INTEGRALI PER SOSTITUZIONE

es 1 $\displaystyle\int \frac{e^x}{e^{2x}-3e^x+2}\,dx =$

pongo $e^x = t$

$x = \log t$

$dx = \frac{1}{t}\,dt$

$$= \int \frac{\cancel{t}}{t^2-3t+2} \cdot \overbrace{\frac{1}{\cancel{t}}\,dt}^{dx} = \int \frac{1}{(t-2)(t-1)}\,dt = \int \left[-\frac{1}{t-1} + \frac{1}{t-2} \right] dt =$$

$$\frac{A}{(t-1)} + \frac{B}{t-2} = \frac{A(t-2)+B(t-1)}{(t-1)(t-2)} \Rightarrow$$

$$\begin{array}{l} t \\ t^0 \end{array} \begin{cases} A+B=0 \\ -2A-B=1 \end{cases} \Rightarrow \begin{cases} A=-1 \\ B=-A=1 \end{cases}$$

$$= -\ln|t-1| + \ln|t-2| + c = \ln\left|\frac{t-2}{t-1}\right| + c = \ln\left|\frac{e^x-2}{e^x-1}\right| + c$$

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es 2 $\displaystyle\int \frac{\sinh x}{\cosh x + 1}\, dx =$

$\sinh x = \dfrac{e^x - e^{-x}}{2}$; $\cosh x = \dfrac{e^x + e^{-x}}{2}$

$$= \int \frac{e^x - e^{-x}}{2} : \left(\frac{e^x + e^{-x}}{2} + 1 \right) dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x} + 2}\, dx$$

$e^x = t$
$x = \ln t$
$dx = \frac{1}{t}\, dt$

$$= \int \frac{t - t^{-1}}{t + t^{-1} + 2} \frac{1}{t}\, dt = \int \frac{t^2 - 1}{\cancel{t}} \cdot \frac{\cancel{t}}{t^2 + 2t + 1} \cdot \frac{1}{t}\, dt =$$

$t^{-1} \to \frac{1}{t}$

$$= \int \frac{(t-1)\cancel{(t+1)}}{t\,(t+1)^{\cancel{2}}}\, dt = \int \frac{t-1}{t(t+1)}\, dt \Rightarrow \int \left(\frac{A}{t} + \frac{B}{t+1} \right) dt$$

$$A(t+1) + Bt = t - 1 \Rightarrow \begin{matrix} t \\ t^0 \end{matrix} \begin{cases} A + B = 1 \\ A = -1 \end{cases} \qquad \begin{cases} A = -1 \\ B = 1 - A = 2 \end{cases}$$

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$$= \int \left(-\frac{1}{t} + \frac{2}{t+1}\right) dt = -\ln|t| + 2\ln|t+1| + c =$$

$$= -\ln|e^x| + 2\ln|e^x+1| + c =$$

$$= -x + 2\ln(e^x+1) + c$$

$$\ln|\underset{>0}{e^x}| = \ln(e^x) = x \ln e = x$$

$$= \ln\left|\frac{(t+1)^2}{t}\right| + c = \ln\frac{(e^x+1)^2}{e^x} + c$$

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es 3 $\int \frac{x+\sqrt{x-1}}{x-5}\,dx =$

sostituzione $\underbrace{t=\sqrt{x-1}} \Rightarrow x=t^2+1 \Rightarrow \underbrace{dx=2t\,dt}$

$$=\int \frac{t^2+1+t}{1+t^2-5}\cdot 2t\,dt = 2\int \frac{t^3+t^2+t}{t^2-4}\,dt$$

$$\frac{t^3+t^2+t}{t^2-4} = \frac{\overline{t^3-4t}+5t+t^2}{t^2-4} = \frac{t(t^2-4)+\overline{t^2+5t}}{t^2-4} =$$

$$= t + \frac{(t^2-4)+4+5t}{t^2-4} = t+1+\frac{5t+4}{t^2-4}$$

$$= 2\int\left(t+1+\frac{5t+4}{(t-2)(t+2)}\right)dx = 2\left(\frac{t^2}{2}+t+\int\frac{5t+4}{(t-2)(t+2)}\,dx\right) =$$

