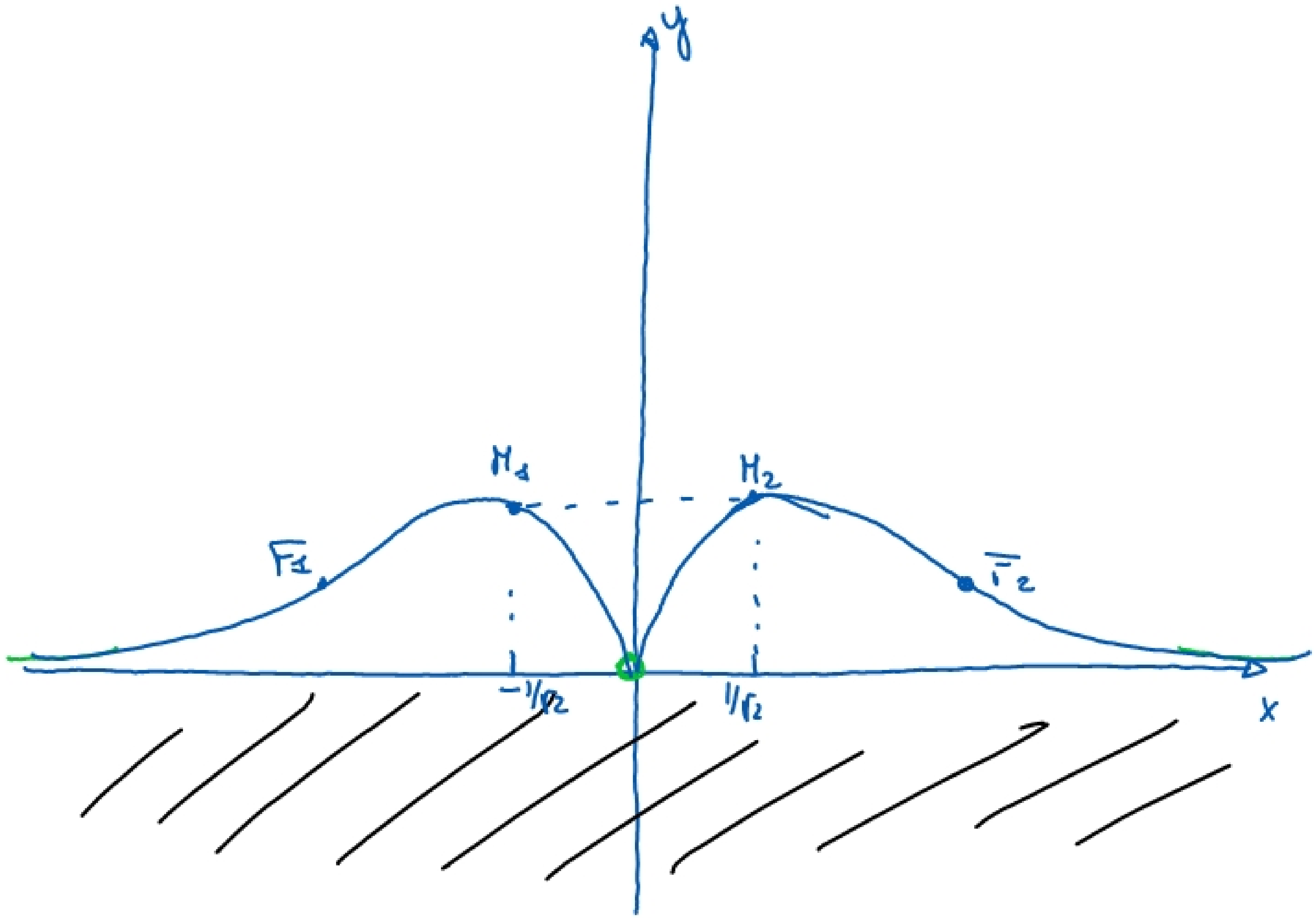
$$f(0) = 0 \qquad (0,0) \in f(x)$$

4. limiti

$$\lim_{\substack{x \to +\infty \\ (x \to -\infty)}} 2|x|\, e^{-(x^2+7)} = \lim_{x \to +\infty} \frac{2|x|}{e^{x^2+7}} = 0$$

