SUCCESSIONI

$\forall n \in \mathbb{N} : \{a_n\} \quad \mathbb{N} \to \mathbb{R}$

- LIMITATA
  - CONVERGENTE $\left\{\frac{1}{n}\right\}$
  - COSTANTE $\{\operatorname{sen} n\pi\}$
  - INDETERMINATA
    - → OSCILLANTE $\{(-1)^n\}$
    - → IRREGOLARE
    - → $\nexists$, non ammette limite
- ILLIMITATA
  - INDETERMINATA
  - DIVERGENTE $\{n\}$

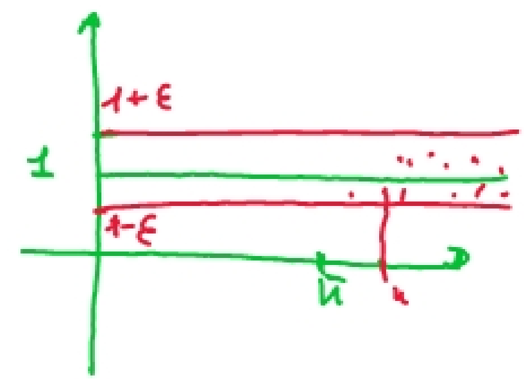
<u>es 1</u> Verificare il limite per la seguente successione:

$$\lim_{n\to+\infty}\left(1+\frac{1}{2^n}\right)=1$$

def: $\forall \varepsilon>0 \quad \exists \bar{n}: \quad \forall n>\bar{n}$

$$|a_n - L| < \varepsilon$$

$$\Big|\underbrace{1+\frac{1}{2^n}}\;-1\Big| < \varepsilon$$



$$\left|\cancel{1}+\frac{1}{2^n}-\cancel{1}\right|<\varepsilon$$

$$\left|\frac{1}{2^n}\right|<\varepsilon \quad \Rightarrow \quad 0<\frac{1}{2^n}<\varepsilon$$

$$0<\frac{1}{2^n}-\varepsilon<0 \quad ?$$

$$\frac{1-2^n\varepsilon}{2^n}<0 \qquad 2^n\varepsilon>1$$

$$2^n \to >0 \qquad 2^n>\frac{1}{\varepsilon}$$

$$\to \bar{n}: \quad n>\log_2\frac{1}{\varepsilon}\in R$$

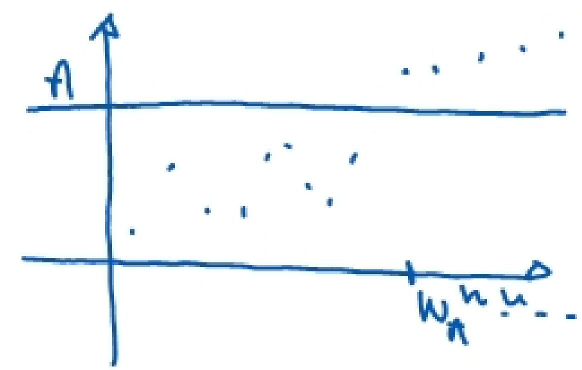
es: $a_n = n^n$ (diverge positivamente)

$a_n = n^n \geq n$

$n_A \geq A$ affinché:

$\forall A \in \mathbb{R}^+ \; \exists n_A \in \mathbb{N} : \forall n \in \mathbb{N} \; n \geq n_A : a_n = n^n \geq A$

$\underline{a_n} = n^n \geq n \geq n_A \geq \underline{A}$



(Teorema del confronto)

es n: Verificare il seguente limite:

$$\lim_{n\to+\infty} \frac{n}{2n+5} = 1/2$$

$$\forall \varepsilon>0 \quad \exists\, \bar{n} \in \mathbb{N} \ : \ \forall n>\bar{n} \ : \ \left|\frac{n}{2n+5} - \frac{1}{2}\right| \leq \varepsilon$$

$$\left|\frac{\cancel{2n} - \cancel{2n} - 5}{2(2n+5)}\right| \overset{?}{\leq} \varepsilon$$

$$\left|\frac{-5}{\underbrace{2(2n+5)}_{>0}}\right| \leq \varepsilon \quad \Rightarrow \quad \frac{5}{2(2n+5)} \leq \varepsilon$$

denominatore come $(2(2n+5)>0)$

$$5 - 2\varepsilon(2n+5) \leq 0$$

$$5 - 4n\varepsilon - 10\varepsilon \leq 0$$

$$4n\varepsilon \geq 5 - 10\varepsilon$$

$$n \geq \frac{5-10\varepsilon}{4\varepsilon} = \frac{5}{4\varepsilon} - \frac{5}{2}$$

