TEOREMA DI L'HÔPITAL

forme di indeterminazione $\frac{0}{0}$, $\frac{\infty}{\infty}$ per $x \to x_0$

$x \to \pm\infty$

forniscono condizioni sufficienti per l'esistenza dei limiti:

Teo: $f, g : [x_0, x_0 + h] \to \mathbb{R}$ $h > 0$ continue t.c.

$f(x_0) = g(x_0) = 0$

derivabili per $x \in (x_0, x_0 + h)$ con $g'(x) \neq 0$

se esiste finito $\lim_{x \to x_0} \frac{f'(x)}{g'(x)}$ allora $= \lim_{x \to x_0} \frac{f(x)}{g(x)}$

attenzione: il limite f/g può esistere senza che esista il limite di f'/g'

Risultato analogo per $x \to \pm\infty$

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es 1

$$\lim_{x \to 0} \frac{e^{-x}-1}{x^2} = \frac{1-1}{0} = \frac{0}{0}$$

$$\underset{H}{=} \lim_{x \to 0} \frac{-e^{-x}}{2x} = \frac{-1}{0} = -\infty$$

es 2

$$\lim_{x \to 1} (2x-2) \ln(x-1) = 0 \cdot (-\infty)$$

$$= \lim_{x \to 1} \frac{\ln(x-1)}{\frac{1}{2x-2}} = \frac{-\infty}{\infty} =$$

$$\frac{1}{2x-2} = \frac{1}{2(x-1)}$$

$$\underset{H}{=} \lim_{x \to 1} \frac{\frac{1}{x-1}}{-\frac{1}{4(x-1)^2}} =$$

$$D\left(\frac{1}{t}\right) = -\frac{1}{t^2}$$

$$= \lim_{x \to 1} \left(-\frac{4(x-1)^{\cancel{2}}}{\cancel{(x-1)}}\right) = 0$$

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es 3

$$\lim_{x\to+\infty} \frac{\ln x}{x^2} = \frac{\infty}{\infty} =$$

$$\underset{H}{=} \lim_{x\to+\infty} \frac{\frac{1}{x}}{2x} = \lim_{x\to+\infty} \frac{1}{2x^2} = 0$$

es 4

$$\lim_{x\to 0^+} \frac{\sqrt{x}}{e^x-1} = \frac{0}{1-1} = \frac{0}{0}$$

$$D(\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

$$\underset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{2\sqrt{x}}}{e^x} =$$

$$= \lim_{x\to 0^+} \frac{1}{2\sqrt{x}\, e^x} = \infty$$

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es 5

$$\lim_{x\to 0^+} \frac{\ln(x+1)}{\frac{1}{\ln x}} = \frac{0}{0}$$

$$D\left(\frac{1}{\ln x}\right) = \cancel{0} - \frac{1}{\ln^2 x} \cdot D\ln x =$$

$$= -\frac{1}{\ln^2 x} \cdot \frac{1}{x}$$

$$\underset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x+1}}{-\frac{1}{x\ln^2 x}} =$$

$$= \lim_{x\to 0^+} \frac{-x\ln^2 x}{\underset{\to 1}{x+1}} = 0$$

es 6

$$\lim_{x\to 2} \frac{1-\sqrt{5-x^2}}{x-2} \underset{H}{=}$$

$$= \lim_{x\to 2} \frac{-\frac{1}{2\sqrt{5-x^2}} \cdot (-2x)}{1} =$$

$$= \lim_{x\to 2} \frac{2x}{2\sqrt{5-x^2}} = \frac{4}{2\cdot 1} = 2$$

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es 7

$$\lim_{x\to 0}\frac{2-(e^x+e^{-x})\cos x}{x^4}=\frac{2-2}{0}=\frac{0}{0}$$

$$\overset{H}{=}\lim_{x\to 0}\frac{-(\overset{1-1=0}{e^x-e^{-x}})\cos x+(e^x+e^{-x})(\overset{0}{\mathrm{sen}\,x})}{4x^3}=\frac{0}{0}$$

$$\overset{H}{=}\lim_{x\to 0}\frac{\overbrace{-(e^x+e^{-x})\cos x}+(e^x-e^{-x})(\mathrm{sen}\,x)+(e^x-e^{-x})\,\mathrm{sen}\,x+\overbrace{(e^x+e^{-x})(\cos x)}}{12x^2}=$$

$$=\lim_{x\to 0}\frac{2(e^x-e^{-x})\,\mathrm{sen}\,x}{12x^2}=\frac{0}{0}$$

$$\overset{H}{=}\lim_{x\to 0}\frac{\cancel{2}\left[(e^x+e^{-x})\,\mathrm{sen}\,x+(e^x-e^{-x})\cos x\right]}{\cancel{24}\,\overset{12}{}\,x}=\frac{0}{0}$$

$$\overset{H}{=}\lim_{x\to 0}\frac{\overbrace{(e^x-e^{-x})\,\mathrm{sen}\,x}+(e^x+e^{-x})\cos x+(e^x+e^{-x})\cos x\overbrace{-(e^x-e^{-x})\,\mathrm{sen}\,x}}{12}=$$

$$=\lim_{x\to 0}\frac{\cancel{2}(e^x+e^{-x})\overset{1}{\cos x}}{\cancel{12}\;6}=\frac{2}{6}=\frac{1}{3}$$

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es 8 $\lim_{x\to 1} \dfrac{\ln x}{2x - x\cdot 2^x} = f.i.$

$$\underset{H}{=} \lim_{x\to 1} \frac{1/x}{2 - 2^x - x\,2^x \log_2 e} = \lim_{x\to 1} \frac{1}{x}\,\frac{1}{2 - 2^x(1 - x\log_2 e)} =$$

$$= \frac{1}{2 - 2(1 - \log_2 e)}$$

es 9 $\lim_{x\to 0} \dfrac{\operatorname{sen} 4x}{\operatorname{sen} 3x} = f.i.$

$$\underset{H}{=} \lim_{x\to 0} \frac{\underset{\to 1}{\cos 4x}\cdot 4}{\underset{\to 1}{\cos 9x}\cdot 9} = 4/9$$

limiti notevoli:

$$\left(= \lim_{x\to 0} \frac{\operatorname{sen} 4x}{4x}\cdot\frac{4x}{9x}\cdot\frac{9x}{\operatorname{sen} 9x}\right)$$

$$\left(\underset{\text{Taylor}}{=} \lim_{x\to 0} \frac{4x}{9x} = 4/9\right.$$

Sviluppando al 1° ord. (sen t)

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