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divisione polinomiale

resto

$$(t^3+t^2+t) : (t^2-4) \Rightarrow (t+1) + \frac{\boxed{5t+4}}{t^2-4}$$

$$
\begin{array}{cccc|l}
t^3 & +t^2 & +t & +0 & t^2-4 \\
-t^3 & & +4t & & \\
\hline
// & t^2 & +5t & & t+1 \\
 & -t^2 & & +4 & \\
\hline
 & // & 5t & +4 &
\end{array}
$$

divisore

$$\int \frac{5t+4}{(t-2)(t+2)}\,dt =$$

$$\frac{A(t+2)+B(t-2)}{(t+2)(t-2)} = \frac{A}{t-2} + \frac{B}{t+2} = \frac{5t+4}{(t-2)(t+2)}$$

$$\begin{matrix} t \\ t^0 \end{matrix} \begin{cases} A+B=5 \\ \cancel{2}A-\cancel{2}B=\cancel{4}\,2 \end{cases} \qquad \begin{cases} A=7/2 \\ B=5-A=5-\frac{7}{2}=3/2 \end{cases}$$

$$= \int \left(\frac{7}{2}\cdot\frac{1}{t-2} + \frac{3}{2}\cdot\frac{1}{t+2} \right) dt = \frac{7}{2}\ln|t-2| + \frac{3}{2}\ln|t+2| + c_1$$

$$= 2\left(\frac{t^2}{2} + t + \frac{7}{2}\ln|t-2| + \frac{3}{2}\ln|t+2| + c \right) =$$

$$= 2\left(\frac{(\sqrt{x-1})^2}{2} + \sqrt{x-1} + \frac{7}{2}\ln\left|\sqrt{x-1}-2\right| + \frac{3}{2}\ln\left|\sqrt{x-1}+2\right| + c \right)$$

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es4 $\int \frac{1}{\sqrt{2x}\left(\sqrt[3]{2x}+1\right)}\,dx =$

$\sqrt{2x}$
$\sqrt[3]{2x}$ $\Rightarrow$ eliminare i radicali $\Rightarrow$ $2x = t^6$

$x = \frac{t^6}{2}$

$\sqrt{2x} = \sqrt{t^6} = t^3$

$\sqrt[3]{2x} = \sqrt[3]{t^6} = t^2$ $\Leftarrow$ $dx = \frac{6t^5}{2} = 3t^5\,dt$

$$= \int \frac{1}{\cancel{t^3}\left(t^2+1\right)} \cdot \overbrace{3t^{\cancel{5}\,2}\,dt}^{dx} = 3\int \frac{t^2}{t^2+1}\,dt$$

$$= 3\int \frac{t^2+1-1}{t^2+1}\,dt = 3\int\left(1-\frac{1}{t^2+1}\right)dt = 3\left(t - \operatorname{arctg} t\right)+c =$$

$= 3 \ldots$

$t^6 = 2x \qquad t = \sqrt[6]{2x}$

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es 5 $\int \sqrt{1-x^2}\, dx =$

Funzione $x = \operatorname{sen} t$
(non è iniettiva)

$$x = \operatorname{sen} t \qquad \Rightarrow \qquad t = \arcsin x$$
$$dx = \cos t \, dt \qquad\qquad \cos t = \sqrt{1-\operatorname{sen}^2 t} = \sqrt{1-x^2}$$

$$= \int \sqrt{1-\operatorname{sen}^2 t} \cdot \cos t \, dt = \int \cos t \cdot \cos t \, dt = \int \cos^2 t \, dt$$

SOSTITUZIONI

$$\cos 2t = \begin{cases} 1 - 2\operatorname{sen}^2 t \\ \cos^2 t - \operatorname{sen}^2 t \\ 2\cos^2 t - 1 \end{cases}$$

$$\cos 2t = 2\cos^2 t - 1$$

$$\cos 2t + 1 = 2\cos^2 t$$

$$\cos^2 t = \frac{\cos 2t + 1}{2}$$

$$= \int \frac{1+\cos 2t}{2} \, dt =$$

$$= \int \left[\frac{1}{2} + \frac{1}{2}\frac{2}{2}\cos 2t \right] dt =$$

$$= \frac{1}{2} t + \frac{1}{4} \operatorname{sen} 2t + c =$$

$$= \frac{1}{2} t + \frac{1}{\cancel{4}\,2} \cdot \cancel{2} \operatorname{sen} t \cos t + c = \frac{1}{2} \arcsin x + \frac{1}{2} x \sqrt{1-x^2} + c$$