si prende un $\bar{n}$ intero maggiore di $\left(\frac{5}{4\varepsilon} - \frac{5}{2}\right)$

es. Verificare il seguente limite:

$$\lim_{n \to +\infty} \frac{n}{2n+5} = 1$$

$\forall \varepsilon > 0 \ \exists \bar{n} \in \mathbb{N} : \forall n > \bar{n}$

$$\left| \frac{n}{2n+5} - 1 \right| \leq \varepsilon$$

$$\left| \frac{n-2n-5}{2n+5} \right| = \left| \frac{-(5+n)}{2n+5} \right| = \frac{5+n}{2n+5}$$

$$\frac{5+n}{\underbrace{2n+5}_{>0}} \leq \varepsilon$$

(denominatore >0)

$5 + n - 2n\varepsilon - 5\varepsilon < 0$

$-2n\varepsilon + n < 5\varepsilon - 5$

$n(1-2\varepsilon) < 5(\varepsilon - 1)$

$n < \dfrac{5(\varepsilon-1)}{1-2\varepsilon}$ [vero $\Leftrightarrow \varepsilon > 1$]

$\nexists \ \bar{n}$ che verifichi la disuguaglianza, ovvero la definizione.

es Determinare inf A, sup A ed eventualmente min A, max A essendo

$$A = \left\{ 3\sin\left(\frac{4n+1}{2}\pi\right) + e^{-\frac{1}{n^2+1}},\ n \in \mathbb{N} \right\}$$

(tema d'esame del 18/03/08)

$$a_n = 3\sin\left(\frac{4n+1}{2}\pi\right) + e^{-\frac{1}{n^2+1}}$$

$$b_n = 3\sin\left(\frac{4n+1}{2}\pi\right) = 3\sin\left(\frac{\pi}{2}\right) = 3$$

$$\frac{4n+1}{2}\pi = \left(2n + \frac{1}{2}\right)\pi = 2n\pi + \frac{1}{2}\pi$$

$2k\pi \quad k > 0$

$$c_n = e^{-\frac{1}{n^2+1}}$$

$$\lim_{n\to+\infty} e^{-\frac{1}{n^2+1}} = e^{-\frac{1}{\infty}\sim 0} = 1 \ (= e^0)$$

$$c_0 = e^{-\frac{1}{0+1}} = e^{-1} = \frac{1}{e} > 0 \qquad c_n \nearrow$$

$$0 < \frac{1}{e} < 1$$

$\min A = \inf A = 3 + e^{-1}$

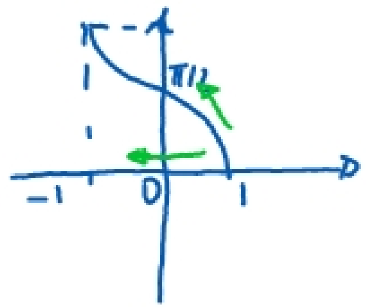
$\sup A = 3 + 1 = 4$

(Tema d'esame 4/9/08)

es: determinare inf A, sup A ed eventualmente min A, max A,

essendo $A = \left\{ \arccos\left(\frac{n+3}{2n^2+5}\right) + 3 \quad n \geq 0 \right\}$

$y = \arccos x$



$b_n = \frac{n+3}{2n^2+5}$ $\qquad b_0 = \frac{3}{5}$

↘ decrescente

$\lim\limits_{n \to +\infty} b_n = \lim\limits_{n \to +\infty} \frac{n+3}{2n^2+5} = \frac{\infty}{\infty} =$

$b_n \to 0$ $\qquad \arccos(b_n) \to \frac{\pi}{2}$

succ. decr. $\qquad$ succ. crescente

2 polinomi

grado N < grado D

$\lim\limits_{n \to +\infty} \arccos\left(\frac{n+3}{2n^2+5}\right) = \arccos 0 = \frac{\pi}{2}$

$= \lim\limits_{n \to +\infty} \frac{\cancel{n}\,(1 + \underbrace{3/n}_{\to 0})}{n^{\cancel{2}}\,(2 + \underbrace{\frac{5}{n^2}}_{\to 0})} = \lim\limits_{n \to +\infty} \frac{1}{2} \cdot \frac{1}{n} = 0$