(più veloce $e^{(\ )}$ rispetto x)

$\Rightarrow$ $y = 0$ asintoto orizzontale



y
M_1
M_2
F_1
F_2
$-1/\sqrt{2}$
$1/\sqrt{2}$
x

5. continuità: la $f(x)$ è continua nel suo dominio

$$f(0)=0$$

6. derivata prima:

$$f(x) = \begin{cases} -2x\, e^{-(x^2+7)} & x<0 \\ 2x\, e^{-(x^2+7)} & x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} -2\left[e^{-(x^2+7)} + x\, e^{-(x^2+7)} \cdot (-2x) \right] = -2\, e^{-(x^2+7)} (1-2x^2) & [x<0] \\ 2\left[e^{-(x^2+7)} + x\, e^{-(x^2+7)} \cdot (-2x) \right] = \underset{+}{2}\, \underbrace{e^{-(x^2+7)}}_{+} \underbrace{(1-2x^2)} & [x \geq 0] \end{cases}$$

essendo $f(x)$ pari, studio il segno solo per $x \geq 0$, e poi "simmetrizzo"

$1-2x^2 \geq 0$

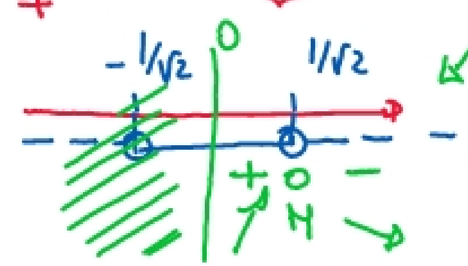


grafico completo $f'(x)$

x		$-1/\sqrt{2}$		0		$1/\sqrt{2} = \sqrt{2}/2$	
segno $f'(x)$	$+$	0	$-$		$+$	0	$-$
andamento $f(x)$	$\nearrow$	MAX	$\searrow$		$\nearrow$	MAX	$\searrow$

ordinate del punto di massimo

$$f\left(\pm\frac{1}{\sqrt{2}}\right) = 2\left|\pm\frac{1}{\sqrt{2}}\right| e^{-\left(\frac{1}{2}+7\right)} = \frac{2}{\sqrt{2}} e^{-15/2} > 0$$

$x=0$ punto critico

$$\lim_{x\to 0^+} 2 \underset{e^{-7}}{e^{-(x^2+7)}} (\underbrace{1-2x^2}_{1}) = 2e^{-7}$$

$$\lim_{x\to 0^-} \left(-2 e^{-(x^2+7)} (1-2x^2)\right) = -2e^{-7}$$

$x=0$ punto di non derivabilità

derivate seconde

$$f''(x) = \begin{cases} -4x\, e^{-(x^2+7)}(3-2x^2) & x \geq 0 \\ 4x\, e^{-(x^2+7)}(3-2x^2) & x < 0 \end{cases}$$

studio per $x \geq 0$ il segno di $f''(x)$

	$-\sqrt{3/2}$	0		$\sqrt{3/2}$	
-4			$-$		$-$
x		0	$+$		$+$
$(3-2x^2)$	0		$+$	0	$-$
$f''(x)$			$-$	0	$+$
$f(x)$			$\cap$	f	$\cup$

$x = 0$ non è punto di flesso

grafico completo $f''(x)$

		$-\sqrt{3/2}$		0		$\sqrt{3/2}$	
$f''(x)$	$+$	0	$-$	0	$-$	0	$+$
$f(x)$	$\cup$	F_1	$\cap$	$\nexists$ non flesso	$\cap$	F_2	$\cup$

(non vi è cambio di concavità)