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es 6 $\int \sqrt{1+x^2}\, dx =$

$$\begin{cases} x = \operatorname{senh} t \\ t = \operatorname{settsenh} x \end{cases}$$

$x = \operatorname{senh} t$

$\sqrt{1+x^2} = \sqrt{1+\operatorname{senh}^2 t}$

$dx = \cosh t \, dt$

Relazione fondamentale:

$\cosh^2 t - \operatorname{senh}^2 t = 1$

$\cosh t = \sqrt{1+\operatorname{senh}^2 t} = \sqrt{1+x^2}$

$$= \int \cosh t \cdot \overbrace{\cosh t \, dt}^{dx} =$$

$$= \int \cosh^2 t \, dt = \int \frac{(e^t + e^{-t})^2}{4}\, dt =$$

$$= \int \frac{e^{2t} + e^{-2t} + 2}{4}\, dt = \frac{1}{4}\int \left(2 \cdot \overbrace{\frac{e^{2t}+e^{-2t}}{2}}^{\cosh(2t)} + 2 \right) dt =$$

$$= \frac{1}{4}\left(\underbrace{\operatorname{senh} 2t}_{2\,\operatorname{senh} t \cdot \cosh t} + 2t \right) + c = \frac{1}{2}\, x \sqrt{1+x^2} + \frac{1}{2} \operatorname{settsenh} x + c$$

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es 7 $\int \sqrt{x^2-1}\,dx$

sostituzione $x = \cosh t$ $\quad \Rightarrow \sqrt{x^2-1} = \sqrt{\cosh^2 t - 1}$

$dx = \operatorname{senh} t\, dt$

$x = \cosh t$ $\qquad \cosh^2 t - \operatorname{senh}^2 t = 1$

$dx = \operatorname{senh} t\, dt$ $\qquad \operatorname{senh} t = \sqrt{\cosh^2 t - 1}$

$$\int \underbrace{\sqrt{\cosh^2 t - 1}}_{\operatorname{senh} t} \cdot \operatorname{senh} t\, dt = \int \operatorname{senh}^2 t\, dt = \int (\cosh^2 t - 1)\, dt =$$

$$= \int \cosh^2 dt - \int 1\, dt = \frac{1}{2} \operatorname{senh} t \cosh t + \frac{1}{2} t - t + c =$$

(vedi il precedente.

$$= \frac{1}{2} x \sqrt{x^2-1} - \frac{1}{2} \log\left(x + \sqrt{x^2-1}\right) + c$$

(N.B. $x = \cosh t$ $\quad t = \log\left(x + \sqrt{x^2-1}\right)$ ← funzione inversa)

(ATTENZIONE all'intervallo $[0, +\infty)$ per invertire)

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es 8 $\displaystyle\int \frac{2}{(1+tg\,x)^2}\,dx = \int \frac{2}{(1+t)^2}\cdot\frac{1}{1+t^2}\,dt =$

$tg\,x = t$

$x = arctg\,t$

$dx = \dfrac{1}{1+t^2}\,dt$

$\displaystyle = \int \frac{2}{(1+t)^2(1+t^2)}\,dt =$

regola per le fratte: $\dfrac{2}{(1+t)^2(1+t^2)} = \dfrac{A}{(1+t)} + \dfrac{B}{(1+t)^2} + \dfrac{Ct+D}{1+t^2}$

$$\begin{cases} A = 1 \\ B = 1 \\ C = -1 \\ D = 0 \end{cases} \Rightarrow = \int \frac{1}{1+t}\,dt + \int \frac{1}{(1+t)^2}\,dt - \int \frac{t}{1+t^2}\,dt =$$

$= \log|1+t| - \dfrac{1}{1+t} - \dfrac{1}{2}\log(1+t^2) + c =$ sostituendo

$= \log|1+tg\,x| - \dfrac{1}{1+tg\,x} - \dfrac{1}{2}\log(1+tg^2 x) + c$

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es9 $\displaystyle\int \frac{\cos x - 3}{\operatorname{sen}^2 x - \cos^3 x + 1} \cdot \underbrace{\operatorname{sen} x \, dx} =$

sostituzione $\quad t = \cos x \qquad (\operatorname{sen}^2 x = 1 - \cos^2 x)$

$\qquad\qquad\qquad dt = -\operatorname{sen} x \cdot dx$

$$= \int \frac{t-3}{1-t^2-t^3+1} \cdot dt = \int \frac{3-t}{t^3+t^2-2} \, dt = \int \frac{3-t}{(t-1)(t^2+2t+2)} \, dt =$$