$\inf A = \min A = a_0 = \arccos \frac{3}{5} + 3$

$\sup A = \frac{\pi}{2} + 3$

CALCOLO DEI LIMITI

$$(A-B)(A+B) = A^2 - B^2$$

1. FORMA DI INDETERMINAZIONE $(+\infty - \infty)$

$$\lim_{n\to+\infty}\left(\sqrt{n+2}-\sqrt{n-1}\right) = \lim_{n\to+\infty}\left(\sqrt{n+2}-\sqrt{n-1}\right)\cdot\frac{\left(\sqrt{n+2}+\sqrt{n-1}\right)}{\sqrt{n+2}+\sqrt{n-1}} =$$

$$= \lim_{n\to+\infty}\frac{n+2-n+1}{\underset{\sqrt{n}}{\sqrt{n+2}}+\underset{\sqrt{n}}{\sqrt{n-1}}} = \lim_{n\to+\infty}\frac{3}{\underset{\downarrow\, +\infty}{\sqrt{n}}\left(\sqrt{1+\underset{\downarrow 0}{\frac{2}{n}}}+\sqrt{1-\underset{\downarrow 0}{\frac{1}{n}}}\right)} = 0$$

$$\lim_{n\to+\infty}\left(\sqrt{n^2+2}-\sqrt{1+n^2+n}\right) =$$

$$= \lim_{n\to+\infty}\frac{\cancel{n^2}+2-1-\cancel{n^2}-n}{\underset{\sqrt{n^2}=n}{\sqrt{n^2+2}}+\underset{\sqrt{n^2}=n}{\sqrt{1+n^2+n}}} = \lim_{n\to+\infty}\frac{1-n}{n\left(\sqrt{1+\frac{2}{n^2}}+\sqrt{\frac{1}{n^2}+1+\frac{1}{n}}\right)} =$$

$n>0$

$$= \lim_{n\to+\infty}\frac{\cancel{n}\left(\overset{\to 0}{\frac{1}{n}}-1\right)}{\cancel{n}\left(\sqrt{1+\underset{\downarrow 0}{\frac{2}{n^2}}}+\sqrt{\underset{\downarrow 0}{\frac{1}{n^2}}+1+\underset{\downarrow 0}{\frac{1}{n}}}\right)} = \frac{-1}{1+1} = -\frac{1}{2}$$

$$\lim_{n\to+\infty}\left(\sqrt{n^3+2}-\sqrt{n^2-1}\right)=$$

$$=\lim_{n\to+\infty}\frac{n^3+2-n^2+1}{\sqrt{n^3+2}+\sqrt{n^2-1}}=\lim_{n\to+\infty}\frac{\overset{n^{3/2}}{n^3}\left(1-\overset{\to 0}{\frac{1}{n}}+\overset{\to 0}{\frac{3}{n^3}}\right)}{\sqrt{n^3}\underbrace{\left(\sqrt{1+\underset{\downarrow 0}{\frac{2}{n^3}}}+\sqrt{\underset{\downarrow 0}{\frac{1}{n}}-\underset{\downarrow 0}{\frac{1}{n^2}}}\right)}_{1}}=+\infty$$

$$=\lim_{n\to+\infty}\underset{\downarrow\, n^{3/2}}{\sqrt{n^3}}\left(\sqrt{1+\overset{\to 0}{\frac{2}{n^3}}}-\underbrace{\sqrt{\frac{1}{n}-\frac{1}{n^3}}}_{\downarrow\ \underbrace{\frac{1}{\sqrt{n}}\cdot\sqrt{1-\frac{1}{n^2}}}_{\downarrow 0}}\right)=+\infty$$

2) Forma di indeterminazione $\frac{\infty}{\infty}$ $\left(\frac{0}{0}\right)$

$$\lim_{n\to+\infty} \frac{3n-1}{n+3} = \lim_{n\to+\infty} \frac{\cancel{n}\left(3-\frac{1}{n}\right)}{\cancel{n}\left(1+3/n\right)} = 3$$

pari grado
$\Rightarrow$ rapporto tra i coeff. di grado massimo

$$\approx \lim_{n\to+\infty} \frac{3n}{n} = 3$$

$$\lim_{n\to+\infty} \frac{n+1}{n^2+1} = \lim_{n\to+\infty} \frac{\cancel{n}\left(1+\frac{1}{n}\right)}{n^2\left(1+\frac{1}{n^2}\right)} = \lim_{n\to+\infty} \frac{1}{n} = 0$$

- grado D > grado N
- prevale il Den.