ES 4

$$f(x) = 7x + 2 + x\log|x| = \begin{cases} 7x + 2 + x\log x & x > 0 \\ 7x + 2 + x\log(-x) & x < 0 \end{cases}$$

dominio $\quad x \neq 0$

non vi sono simmetrie

limiti $\quad \lim_{x \to +\infty} (7x + 2 + x\log x) = +\infty$

$$\lim_{x \to -\infty} (\underbrace{7x + 2}_{-\infty} + \underbrace{x\log(-x)}_{-\infty\ (+\infty)}) = -\infty$$

$$\lim_{x \to 0^\pm} (7x + 2 + x\log|x|) = +2 \qquad x = 0 \text{ asintoto verticale}$$

ricerca asintoto obliquo

$$m = \lim_{x \to \infty} \frac{7x + 2 + x\log|x|}{x} = \infty \qquad \text{non vi sono asintoti obliqui}$$

$$\Big[7 + \underbrace{\frac{2}{x}}_{\to 0} + \underbrace{\log|x|}_{\to \infty}\Big]$$

Studio del segno

$$f(x) = 7x+2+x\log|x| = \begin{cases} 7x+2+x\log x & x>0 \\ 7x+2+x\log(-x) & x<0 \end{cases}$$

per $x>0$: $\quad 7x+2+x\log x \geq 0$

$$x\log x \geq -7x-2$$

$$\log x \geq \frac{-7x-2}{x}$$

$y = \dfrac{-7x-2}{x}$ funzione omografica

asintoto oriz. $\quad y=-7$

asintoto vert. $\quad x=0$

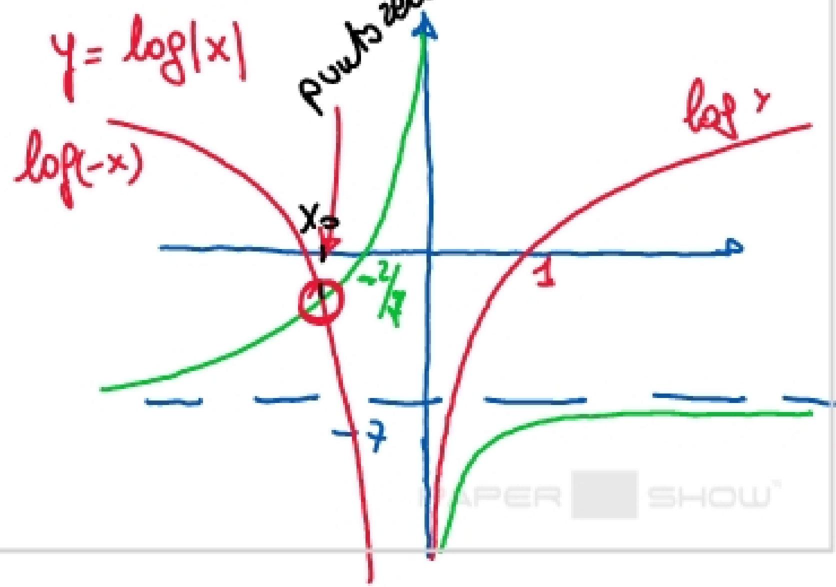
per $x<0$ $\quad 7x+2+x\log(-x) \geq 0$

$$x\log(-x) \geq -7x-2$$

$$\log(-x) \geq \frac{-7x-2}{x}$$

$-1 < x_0 < -2/7$

segno $\quad - \quad \phi \quad +$



derivata prima

$$f'(x) = 7 + \log|x| + \frac{x}{x}\,\mathrm{sgn}(x) = \begin{cases} 7 + \log x + 1 & x > 0 \\ 7 + \log(-x) + 1 & x < 0 \end{cases}$$

