SERIE 1: LA CONVERGENZA

C.N. $\sum_{n=0}^{+\infty} a_n$ converge $\Rightarrow \lim_{n\to+\infty} a_n = 0$

esempi:

es 1 $\quad \sum_{n=1}^{+\infty} (-1)^n \frac{n+1}{n}$ $\quad \lim_{n\to+\infty} (-1)^n \underbrace{\frac{n+1}{n}}_{1} = \begin{cases} +1 & n \text{ pari} \\ -1 & n \text{ dispari} \end{cases}$ $\nexists$

es 2 $\quad \sum_{n=2}^{+\infty} (\log n)^{-\frac{1}{n}}$ $\quad \lim_{n\to+\infty} (\log n)^{-\frac{1}{n}} = \lim_{n\to+\infty} e^{\log(\log n)^{-\frac{1}{n}}} =$

$= \lim_{n\to+\infty} e^{-\frac{1}{n}\cdot\log(\log n)} = 1$ $\quad$ ($\to 0$)

es 3 $\quad \sum_{n=1}^{+\infty} \frac{5}{n} = 5\sum_{n=1}^{+\infty}\frac{1}{n}$ non conv. $\quad \lim_{n\to+\infty} \frac{5}{n} = 0$ $\quad$ (controesempio) $\nLeftarrow$

es 4 $\quad \sum_{n=1}^{+\infty} n \operatorname{sen}\frac{1}{n}$ $\quad \lim_{n\to+\infty} n \operatorname{sen}\frac{1}{n} = 1$

es 5 $\quad \sum_{n=1}^{+\infty} n \operatorname{sen}\frac{1}{n^2}$ $\quad \lim_{n\to+\infty} n \operatorname{sen}\frac{1}{n^2} = \lim_{n\to+\infty} \frac{\overbrace{n^2 \cdot \operatorname{sen}\frac{1}{n^2}}^{1}}{n} =$

$= 0$

CRITERI PER SERIE A TERMINI NON NEGATIVI

(1) data $\sum a_n$, $a_n \geq 0$, se $\{S_n\}$ è monotona non decrescente
$\Rightarrow \sum a_n$ è convergente o divergente, ma non indeterminata

(2) Siano a_n e b_n, t.c. $a_n \geq 0$ e $b_n \geq 0$; sia $c > 0$ t.c. $a_n \leq c \cdot b_n \ \forall n \geq \bar{n}$

CONFRONTO

$\Rightarrow$

- $\sum b_n$ converge $\Rightarrow$ $\sum a_n$ converge (b_n maggiorante di a_n)
- $\sum a_n$ diverge $\Rightarrow$ $\sum b_n$ diverge (a_n minorante di b_n)

es 1 $a_n = \frac{1}{n^3}$; $b_n = \frac{1}{n^2}$; $c_n = \frac{1}{n}$: $\forall n > 1$: $\frac{1}{n^3} < \frac{1}{n^2} < \frac{1}{n}$

$\Rightarrow \sum_{n=1}^{+\infty} \frac{1}{n^2}$ è maggiorante di $\sum_{n=1}^{+\infty} \frac{1}{n^3}$

Serie armonica : $\sum_{n=1}^{+\infty} \frac{1}{n}$ divergente

$\sum_{n=1} \frac{1}{n^2}$ è minorante di $\sum_{n=1} \frac{1}{n}$

es 1 serie geometrica: $\sum_{n=0}^{+\infty} a q^n$ converge se $|q| < 1$

$2^{n-1} \leq n! \leq n^n \Rightarrow \frac{1}{2^{n-1}} \geq \frac{1}{n!} \geq \frac{1}{n^n} \Rightarrow$ la serie ...