$$\approx \lim_{n\to+\infty} \frac{n}{n^2} = 0$$

$$\lim_{n\to+\infty} \frac{n^3+1}{2n-1} = \lim_{n\to+\infty} \frac{n^{\cancel{3}\,2}\left(1+\frac{1}{n^3}\right)}{\cancel{n}\left(2-\frac{1}{n}\right)} = \lim_{n\to+\infty} n^2 = +\infty$$

- grado D < grado N
- prevale il Num

$$\approx \lim_{n\to+\infty} \frac{n^3}{2n} = \lim_{n\to+\infty} n^2 = +\infty$$

$$\lim_{n\to+\infty} \frac{n+(-1)^n}{n-(-1)^n} = \lim_{n\to+\infty} \frac{\cancel{n}\left(1+\frac{(-1)^n}{n}\right)}{\cancel{n}\left(1-\frac{(-1)^n}{n}\right)} = 1$$

$$\frac{(-1)^n}{n} \to 0$$

N.B. $(-1)^n \begin{cases} 1 & n = 2m \text{ (pari)} \\ -1 & n = 2m+1 \text{ (dispari)} \end{cases}$

Riassumendo: CONFRONTANDO 2 POLINOMI IN RAPPORTO:

$\frac{N}{D}$

		LIMITE
se grado N > grado D	prevale il N	$\Rightarrow +\infty$
se grado N = grado D	pari grado	$\Rightarrow$ rapporto tra i coefficienti di grado massimo
se grado N < grado D	prevale il D	$\Rightarrow 0$

$$\lim_{n\to+\infty}\left(\frac{2-3n}{2n+1}\right)^3=\lim_{n\to+\infty}\left(\frac{\cancel{n}\left(\overset{\to 0}{\frac{2}{n}}-3\right)}{\cancel{n}\left(2+\underset{\to 0}{\frac{1}{n}}\right)}\right)^3=\left(-\frac{3}{2}\right)^3=-\frac{27}{8}$$

grado N = grado D

$$\lim_{n\to+\infty}\left(\overset{+\infty}{\log_a\left(n+\sqrt{1+n^2}\right)}\overset{-}{-}\overset{\infty}{\log_a n}\right)=$$

$a>0,\ a\neq 1$

$$=\lim_{n\to+\infty}\left(\log_a\frac{n+\sqrt{1+n^2}}{n}\right)=\lim_{n\to+\infty}\left(\log_a\frac{n+\overset{n}{\sqrt{n^2}}\left(\sqrt{\frac{1}{n^2}+1}\right)}{n}\right)=$$

grado N = grado D

$$=\lim_{n\to+\infty}\left(\log_a\frac{\cancel{n}\left(1+\sqrt{1+\overset{\to 0}{\frac{1}{n^2}}}\right)}{\cancel{n}}\right)=\log_a 2$$

$$\lim_{n\to+\infty} \frac{4(n+1)! - \frac{1}{2^n} + \sin(n^4)}{n!\left[\log(2^n) - \log(3^n)\right]} =$$

(Tema d'esame 3/07/08)

$-1 \leq \sin(n^4) \leq 1$

si raccoglie il termine che "va" più veloc. all'∞

$$= \lim_{n\to+\infty} \frac{(n+1)!\left[4 - \frac{1}{(n+1)!\,2^n} + \frac{\sin(n^4)}{(n+1)!}\right]}{n!\,\log\left(\frac{2^n}{3^n}\right)} =$$

$$= \lim_{n\to+\infty} \frac{(n+1)\cancel{n!}}{\cancel{n!}} \cdot \frac{\left[4 - \overbrace{\frac{1}{(n+1)!\,2^n}}^{\to 0} + \overbrace{\frac{\overbrace{\sin(n^4)}^{[-1,1]}}{(n+1)!}}^{\to 0}\right]}{n\log\left(\frac{2}{3}\right)^n}$$

$$= \lim_{n\to+\infty} \frac{n+1}{n} \cdot \frac{4}{\log 2/3} = \frac{4}{\log 2/3}$$

pari grado $\to (1)$

$$\lim_{n\to+\infty}\frac{\sqrt{1+\frac{1}{n}}-\sqrt{1-\frac{2}{n}}}{7n\sin\frac{1}{n^2}+2n^3\sin\frac{1}{n^4}}=$$

(Tema d'esame 4/03/08)