studio del segno

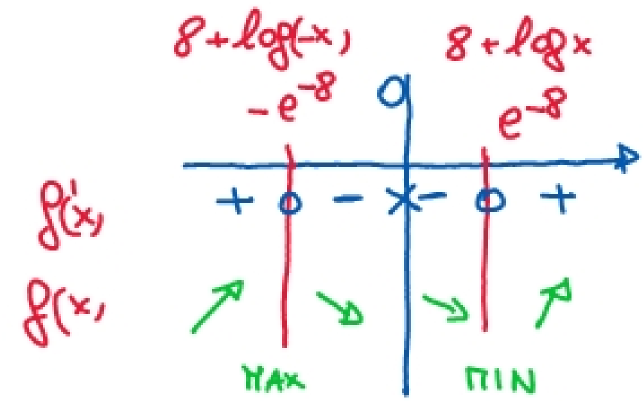
$x > 0$: $\quad 8 + \log x \geq 0$

$$\log x \geq -8 \qquad x \geq e^{-8}$$

$x < 0$ $\quad 8 + \log(-x) \geq 0$

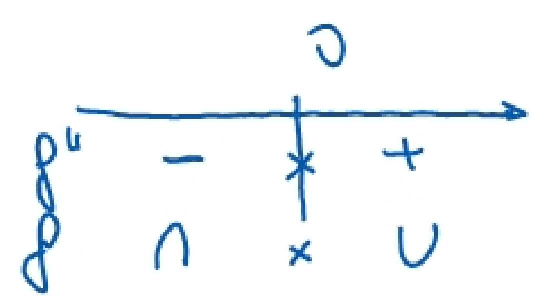
$$\log(-x) \geq -8 \qquad -x \geq e^{-8}$$