SERIE DI RIFERIMENTO:

$$\sum_{n=1}^{+\infty} \frac{1}{n^\alpha}$$ converge per $\alpha > 1$

diverge per $\alpha \leq 1$

$$\sum_{n=2}^{+\infty} \frac{1}{n(\log n)^\alpha}$$ converge per $\alpha > 1$

diverge per $\alpha \leq 1$

$$\sum_{n=2}^{+\infty} \frac{1}{n^\alpha \log n}$$ converge per $\alpha > 1$

diverge per $\alpha \leq 1$

serie armonica : $\sum_{n=1}^{+\infty} \frac{1}{n}$ divergente

serie geometrica : $\sum_{n=0}^{+\infty} a\, q^n$ converge se $|q|<1$, $a \in \mathbb{R}$

es 1 $\sum_{n=1}^{+\infty} \frac{1}{n^2}$

$\forall n \in \mathbb{N} \setminus \{1\}$

$\frac{1}{n^2-1} > \frac{1}{n^2}$

$$\sum_{n=2}^{+\infty} \frac{1}{n^2-1} = \frac{1}{2} \sum_{n=2}^{+\infty} \left(\frac{1}{n-1} - \frac{1}{n+1} \right) \qquad S_n = \frac{1}{2}\left(\frac{3}{2} - \frac{1}{n} - \frac{1}{n+1} \right)$$

$$\lim_{n \to +\infty} S_n = \lim_{n \to +\infty} \frac{1}{2}\left(\frac{3}{2} - \underset{\downarrow 0}{\frac{1}{n}} - \underset{\downarrow 0}{\frac{1}{n+1}} \right) = \frac{3}{4}$$ converge

es 2 $\sum_{n=0}^{+\infty} \operatorname{sen} \frac{3}{2^n}$

$\forall n \in \mathbb{N}$ $\quad \operatorname{sen} \frac{3}{2^n} < \frac{3}{2^n}$

$$\sum_{n=0}^{+\infty} \operatorname{sen} \frac{3}{2^n} < \sum_{n=0}^{+\infty} \frac{3}{2^n} = \qquad \forall n \in \mathbb{N}$$

$$= 3 \sum_{n=0}^{+\infty} \left(\frac{1}{2} \right)^n \leftarrow \text{convergente} \Rightarrow \sum_{n=0}^{+\infty} \operatorname{sen} \frac{3}{2^n} \text{ conv.}$$

(Teo. confronto)

serie geometrica $q = \frac{1}{2}$

es 3 $\displaystyle\sum_{n=0}^{+\infty} \underbrace{\frac{n^2}{1+n^2}}_{<1} \cdot 2^{-n}$ serie a t. p.

$$a_n = \frac{n^2}{1+n^2} \cdot 2^{-n} \le 2^{-n} \rightarrow \sum_{n=0}^{+\infty} \frac{1}{2^n} = \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n$$

serie geom. $p=\frac{1}{2}$

Σa_n convergente $\leftarrow$ convergente

es 4 $\displaystyle\sum_{n=1}^{+\infty} \left(\sqrt{n+1} - \sqrt{n}\right)^3$

? $a_n > 0$ $\quad \sqrt{n+1} - \sqrt{n} > 0$? $\forall n$

$\sqrt{n+1} > \sqrt{n}$ vero $\forall n$

$n+1 > n \quad 0 < 1$ vero

$$a_n = \left(\sqrt{n+1} - \sqrt{n}\right)^3 = \left(\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}\right)^3 = \left(\frac{1}{\sqrt{n+1}+\sqrt{n}}\right)^3 < \frac{1}{n^{3/2}}$$

grado den. 1/2

$\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n^{3/2}}$ serie armonica generalizzata, $\alpha = \frac{3}{2} > 1$ converge.

es 5

$$\sum_{n=1}^{+\infty} \frac{1}{n+\sqrt{n}}$$ serie a T. p.

$$a_n = \frac{1}{n+\sqrt{n}} \geq \frac{1}{n+n} = \frac{1}{2n} = \frac{1}{2}\cdot\frac{1}{n}$$

$$\sum_{n=1}^{+\infty} \frac{1}{2}\cdot\frac{1}{n} = \frac{1}{2}\cdot\sum_{n=1}^{+\infty}\frac{1}{n}$$ serie armonica $\Downarrow$ divergente

$$\sum_{n=1}^{+\infty} \frac{1}{n+\sqrt{n}}$$ diverge $\Leftarrow$

es 6.