RAZIONALIZZAZIONE INVERSA $(+\infty,-\infty)$

$$=\lim_{n\to+\infty}\frac{\cancel{1}+\frac{1}{n}\cancel{-1}+\frac{2}{n}}{\underbrace{\Big(\sqrt{1+\underset{\to 0}{\frac{1}{n}}}+\sqrt{1-\underset{\to 0}{\frac{2}{n}}}\Big)}_{2}\left(7n\sin\frac{1}{n^2}+2n^3\sin\frac{1}{n^4}\right)}=$$

$$=\lim_{n\to+\infty}\frac{3}{2}\cdot\frac{1}{n\left(7n\sin\frac{1}{n^2}+2n^3\sin\frac{1}{n^4}\right)}=$$

$$=\frac{3}{2}\lim_{n\to+\infty}\frac{1}{7\underbrace{n^2\sin\frac{1}{n^2}}_{1}+2\underbrace{n^4\sin\frac{1}{n^4}}_{1}}=$$

$$=\frac{\cancel{3}^{\,1}}{2}\cdot\frac{1}{\cancel{9}_{\,3}}=1/6$$

$$\lim_{n\to+\infty}\frac{\sin\frac{1}{n}}{\frac{1}{n}}=1 \qquad \left(\tfrac{1}{n}\to 0\right)$$

$$=\lim_{n\to+\infty}n\cdot\sin\frac{1}{n}$$

$$\lim_{n\to+\infty} \frac{\sin^2 n^2 + n^{2n} + 3\log(n+2)}{12(n+1)^{2n} - (2n)! - 2e^{2n}} =$$

(Tema d'esame 10/10/07)

$$= \lim_{n\to+\infty} \frac{n^{2n}}{(n+1)^{2n}} \cdot \frac{\left[\underbrace{\frac{\sin^2 n^2}{n^{2n}}}_{\to 0} + 1 + 3\underbrace{\frac{\log(n+2)}{n^{2n}}}_{\to 0}\right]}{\left[12 - \underbrace{\frac{(2n)!}{(n+1)^{2n}}}_{\to 0} - 2\underbrace{\frac{e^{2n}}{(n+1)^{2n}}}_{\to 0}\right]} =$$

(confronto tra infiniti)

$$= \frac{1}{12} \lim_{n\to+\infty} \left(\frac{n}{n+1}\right)^{2n} =$$

$$= \frac{1}{12} \lim_{n\to+\infty} \left(\frac{n+1}{n}\right)^{-2n} = \frac{1}{12} \lim_{n\to+\infty} \Bigg[\underbrace{\left(1+\frac{1}{n}\right)^{n}}_{e}\Bigg]^{-2} = \frac{1}{12} \cdot e^{-2}$$

limite notevole $\lim_{n\to+\infty} \left(1+\frac{1}{n}\right)^n = e$

$$a_n = \sqrt[n]{3^n + 7^n}$$

$$\lim_{n \to +\infty} \sqrt[n]{3^n + 7^n} = \lim_{n \to +\infty} \left(3^n + 7^n\right)^{\frac{1}{n}} =$$

$$= \lim_{n \to +\infty} \left(7^n\right)^{\frac{1}{n}} \left(\frac{3^n}{7^n} + 1\right)^{\frac{1}{n}} =$$

$$= \lim_{n \to +\infty} 7 \underbrace{\left(1 + \underbrace{\left(\frac{3}{7}\right)^n}_{\to 0}\right)^{\frac{1}{n} \to 0}}_{1^0} = 7$$

- esponenziale con base $0 < a < 1$
  $a = \frac{3}{7}$

$$\lim_{n\to+\infty} \frac{e^{3\sqrt{\log^2 n + \log n + 1}}}{n^3} =$$

$$= \lim_{n\to+\infty} \frac{e^{3\sqrt{\log^2 n + \log n + 1}}}{e^{\log n^3}} = \lim_{n\to+\infty} e^{3\left(\sqrt{\log^2 n+\log n+1} - \log n\right)} =$$

$$e^{\log n^3} \to e^{3\log n}$$

$$= e^{3\cdot\frac{1}{2}} = e^{3/2}$$

(*)

$$\lim_{n\to+\infty}\left(\sqrt{\log^2 n + \log n + 1} - \log n\right) = \lim_{n\to+\infty} \frac{\cancel{\log^2 n} + \log n + 1 - \cancel{\log^2 n}}{\sqrt{\log^2 n + \log n + 1} + \log n} =$$

$$= \lim_{n\to+\infty} \frac{\cancel{\log n}\left(1 + \underbrace{\frac{1}{\log n}}_{\to 0}\right)}{\cancel{\log n}\left(\sqrt{1 + \underbrace{\frac{1}{\log n}}_{\to 0} + \underbrace{\frac{1}{\log^2 n}}_{\to 0}} + 1\right)} = \frac{1}{2}$$