$$x \leq -e^{-8}$$

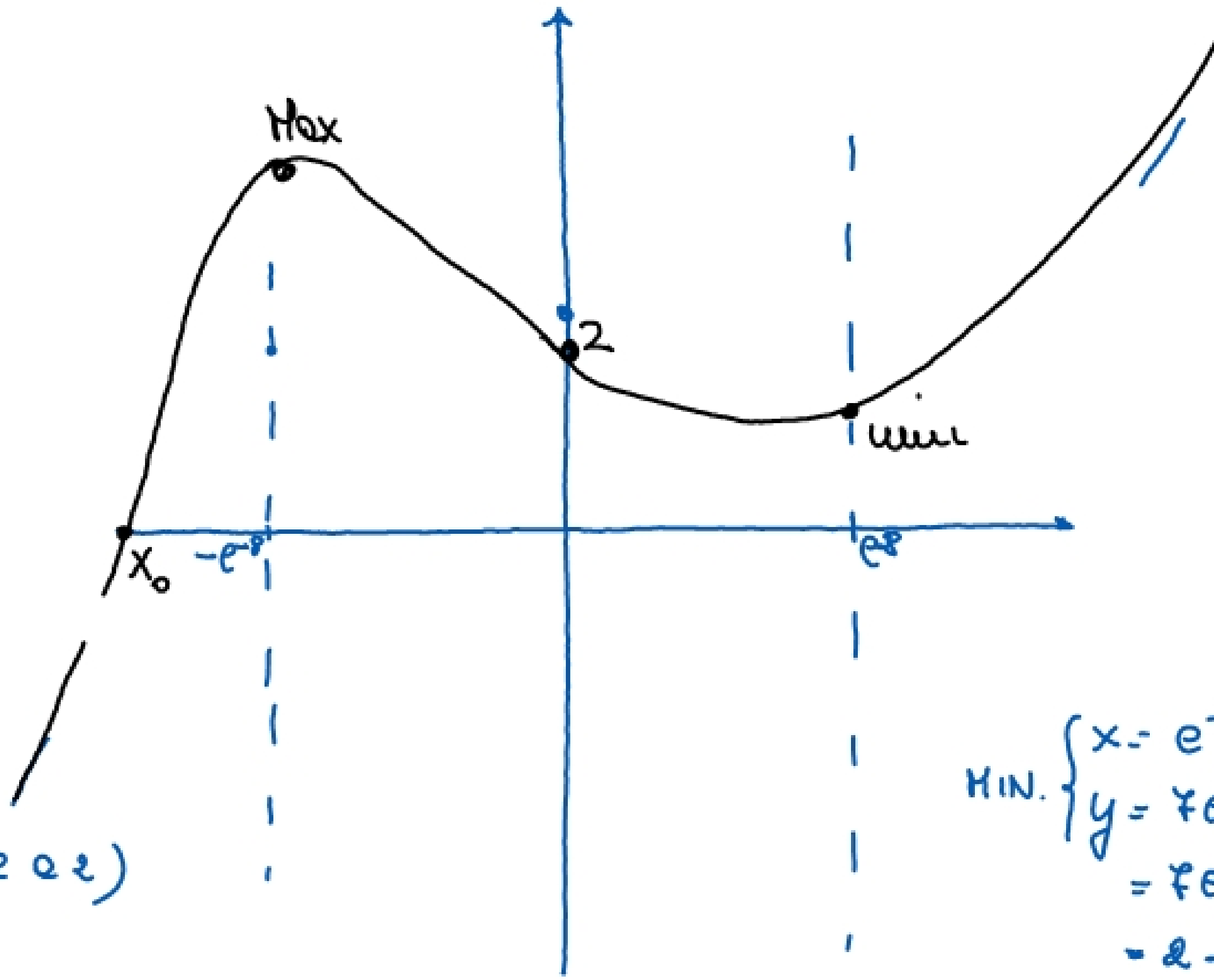


derivata seconda

$$f''(x) = \begin{cases} \frac{1}{x} & x > 0 \\ \frac{1}{-x}\cdot(-1) & x < 0 \end{cases} = \frac{1}{x} \quad \text{con } x \neq 0$$



GRAFICO



$$\begin{cases} x = 0 \\ y \to 2 \end{cases} \quad \text{(tende a 2)}$$

$$\text{MIN.} \begin{cases} x = e^{-8} \\ y = 7e^{-8} + 2 + e^{-8} \cdot \log(e^{-8}) = \\ \quad = 7e^{-8} + 2 + e^{-8} \cdot (-8) = \\ \quad = 2 - e^{-8} \end{cases}$$

$$\text{MAX} \begin{cases} x = -e^{-8} \\ y = 7(-e^{-8}) + 2 + e^{-8} \log(e^{-8}) = \\ \quad = -7e^{-8} + 2 + e^{-8}(-8) = \\ \quad = 2 - 15e^{-8} \end{cases}$$

T.E. 5

$$f(x) = \sqrt[3]{e^{2x} - e} + 3\sqrt[3]{e}$$

dominio $\forall x \in R$

segno $\sqrt[3]{e^{2x} - e} + 3\sqrt[3]{e} \geq 0$

$$\sqrt[3]{e^{2x} - e} \geq -3\sqrt[3]{e}$$

$$\sqrt[3]{e^{2x} - e} \geq \sqrt[3]{-27e}$$

$$e^{2x} - e \geq -27e$$

$$e^{2x} \geq -26e \qquad -26e < 0 \ \forall x \in R \Rightarrow f(x) > 0 \ \forall x \in R,$$

dom R:

non vi sono simmetrie

limiti $\lim_{x \to +\infty} \left(\underbrace{\sqrt[3]{e^{2x} - e}}_{\to +\infty} + 3\sqrt[3]{e} \right) = +\infty$

$$\lim_{x \to -\infty} \left(\sqrt[3]{\underbrace{e^{2x}}_{\to 0} - e} + 3\sqrt[3]{e} \right) = -\sqrt[3]{e} + 3\sqrt[3]{e} = 2\sqrt[3]{e} > 0$$

$y = 2\sqrt[3]{e}$ asintoto orizzontale sinistro

(N.B. l'asintoto obliquo a destra non esiste dato che come ordine di infinito l'esponenziale prevale su x)

derivata prima

$$f'(x) = \frac{1}{3}\left(e^{2x}-e\right)^{-2/3} \cdot 2e^{2x} = \frac{2e^{2x}}{3\sqrt[3]{(e^{2x}-e)^2}} \Rightarrow f'(x) > 0 \quad \forall x \in \mathbb{R} \setminus \left\{\tfrac{1}{2}\right\}$$

dom $f'(x)$: $\quad e^{2x}-e \neq 0 \qquad e^{2x} \neq e \qquad 2x \neq 1 \qquad x \neq \tfrac{1}{2}$

$$\lim_{x \to \frac{1}{2}^{\pm}} \left(\frac{2e^{2x}}{\underbrace{3\sqrt[3]{(e^{2x}-e)^2}}_{\to 0}}\right) = +\infty$$