(identità polinomiale → scomposizione in fratti)

$$\frac{A}{t-1} + \frac{Bt+C}{t^2+2t+2}$$

$$= \int \left\{ \frac{2/5}{t-1} + \frac{-2/5\, t - 11/5}{t^2+2t+2} \right\} dt = \frac{1}{5} \int \left(\frac{2}{t-1} - \frac{2t+11}{t^2+2t+2} \right) dt =$$

$$= \frac{1}{5} \int \left(\frac{2}{t-1} - \frac{2t+2}{t^2+2t+2} - \frac{9}{\underset{t^2+2t+1+1}{(t+1)^2+1}} \right) dt =$$

$$= \frac{1}{5} \left(2 \ln|t-1| - \ln(t^2+2t+2) - 9 \operatorname{arctg}(t+1) \right) + C =$$

$$= \frac{1}{5} \left(2 \ln|\cos x - 1| - \ln(\cos^2 x + 2\cos x + 2) - 9 \operatorname{arctg}(\cos x + 1) \right) + C$$

es 10 $\displaystyle\int \frac{1}{4\operatorname{sen}x + 3\cos x}\,dx =$

$t = \operatorname{tg}\frac{x}{2} \rightarrow x = 2\operatorname{arctg} t$

$dx = \frac{2}{1+t^2}\,dt$

$\Rightarrow$ FORMULE PARAMETRICHE (FORMULE DI BISEZIONE)

$\operatorname{sen}\alpha = \dfrac{2\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}} \longrightarrow \dfrac{2t}{1+t^2}$

$\cos\alpha = \dfrac{1-\operatorname{tg}^2\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}} \longrightarrow \dfrac{1-t^2}{1+t^2}$

$$= \int \frac{1}{4\cdot\frac{2t}{\cancel{1+t^2}} + 3\,\frac{1-t^2}{\cancel{1+t^2}}} \cdot \overbrace{\frac{2}{\cancel{1+t^2}}\cdot dt}^{dx} = \int \frac{2}{8t+3-3t^2}\,dt =$$

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$$= -2\int \frac{1}{(3t+1)(t-3)}\,dt =$$

fratte: $\frac{1}{(3t+1)(t-3)} = \frac{A}{3t+1} + \frac{B}{t-3} = \frac{A(t-3)+B(3t+1)}{(3t+1)(t-3)}$

$$\begin{cases} A+3B=0 \\ -3A+B=1 \end{cases} \Rightarrow \begin{cases} A=-3/10 \\ B=1/10 \end{cases}$$

$$= -2\int\left(-\frac{3}{10}\cdot\frac{1}{3t+1} + \frac{1}{10}\,\frac{1}{t-3}\right)dt = \frac{2}{10}\log|3t+1| - \frac{2}{10}\log|t-3| + c =$$

$$= \frac{1}{5}\log\left|\frac{3t+1}{t-3}\right| + c = \frac{1}{5}\log\left|\frac{3\,tg\frac{x}{2}+1}{tg\frac{x}{2}-3}\right| + c$$

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T. E.

$$\int_0^1 \frac{3\,\operatorname{arctg} e^x}{\cosh x}\,dx \overset{(*)}{=} 3\,\operatorname{arctg}^2 e^x \Big|_0^1 = 3\,\operatorname{arctg}^2 e - 3\,\underbrace{\operatorname{arctg}^2 1}_{(\pi/4)^2}$$

$$= \int_0^1 3\,\frac{\operatorname{arctg} e^x}{\frac{e^x+e^{-x}}{2}} = \int_0^1 3\cdot\frac{2 e^x}{e^{2x}+1}\cdot \operatorname{arctg} e^x\,dx =$$

$$D(e^x)$$

$$\frac{e^x+\frac{1}{e^x}}{2} = \frac{e^{2x}+1}{2e^x}$$

pongo $e^x = t$

$x = \log t \Rightarrow$

$dx = \frac{1}{t}\,dt$

$$= \int 6\,\frac{\cancel{t}\cdot \operatorname{arctg} t}{t^2+1}\,\frac{1}{\cancel{t}}\,dt =$$