$$\sum_{n=2}^{+\infty} \frac{\log n}{n\sqrt{n+1}}$$

$$a_n = \frac{\log n}{n\sqrt{n+1}} < \frac{\log n}{n\cdot\sqrt{n}} < \frac{n^{\alpha - \frac{1}{4}}}{n^{3/2}} < \frac{1}{n^{5/4}}$$

[N.B. $\forall \alpha>0\ \exists \bar{n}\in\mathbb{N}: \forall n>\bar{n}\quad \log n < n^{\alpha}$] Attenzione $\log n < 1$ NO!!

$$\sum_{n=1}^{+\infty}\frac{1}{n^{5/4}}$$ è convergente (serie armonica generalizzata)

$$\frac{5}{4} > 1$$

$$\Rightarrow \sum_{n=2}^{+\infty}\frac{\log n}{n\sqrt{n+1}}$$ convergente

es7: Se $\alpha \in \mathbb{R}$, la serie $\displaystyle\sum_{n=2}^{+\infty}\left(\frac{e^{\alpha n}}{n^2}+\frac{1}{n^{\alpha+3}\log n}\right)$ converge $\iff \alpha$?

$$=\sum_{n=2}^{+\infty}\frac{e^{\alpha n}}{n^2}+\sum_{n=2}^{+\infty}\frac{1}{n^{\alpha+3}\cdot\log n}$$

(se convergono entrambe, $\Rightarrow$ converge la serie "somma")

1) $\displaystyle\sum_{n=2}^{+\infty}\frac{e^{\alpha n}}{n^2}$ se $\alpha>0$ $\displaystyle\lim_{n\to+\infty}\frac{e^{\alpha n}}{n^2}=+\infty$ divergente

(*) se $\underbrace{\alpha\le 0}$ $\displaystyle b_n=\frac{e^{\alpha n}}{n^2}=\frac{1}{n^2\underbrace{e^{-\alpha n}}_{\geq 1}}\le\frac{1}{n^2}\quad\forall n\ge 2$

$(-\alpha)>0 \Longrightarrow$

$\sum b_n$ converge $\Longleftarrow$ $\sum\frac{1}{n^2}$ serie armonica generalizzata $\alpha=2$ converge.

2) $\displaystyle\sum_{n=2}^{+\infty}\frac{1}{n^{\alpha+3}\log n}$ convergente se $\alpha+3>1$

$\Downarrow$ $\alpha>-2$ (*)

(1+2) $\begin{cases}\alpha\le 0\\ \alpha>-2\end{cases} \Longrightarrow \alpha\in\,]-2,0] \Longrightarrow \sum$ converge

CRITERI:

① DELLA RADICE ASINTOTICO

$a_n \geq 0$

$$\lim_{n \to +\infty} \sqrt[n]{a_n} = L \begin{cases} L < 1 & \text{converge} \\ L = 1 & \text{inefficace} \\ L > 1 & \text{diverge} \end{cases} \qquad \sum_{n=0}^{+\infty} a_n$$

② DEL RAPPORTO ASINTOTICO

$a_n > 0$

$$\lim_{n \to +\infty} \frac{a_{n+1}}{a_n} = L \begin{cases} L < 1 & \text{converge} \\ L = 1 & \text{inefficace} \\ L > 1 & \text{diverge} \end{cases} \qquad \sum_{n=0}^{+\infty} a_n$$