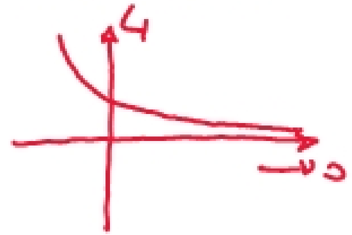
$$\lim_{n\to+\infty} \frac{n^n+3^n}{4^{n\log n}} =$$

(*) $n\log n = \log n^n$

$$= \lim_{n\to+\infty} \left(\frac{e^{\overbrace{\log n^n}}}{4^{\underbrace{n\log n}_{*}}} + \frac{3^n}{4^{n\log n}} \right) =$$

$$= \lim_{n\to+\infty} \left(\left(\frac{e}{4}\right)^{\overbrace{n\log n}^{\to+\infty}} + \left(\frac{3}{4^{\log n}}\right)^{n} \right) =$$

$0 < \frac{e}{4} < 1$



$0 < \frac{3}{4^{\log n}} < \frac{3}{4} < 1 \quad \forall n \geq 3 > e = 2{,}7\dots$

(Teorema del confronto)

$= 0 + 0 = 0$

$$\lim_{n\to+\infty} n^2\left(\log(n+2)+\log\frac{1}{n}\right)\cdot \operatorname{sen}\frac{1}{n} =$$

(Tema d'esame del 9-12-04)

$$= \lim_{n\to+\infty} \left(\log\left((n+2)\cdot\frac{1}{n}\right)\right) n^{\cancel{2}}\cdot \frac{\operatorname{sen}\frac{1}{n}}{\underbrace{\frac{1}{n}\cdot\cancel{n}}_{1}} =$$

$$= \lim_{n\to+\infty} n\cdot \log\frac{n+2}{n} =$$

$$= \lim_{n\to+\infty} \log\left(\frac{n+2}{n}\right)^{n} = \lim_{n\to+\infty} \log\left[\underbrace{\left(1+\frac{2}{n}\right)^{\frac{n}{2}}}_{\to e}\right]^{2} =$$

$$= \log e^{2} = 2\cdot\underbrace{\log e}_{1} = 2$$

⇒ Lim. I ⇒

$$\lim_{n\to+\infty} n\log\left(1+\frac{1}{n}\right)=1$$

$$= \lim_{n\to+\infty} \frac{\log\left(1+\frac{1}{n}\right)}{\frac{1}{n}} = 1$$

$$\lim_{n\to+\infty}\left(\frac{\log\left(1+\frac{1}{2n}\right)}{7\log n\cdot \operatorname{sen}\frac{1}{2n}}+\frac{\log(n+2)+n^{-\frac{1}{2}}}{2\log n}\right)=$$

(Terma d'esame 4_07_05)

$$=\lim_{n\to+\infty}\left(\frac{2n\,\log\left(1+\frac{1}{2n}\right)}{7\log n\ \underbrace{\dfrac{\operatorname{sen}\frac{1}{2n}}{\frac{1}{2n}}}_{1}}+\frac{\log\left(n\left(1+\frac{2}{n}\right)\right)+\frac{1}{\sqrt{n}}}{2\log n}\right)=$$

$\frac{2}{n}\to 0$, $\frac{1}{\sqrt{n}}\to 0$

$$=\lim_{n\to+\infty}\left(\frac{\log\left(1+\frac{1}{2n}\right)^{2n}\to\log e=1}{7\log n}+\frac{\cancel{\log n}}{2\cancel{\log n}}\right)$$

$$=0+\frac{1}{2}=\frac{1}{2}$$

$$\log n\left(1+\frac{2}{n}\right)=$$

$$=\log n+\underbrace{\log\left(1+\frac{2}{n}\right)}_{0}\quad n\to\infty$$

$\frac{2}{n}\to 0$

$$\lim_{n \to +\infty} \left(\log_a \left(n - \sqrt{n^2-1} \right) + \log_a n \right) =$$

$a > 0 \quad a \neq 1$

$$= \lim_{n \to +\infty} \log_a \left(\left(n - \sqrt{n^2-1} \right) \cdot n \right) =$$

$$\left(n - \sqrt{n^2-1} \right) \frac{\left(n + \sqrt{n^2-1} \right)}{\left(n + \sqrt{n^2-1} \right)}$$

$$= \lim_{n \to +\infty} \log_a \left(n \, \frac{\cancel{n^2} - \cancel{n^2} + 1}{n + \sqrt{n^2-1}} \right) =$$