$$= 6 \int \frac{\operatorname{arctg} t}{t^2+1}\,dt = \cancel{6}^3\cdot\frac{\operatorname{arctg}^2 t}{\cancel{2}_1} + c \quad (*)$$

$$\frac{1}{t^2+1} = D(\operatorname{arctg} t)$$

4.E. $$\int_0^{\pi/2} \frac{2\,\mathrm{sen}\,2x + \cos x}{1+\mathrm{sen}^2 x}\,dx =$$

$$D\left(1+\mathrm{sen}^2 x\right) = 2\,\mathrm{sen}\,x\,\cos x$$

$$= \int_0^{\pi/2} \left[\frac{2\cdot \overbrace{2\,\mathrm{sen}\,x\,\cos x}^{\mathrm{sen}\,2x}}{1+\mathrm{sen}^2 x} + \frac{\overbrace{\cos x}^{D(\mathrm{sen}\,x)}}{1+\mathrm{sen}^2 x} \right] dx =$$

$$= 2 \log\left(1+\mathrm{sen}^2 x\right) + \mathrm{arctg}\left(\mathrm{sen}\,x\right) \Big|_0^{\pi/2} =$$

$$= 2\log\Big(1+\big(\underbrace{\mathrm{sen}\,\tfrac{\pi}{2}}_{1}\big)^2\Big) + \mathrm{arctg}\big(\underbrace{\mathrm{sen}\,\tfrac{\pi}{2}}_{1}\big) - \underbrace{2\log\big(1+(\mathrm{sen}\,0)^2\big)}_{0} - \underbrace{\mathrm{arctg}(\mathrm{sen}\,0)}_{0} =$$

$$= 2\log 2 + \frac{\pi}{4}$$

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INTEGRAZIONE PER PARTI

$$\int f'(x)\cdot g(x)\,dx = f(x)\cdot g(x) - \int f(x)\cdot g'(x)\,dx$$

es 1 $\displaystyle\int_0^1 \arcsin x\,dx = \int_0^1 \underbrace{1}_{1=D(x)}\cdot\underbrace{\arcsin x}_{D(\arcsin x)=\frac{1}{\sqrt{1-x^2}}}\,dx$

$$= x\cdot\arcsin x\,\Big|_0^1 - \int \frac{-2x}{-2}\cdot\frac{1}{\sqrt{1-x^2}}\,dx =$$

$$= x\arcsin x\,\Big|_0^1 + \sqrt{1-x^2}\,\Big|_0^1 = \frac{\pi}{2} - 1$$

$$D\left(\sqrt{1-x^2}\right) = \frac{1}{2\sqrt{1-x^2}}\cdot(-2x)$$

es 2 $\displaystyle\underbrace{\int \operatorname{sen}^2 x\,dx}_{I} = \int \underbrace{\operatorname{sen} x}_{D(-\cos x)}\cdot\operatorname{sen} x\,dx =$

$$= -\operatorname{sen} x\cos x - \int (-\cos x)\cdot\cos x\,dx = -\operatorname{sen} x\cos x + \int \underbrace{\cos x\cdot\cos x}_{\cos^2 x}\,dx = -\operatorname{sen} x\cos x + \int (1-\operatorname{sen}^2 x)\,dx =$$

$$= \underbrace{-\operatorname{sen} x\cos x + x - \underbrace{\int \operatorname{sen}^2 x\,dx}_{I}}$$

$$\int \operatorname{sen}^2 x \, dx = -\operatorname{sen} x \cos x + x - \int \operatorname{sen}^2 x \, dx$$

$$2 \int \operatorname{sen}^2 x \, dx = -\operatorname{sen} x \cos x + x$$

$$\int \operatorname{sen}^2 x \, dx = -\frac{1}{2} \operatorname{sen} x \cos x + \frac{x}{2}$$

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es 3 $\displaystyle\int_2^4 \arcsin \frac{1}{\sqrt{2+2|x-3|}}\,dx =$

$x-3=t$ $\quad (x=t+3)$ $\quad x=2 \rightarrow t=-1$

$dx = dt$ $\quad\quad x=4 \rightarrow t=1$

$\displaystyle = \int_{-1}^{1} \arcsin \frac{1}{\sqrt{2+2|t|}}\,dt =$ $\leftarrow$ la funzione è pari il dominio è simmetrico rispetto all'origine