(3) CRITERIO DEL CONFRONTO ASINTOTICO

$\sum a_n$ e $\sum b_n$ serie a termini non negativi

con $b_n > 0$, $\forall n \geq m$ esiste $\lim_{n \to +\infty} \frac{a_n}{b_n} = L$

$$\Rightarrow \begin{cases} L \in]0, +\infty[\text{ e } \sum a_n \text{ conv} \iff \sum b_n \text{ conv.} \\ L = 0 \text{ e } \sum b_n \text{ conv.} \Rightarrow \sum a_n \text{ conv.} \\ L = +\infty \text{ e } \sum b_n \text{ div.} \Rightarrow \sum a_n \text{ div.} \end{cases}$$

es1 Sia $\alpha \in \mathbb{R}$, la serie $\sum_{n=0}^{+\infty} \frac{n^2}{e^{6n\alpha}}$ converge $\Leftrightarrow \alpha = ?$

1. a_n è a termini positivi

2. $\sqrt[n]{a_n} = \sqrt[n]{\frac{n^2}{e^{6n\alpha}}} = \frac{n^{2/n}}{e^{6\alpha}} \Rightarrow \sqrt[n]{n} \cdot \sqrt[n]{n}$

$$\lim_{n\to+\infty} \frac{n^{2/n}}{e^{6\alpha}} = \lim_{n\to+\infty} \frac{\overbrace{\sqrt[n]{n}}^{\to 1} \cdot \overbrace{\sqrt[n]{n}}^{\to 1}}{e^{6\alpha}} = \frac{1}{e^{6\alpha}}$$

se $\frac{1}{e^{6\alpha}} < 1$ $\quad e^{6\alpha} > 1 = e^0 \Leftrightarrow 6\alpha = 0$, $\alpha = 0$

$\quad \hookrightarrow \boxed{\alpha > 0}$ $\quad \Sigma\, a_n$ converge

se $\frac{1}{e^{6\alpha}} > 1$ $\Rightarrow \alpha < 0$ $\quad \Sigma\, a_n$ diverge

se $\frac{1}{e^{6\alpha}} = 1$ $\Rightarrow \alpha = 0$ $\quad \Sigma\, a_n$? Test inefficace se $\alpha = 0$ $\quad \displaystyle\sum_{n=0}^{+\infty} \frac{n^2}{\underbrace{e^0}_{=1}} = \sum_{n=0}^{+\infty} n^2$ serie divergente

se $\alpha < 0$ C.N. non è verificata $\Rightarrow$ divergenza

se $\alpha > 0$ $\sum a_n$ convergente

2) CRIT. RAPP.

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{e^{6\alpha(n+1)}} \cdot \frac{e^{6\alpha n}}{n^2} =$$

$$= \frac{(n+1)^2}{n^2} \cdot \frac{\cancel{e^{6\alpha n}}}{\cancel{e^{6\alpha n}} \cdot e^{6\alpha}}$$

$$\lim_{n\to+\infty} \underbrace{\frac{(n+1)^2}{n^2}}_{\to 1} \cdot \frac{1}{e^{6\alpha}} = \frac{1}{e^{6\alpha}}$$ devo discutere

es2 Trovare $\sup \left\{ \alpha \in \mathbb{R}^+ : \sum_{n=1}^{+\infty} n! \left(\frac{\alpha}{n}\right)^n \text{ converge} \right\}$ $\qquad \alpha > 0$

1. serie a t. p.

2. $a_n = n! \left(\frac{\alpha}{n}\right)^n \qquad \lim_{n\to+\infty} \frac{a_{n+1}}{a_n} =$

$a_{n+1} = (n+1)! \left(\frac{\alpha}{n+1}\right)^{n+1} \qquad = \lim_{n\to+\infty} (n+1)! \left(\frac{\alpha}{n+1}\right)^{n+1} \cdot \frac{1}{n!} \cdot \left(\frac{n}{\alpha}\right)^n =$

$$= \lim_{n\to+\infty} \frac{(n+1)\, n!}{n!} \cdot \frac{\alpha^n \cdot \alpha}{\alpha^n} \cdot \frac{n^n}{(n+1)^n \cdot (n+1)} =$$