$x = \frac{1}{2}$ punto a tangenza verticale (di non derivabilità)

		$1/2$	
$f'(x)$	$+$	$\times$	$+$
$f(x)$	$\nearrow$	?	$\nearrow$

derivata seconda

$$f'(x) = \frac{2}{3} e^{2x} \left(e^{2x} - e\right)^{-2/3}$$

$$f''(x) = \frac{2}{3}\left[2 e^{2x}\left(e^{2x}-e\right)^{-2/3} - \frac{2}{3}\left(e^{2x}-e\right)^{-5/3} 2 e^{2x}\cdot e^{2x}\right] =$$

$$= \frac{2}{3}\cdot 2\cdot e^{2x}\left(e^{2x}-e\right)^{-2/3}\left[1-\frac{2}{3}\left(e^{2x}-e\right)^{-1} e^{2x}\right] =$$

$$= \frac{4}{3} e^{2x}\left(e^{2x}-e\right)^{-2/3}\left[\frac{3(e^{2x}-e)-2e^{2x}}{3\left(e^{2x}-e\right)}\right] =$$

$$= \frac{4}{9}\,\frac{\overset{+}{e^{2x}}\left(e^{2x}-3e\right)}{\underset{+}{\sqrt[3]{(e^{2x}-e)^2}}\left(e^{2x}-e\right)}$$

dom $f''(x)$: $\forall x \in \mathbb{R} \setminus \{1/2\}$

$e^{2x} - 3e \geq 0$

$e^{2x} \geq 3e$

$2x \geq \log 3e = \log 3 + 1$

$x \geq \frac{1}{2}(\log 3 + 1)$

		$1/2$		$\frac{1}{2}(\log 3 + 1)$	
$e^{2x}-3e\geq 0$	$-$		$-$	0	$+$
$e^{2x}-e>0$	$-$	$\times$	$+$		$+$
$f''(x)$	$+$	$\times$	$-$	0	$+$
$f(x)$	$\cup$	$\times$	$\cap$	fl.	$\cup$