$n + \sqrt{n^2}$ raccoglimento

$$= \lim_{n \to +\infty} \log_a \left(\frac{\cancel{n}}{\cancel{n} \left(1 + \sqrt{1 - \frac{1}{n^2}} \right)} \right) =$$

$$= \log_a \frac{1}{2}$$

$$\lim_{n \to +\infty} \left(\sqrt{n^2+4n+3} - n \right) =$$

$$= \lim_{n \to +\infty} \frac{n^2+4n+3-n^2}{\sqrt{n^2+4n+3} + n} =$$

$$= \lim_{n \to +\infty} \frac{\cancel{n}\left(4 + \overset{\to 0}{\frac{3}{n}}\right)}{\cancel{n}\left(\sqrt{1 + \underbrace{\frac{4}{n}}_{\to 0} + \underbrace{\frac{3}{n^2}}_{\to 0}} + 1\right)} =$$

$$= \frac{4}{2} = 2$$

$$\lim_{n \to +\infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n} \right) =$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$= \lim_{n \to +\infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n} \right) \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n^2} + \sqrt[3]{n(n+1)}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n^2} + \sqrt[3]{n(n+1)}} =$$

$$= \lim_{n \to +\infty} \frac{\cancel{n} + 1 - \cancel{n}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n^2} + \sqrt[3]{n(n+1)}} = 0$$

$$\lim_{n\to+\infty} \sqrt[n]{3^n + (1-(-1)^n)7^n} =$$

successione che non ammette limite

questa successione ammette 2 sottosuccessioni che ammettono limite

n pari $(-1)^{2m} = 1$

$$= \lim_{n\to+\infty} \sqrt[n]{3^n + \underbrace{(1-1)}_{0} 7^n} = \lim_{n\to+\infty} \sqrt[n]{3^n} = 3$$

b_n costante

n dispari $(-1)^{2m+1} = -1$

$$= \lim_{n\to+\infty} \sqrt[n]{3^n + 2\cdot 7^n} = \lim_{n\to+\infty} \underbrace{(7^n)^{\frac{1}{n}}}_{7} \underbrace{\left(\left(\frac{3}{7}\right)^{n} + 2\right)^{\frac{1}{n}}}_{2^0 = 1} = 7$$

$\left(\frac{3}{7}\right)^n \to 0$, $\frac{1}{n} \to 0$

$$\lim_{n\to+\infty} \frac{\log(n+3)! - \log n!}{\log(2n^6)} =$$

$$= \lim_{n\to+\infty} \frac{\log \frac{(n+3)!}{n!}}{\log 2 + \log n^6} =$$

$$= \lim_{n\to+\infty} \frac{\log \frac{(n+3)(n+2)(n+1)\cdot \cancel{n!}}{\cancel{n!}}}{\log 2 + 6\log n} =$$

$$= \lim_{n\to+\infty} \frac{\log\left[(n^3)\cdot\left(1+\overset{\to 0}{\frac{3}{n}}\right)\left(1+\overset{\to 0}{\frac{2}{n}}\right)\left(1+\overset{\to 0}{\frac{1}{n}}\right)\right]}{\log n\left(\frac{\log 2}{\log n}+6\right)} \approx$$

$$\approx \lim_{n\to+\infty} \frac{3\log n}{6\log n} = 3/6 = \frac{1}{2}$$

$$\lim_{n\to+\infty} \frac{(n-1)!\; n^{n+1} - (n+1)!\; n^{n-1}}{n^n \left((n-1)! + \log n\right)} =$$

$$= \lim_{n\to+\infty} \frac{(n-1)!\; n^{n+1} - (n+1)\, n\, (n-1)!\; n^{n-1}}{n^n \cdot (n-1)! \left(1 + \frac{\log n}{(n-1)!}\right)} =$$

$$= \lim_{n\to+\infty} \frac{(n-1)!\; n^{n-1} \left[n^{n+1-n+1} - n(n+1) \right]}{n^n\, (n-1)! \left(1 + \frac{\log n}{(n-1)!}\right)} =$$

$$= \lim_{n\to+\infty} \frac{\cancel{(n-1)!}\; n^{n-1}}{n^n\, \cancel{(n-1)!}} \underbrace{\left(1 + \overbrace{\frac{\log n}{(n-1)!}}^{\to 0}\right)^{-1}}_{1} \left(\cancel{n^2} - \cancel{n^2} - n\right) =$$