$[\alpha > 0]$

$$= \lim_{n\to+\infty} \alpha \left(\frac{n}{n+1}\right)^n = \alpha \cdot \lim_{n\to+\infty} \left[\left(1 + \frac{1}{n}\right)^n\right]^{-1} = \frac{\alpha}{e}$$

se $\frac{\alpha}{e} > 1 \Rightarrow \alpha > e \Rightarrow \sum a_n$ diverge (positivamente) $\quad \sup. +\infty$

$\frac{\alpha}{e} < 1 \Rightarrow \alpha < e \Rightarrow \sum a_n$ converge $\quad \exists \sup_\alpha < +\infty$

$\frac{\alpha}{e} = 1$ criterio inefficace $\sum_{n=1}^{+\infty} n! \left(\frac{e}{n}\right)^n \Rightarrow$ $\quad \alpha = e$

$\alpha = e$

$$\sum_{n=1}^{+\infty} n!\left(\frac{e}{n}\right)^n$$

$$\left(\lim_{n\to+\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to+\infty} \frac{n!}{n^n} = (*)\right)$$

$$a_{n+1} = (n+1)!\left(\frac{e}{n+1}\right)^{n+1} \qquad = +\infty$$

$$a_n = n!\left(\frac{e}{n}\right)^n$$

$$(*) = \lim_{n\to+\infty} (n+1)!\left(\frac{e}{n+1}\right)^{n+1}\cdot\left(\frac{n}{e}\right)^n\cdot\frac{1}{n!} =$$

$$= \lim_{n\to+\infty} (n+1)\,\cancel{n!}\,\frac{e^{n+1}}{(n+1)^{n+1}}\cdot\frac{n^n}{e^n}\cdot\frac{1}{\cancel{n!}} =$$

$$= \lim_{n\to+\infty} \frac{\cancel{(n+1)}\,\cancel{e^n}\cdot e}{(n+1)^n\,\cancel{(n+1)}}\cdot\frac{n^n}{\cancel{e^n}} = \lim_{n\to+\infty} e\cdot\underbrace{\left(\frac{n}{n+1}\right)^n}_{e^{-1}} = 1$$

$$\sum_{n=1}^{+\infty} n! \left(\frac{e}{n}\right)^n$$

$$a_n \qquad \lim_{n\to+\infty} \frac{n!\, e^n}{n^n} =$$

$$= \lim_{n\to+\infty} \underbrace{\cancel{n^n}\cdot \cancel{e^{-n}} \sqrt{2\pi n}\; \overbrace{e^{\frac{\vartheta_n}{12n} \to 0}}^{1}}_{F.S.} \cdot \frac{\cancel{e^n}}{\cancel{n^n}} =$$

$$= +\infty$$

es 3 Studiare la convergenza della serie $\sum_{n=1}^{+\infty} \left(\frac{n}{n+1}\right)^{n^2}$

criterio della radice:

$$\lim_{n\to+\infty} \sqrt[n]{a_n} = \lim_{n\to+\infty} \sqrt[\cancel{n}]{\left(\frac{n}{n+1}\right)^{n^{\cancel{2}}}} = \lim_{n\to+\infty} \left(\frac{n}{n+1}\right)^n$$

$$= \lim_{n\to+\infty} \left(\frac{n+1}{n}\right)^{-n} = \lim_{n\to+\infty} \left(\left(1+\frac{1}{n}\right)^n\right)^{-1} = e^{-1} = \frac{1}{e} < 1$$

$\longrightarrow$ conv.

$$= \lim_{n\to+\infty} e^{\log\left(\frac{n}{n+1}\right)^n} = \lim_{n\to+\infty} e^{\log\left(\frac{n+1}{n}\right)^{-n}} =$$

$$= \lim_{n\to+\infty} e^{-\log\underbrace{\left(1+\frac{1}{n}\right)^n}_{e}} = e^{-1}$$

es 3 $\displaystyle\sum_{n=1}^{+\infty} \underbrace{\left(e^{\frac{1}{n^2}}-1\right)}_{>0}$

confronto $b_n = \frac{1}{n^2}$ serie armonica generalizzata

$\sum b_n = \sum \frac{1}{n^2}$ conv.

$$\lim_{n\to+\infty} \frac{e^{\frac{1}{n^2}}-1}{\frac{1}{n^2}} = 1$$

LIM. NOT.

$$\lim_{x\to 0} \frac{e^x-1}{x} = 1$$

$$\lim_{n\to+\infty} \frac{e^{\frac{1}{n}}-1}{1/n} = 1$$

$\sum b_n$ conv. $\Rightarrow$ $\sum a_n$ conv.

es 4 $\displaystyle\sum_{n=0}^{+\infty} \frac{n+3}{n^3+n^2+4}$ t.p.