$e^{2x} - e > 0$

$e^{2x} > e$

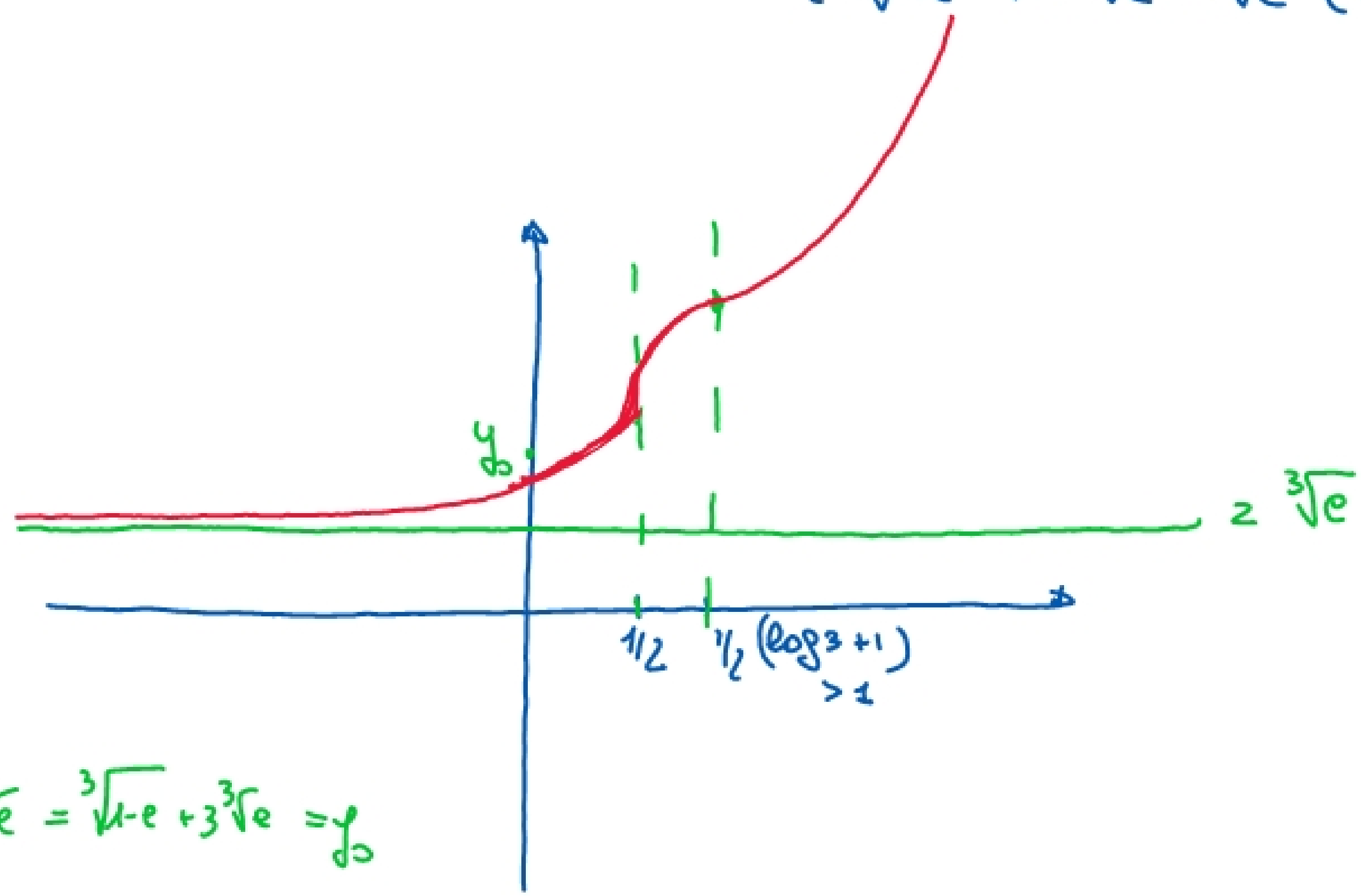
$2x > 1$

$x > 1/2$

flesso : $f\left(\frac{1}{2}(\log 3+1)\right) = \sqrt[3]{e^{2\cdot\frac{1}{2}(\log 3+1)} - e} + 3\sqrt[3]{e} = \sqrt[3]{3e-e} + 3\sqrt[3]{e} =$

$= \sqrt[3]{2e} + 3\sqrt[3]{e} = \sqrt[3]{e}\left(\sqrt[3]{2}+3\right)$

grafico



$$\begin{cases} x = 0 \\ y = \sqrt[3]{e^0 - e} + 3\sqrt[3]{e} = \sqrt[3]{1-e} + 3\sqrt[3]{e} = y_0 \end{cases}$$

TE 6

$$f(x) = \begin{cases} \sqrt[3]{1-x}\ \sqrt[3]{x^2} & x \geq 0 \quad [f_1(x)] \\ \frac{1}{x} & x < 0 \quad [f_2(x)] \end{cases}$$

dominio $D \equiv \mathbb{R}$ perché $f_1(x)$ esiste per ogni valore reale
$f_2(x)$ esiste per $x \neq 0$ che non appartiene al suo dominio, sottodominio di definizione.

non vi sono simmetrie

limiti $\lim_{x \to -\infty} \frac{1}{x} = 0 \Rightarrow y = 0$ asintoto orizzontale sinistro

$\lim_{x \to 0^-} \frac{1}{x} = -\infty \Rightarrow x = 0$ asintoto verticale sinistro