$\displaystyle = 2\int_0^1 \arcsin \frac{1}{\sqrt{2+2t}}\,dt = 2\int_{\sqrt{2}}^{2} y \cdot \arcsin \frac{1}{y}\,dy =$

$y = \sqrt{2+2t}$ $\quad\quad t=0 \quad\quad y=\sqrt{2}$

$2+2t = y^2$ $\quad\quad \Rightarrow$

$\quad\quad t=1 \quad\quad y=\sqrt{2+2}=2$

$2\,dt = 2\,y\,dy$

$dt = y\,dy$

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$$\int \underbrace{y}_{f'}\,\underbrace{\arcsin\frac{1}{y}}_{g}\,dy =$$

$$= \frac{y^2}{2}\arcsin\frac{1}{y} - \int \frac{y^2}{2}\cdot\frac{1}{\sqrt{1-\frac{1}{y^2}}}\cdot\left(-\frac{1}{y^2}\right)dy =$$

$$= \frac{y^2}{2}\arcsin\frac{1}{y} + \frac{1}{2}\int\frac{2y}{2\sqrt{y^2-1}}\,dy =$$

$$= \frac{y^2}{2}\arcsin\frac{1}{y} + \frac{1}{2}\sqrt{y^2-1} + C$$

Infine

$$= \cancel{2}\cdot\frac{1}{\cancel{2}}\left(y^2\arcsin\frac{1}{y} + \sqrt{y^2-1}\right)\Bigg|_{\sqrt{2}}^{2} = 4\arcsin\frac{1}{2} + \sqrt{3} - 2\arcsin\frac{1}{\sqrt{2}} - 1 =$$

$$= \frac{\pi}{\cancel{6}_3}\cdot\cancel{4}^2 + \sqrt{3} - \cancel{2}\cdot\frac{\pi}{\cancel{4}_2} - 1 = \sqrt{3} - 1 + \frac{\pi}{6}$$

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(N.B. sostituzione)

es 4 $\displaystyle\int \frac{\sqrt{x}}{2+\sqrt{x}}\,dx =$

si pone $\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$$= \int \frac{t}{2+t}\cdot 2t\,dt =$$

$$= \int \frac{2\,t^2}{2+t}\,dt = 2\int \frac{t^2+4-4}{2+t}\,dt =$$

$$= 2\int\left[\frac{t^2-4}{t+2} + \frac{4}{2+t}\right]dt =$$

$$= 2\int\left[\frac{(t-2)\cancel{(t+2)}}{\cancel{t+2}} + \frac{4}{2+t}\right]dt = 2\left(\frac{t^2}{2} - 2t + 4\ln|2+t|\right) + c =$$

$$= 2\left(\frac{x}{2} - 2\sqrt{x} + 4\ln|2+\sqrt{x}|\right) + c$$

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T.E

$$\int_{\log 2}^{6} \frac{e^{2x}}{\sqrt{e^x - 1}}\, dx = \int_{\log 2}^{6} 2 \cdot e^x \cdot \frac{e^x}{2\sqrt{e^x-1}}\, dx \underset{\text{p.p.}}{=}$$

$$\left(D\left(\sqrt{e^x-1}\right) = \frac{1}{2\sqrt{e^x-1}} \cdot e^x \right)$$

$$= 2 \left\{ e^x \sqrt{e^x-1} \Big|_{\log 2}^{6} - \int_{\log 2}^{6} e^x \sqrt{e^x-1}\, dx \right\} =$$

$$D\left((e^x-1)^{3/2}\right) = \frac{3}{2} (e^x-1)^{1/2} \cdot e^x$$

$$= 2 \left\{ e^x \sqrt{e^x-1} \Big|_{\log 2}^{6} - \frac{2}{3} (e^x-1)^{3/2} \Big|_{\log 2}^{6} \right\} =$$

$$= 2 \left\{ e^6 \sqrt{e^6-1} - \frac{2}{3} (e^6-1)^{3/2} - e^{\log 2} (e^{\log 2}-1)^{1/2} + 2/3 (e^{\log 2}-1)^{3/2} \right\} =$$

$$= 2\, e^6 (e^6-1)^{1/2} - \frac{4}{3} (e^6-1)^{3/2} - 8/3$$

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