$$\approx \lim_{n\to+\infty} \frac{n^{n-1}(-n)}{n^n} = -1$$

$$\lim_{n\to+\infty} \frac{(n!)^{n-1} - \left((n-1)!\right)^{n}}{\left((n-1)!\,(n-10)\right)^{n-1}} =$$

$$= \lim_{n\to+\infty} \frac{\cancel{\left((n-1)!\right)^{n-1}} \left(\frac{n^{n-1}\cancel{\left((n-1)!\right)^{n-1}}}{\cancel{\left((n-1)!\right)^{n-1}}} - \frac{\cancel{\left((n-1)!\right)^{n-1}}\left((n-1)!\right)}{\cancel{\left((n-1)!\right)^{n-1}}} \right)}{\cancel{\left((n-1)!\right)^{n-1}}\;(n-10)^{n-1}} =$$

$$= \lim_{n\to+\infty} \frac{n^{n-1} - (n-1)!}{(n-10)^{n-1}} = e^{10}$$

$$= \lim_{n\to+\infty} \frac{n^{n-1}}{(n-10)^{n-1}} \Big(\underbrace{1 - \frac{(n-1)!}{n^{n-1}}}_{\to 1} \Big) \approx \lim_{n\to+\infty} \left(\frac{n}{n-10}\right)^{n-1} =$$

$$= \lim_{n\to+\infty} \left(\frac{n-10}{n}\right)^{1-n} =$$

$$= \lim_{n\to+\infty} \underbrace{\left(1-\frac{10}{n}\right)^{1}}_{1} \Big[\underbrace{\left(1-\frac{10}{n}\right)^{-n/10}}_{e} \Big]^{10} =$$

$$0 < \frac{\overbrace{(n-1)}^{m}!}{n^{n-1}} = \frac{m!}{(m+1)^m} < \frac{m!}{m^m} \quad m \to \infty$$

$$a_n = \frac{n \sin n}{n^2+1}$$

$\nexists \lim_{n \to +\infty} \sin n$

Teo dei Carabinieri

$$-1 \leq \sin n \leq 1 \qquad \cdot n > 0$$

$$-n \leq n \sin n \leq n \qquad : (n^2+1) > 0$$

$$\underbrace{\frac{-n}{n^2+1}}_{0} \leq \frac{n}{n^2+1} \sin n \leq \underbrace{\frac{n}{n^2+1}}_{0}$$

$n \to \infty$

$$\Downarrow$$

0

$$\lim_{n \to +\infty} \frac{n \sin n}{n^2+1} = 0$$

$$\lim_{n \to +\infty} \frac{n + \sin n}{\operatorname{arctg} n - n} =$$

$$-1 \le \sin n \le 1 \quad n \in \mathbb{N}$$

$$\underbrace{-\frac{1}{n}}_{\downarrow\, 0} \le \frac{\sin n}{n} \le \underbrace{\frac{1}{n}}_{\downarrow\, 0}$$

$$= \lim_{n \to +\infty} \frac{\cancel{n}\left(1 + \overset{\to 0}{\frac{\sin n}{n}}\right)}{\cancel{n}\left(\underbrace{\frac{\operatorname{arctg} n}{n}}_{\downarrow\, 0} - 1\right)} = -1$$

$b_n = \frac{\sin n}{n} \to 0$ teo. dei carabinieri

$$\lim_{n \to +\infty} \operatorname{arctg} n = \frac{\pi}{2}$$

LIMITI NOTEVOLI

SUCCESSIONI

$$\lim_{n \to +\infty} \frac{\operatorname{sen} \frac{1}{n}}{1/n}$$

$\frac{1}{n} \rightsquigarrow 0$

FUNZIONI

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$$

$$\lim_{n\to+\infty} \frac{n^{n-3}+(n-3)^n}{6n^n+7n!} =$$

$$= \lim_{n\to+\infty} \frac{\cancel{n^n}\left(\overset{\to 0}{n^{-3}}+\left(\frac{n-3}{n}\right)^n\right)}{\cancel{n^n}\left(6+7\underbrace{\frac{n!}{n^n}}_{\to 0}\right)} = \frac{e^{-3}}{6}$$

$$\lim_{n\to+\infty}\left(\frac{n-3}{n}\right)^n = \lim_{n\to+\infty}\left[\left(1-\frac{3}{n}\right)^{-\frac{n}{3}}\right]^{-3} = e^{-3}$$

$$-\frac{3}{n} = +\frac{1}{-\frac{n}{3}}$$