$\lim_{x \to +\infty} \sqrt[3]{(1-x)(x^2)} = -\infty$

ricerca dell'asintoto obliquo a destra

$$m = \lim_{x \to +\infty} \frac{\sqrt[3]{x^2(1-x)}}{x} = \lim_{x \to +\infty} \frac{-x\sqrt[3]{-\frac{1}{x}+1}}{x} = -1$$

$$q = \lim_{x \to +\infty} \left(\sqrt[3]{x^2(1-x)} + x \right) = (*)$$

$$(*) = \lim_{x\to+\infty} \frac{\left(\sqrt[3]{(1-x)x^2} + x\right)\left(\sqrt[3]{[(1-x)x^2]^2} + x^2 - x\sqrt[3]{(1-x)x^2}\right)}{\sqrt[3]{[(1-x)x^2]^2} + x^2 - x\sqrt[3]{(1-x)x^2}} =$$

(razionalizzazione inversa, col cubo)

$$= \lim_{x\to+\infty} \frac{x^2(1-x) + x^3}{3x^2} = \lim_{x\to+\infty} \frac{x^2}{3x^2} = \frac{1}{3}$$

(al denominatore sono tutti monomi del tipo x^2)

$y = -x + \frac{1}{3}$

asintoto obliquo dx

continuità:

la funzione è discontinua in $x = 0$
(punto di infinito)

derivata prima

$$f_1'(x) = \frac{2}{3} x^{-\frac{1}{3}} (1-x)^{\frac{1}{3}} + x^{2/3} \frac{1}{3} (1-x)^{-2/3}(-1) =$$

$$= \frac{2\sqrt[3]{(1-x)}}{3\sqrt[3]{x}} - \frac{\sqrt[3]{x^2}}{3\sqrt[3]{(1-x)^2}}$$

dominio $f_1'(x)$: $x \neq 0$ e $x \neq 1 \rightarrow$ punto di non derivabilità

$\downarrow$

$$\lim_{x\to 1^-} f_1'(x) = +\infty \qquad \lim_{x\to 1^+} f_1'(x) = -\infty$$

tg. verticale

$$f_2'(x) = -\frac{1}{x^2} \qquad \lim_{x\to 0^-} f_2'(x) = -\infty$$

$$\lim_{x\to 0^+} f_1'(x) = +\infty$$

(punto a tangente verticale)

Studio del segno per $f_1'(x)$

$(x>0)$

$$f_1'(x) = \frac{2(1-x)-x}{2\sqrt[3]{(1-x)^2 x}} = \frac{2-3x}{3\sqrt[3]{x(1-x)^2}} \geq 0$$

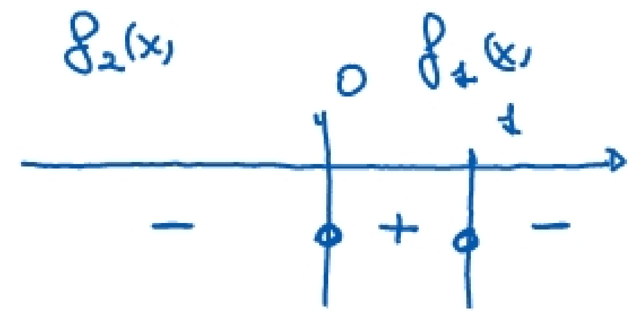
N $2-3x \geq 0$ $\quad x \leq 2/3$

D $\sqrt[3]{x} > 0$ $\quad x > 0$

		0		$2/3$		1	
$f'(x)$	$-$	$\times$	$+$	0	$-$	$\times$	$-$
$f(x)$	$\searrow$	$\times$	$\nearrow$	Max	$\searrow$	$\times$	$\nearrow$
	$f_2'(x)$		$f_1'(x)$				

(N.B. si tralascia lo studio delle derivate seconde)

STUDIO DEL SEGNO



$x < 0 \qquad f_2(x) = \frac{1}{x}$

$x \geq 0 \qquad f_2(x) = \sqrt[3]{1-x}\ \sqrt[3]{x^2}$

$\overset{+}{=0}$ in $x=0$

GRAFICO