$\sum b_n = \sum \frac{1}{n^2}$ conv.

$$\lim_{n\to+\infty} \frac{\frac{n+3}{n^3+n^2+4}}{\frac{1}{n^2}} = \lim_{n\to+\infty} \frac{n+3}{n^3+n^2+4}\cdot n^2 = \lim_{n\to+\infty} \frac{n^3+3n^2}{n^3+n^2+4} =$$

$$= 1 \quad \Rightarrow \quad \sum a_n \text{ conv.}$$

es 5. $\displaystyle\sum_{n=1}^{+\infty} \frac{4^n (n^2 + \operatorname{sen}(e^n))}{3^{2n}}$ S.T.P.

$-1 \le \operatorname{sen}(e^n) \le 1$

$n^2 + \operatorname{sen} e^n \ge n^2 - 1 \ge 0 \quad \forall n > 1$

$(n^2 + \operatorname{sen} e^n) \sim n^2 \quad \forall n > \bar{n}$

$$a_n = \frac{4^n (n^2 + \operatorname{sen} e^n)}{3^{2n}} \sim \frac{4^n n^2}{3^{2n}} = b_n$$

$$\lim_{n\to+\infty} \sqrt[n]{b_n} = \lim_{n\to+\infty} \sqrt[n]{\frac{4^n n^2}{3^{2n}}} = \frac{4}{3^2} \lim_{n\to+\infty} \sqrt[n]{n^2} = \boxed{= \frac{4}{9} < 1}$$

$$\sqrt[n]{n^2} = \underbrace{\sqrt[n]{n}}_{\downarrow 1} \cdot \underbrace{\sqrt[n]{n}}_{\downarrow 1}$$

es 6. $\displaystyle\sum_{n=2}^{+\infty} \frac{\ln n}{n}$ S.T.P. $n \ge 1$

$$\underbrace{\frac{\ln n}{n}}_{\substack{a_n \\ \text{divergente}}} > \underbrace{\frac{1}{n}}_{\substack{b_n \\ \text{divergente}}} \quad \text{vera} \quad \forall n > \bar{n} \ (1)$$

criterio del confronto

$\sum \frac{1}{n}$ diverge $\Rightarrow \sum \frac{\ln n}{n}$ diverge

es 7 $\displaystyle\sum_{n=1}^{+\infty} \frac{\log n}{n^4}$ T. P. $\displaystyle\sum_n \frac{1}{n^3}$ s. a. g. $(\alpha > 1)$

serie di partenza

conv.

$$\lim_{n\to+\infty} \frac{\frac{\log n}{n^4}}{\frac{1}{n^3}} = \lim_{n\to+\infty} \frac{\log n}{n^4 \cdot \frac{1}{n^3}} = \lim_{n\to+\infty} \frac{\log n}{n} = 0$$

↑ serie di confronto

es 8 $\displaystyle\sum_{n=1}^{+\infty} \frac{5n+4^n}{\ln n + 5^n} =$ S.T.P. $(n \geq 1)$ $\qquad b_n = \left(\frac{4}{5}\right)^n \to \sum \left(\frac{4}{5}\right)^n$

si comporta

$$\Rightarrow \frac{5n+4^n}{\ln n+5^n} \sim \frac{4^n}{5^n} = \left(\frac{4}{5}\right)^n$$

$$\left[\frac{5n+4^n}{\ln n+5^n} = \frac{4^n\left(\frac{5n}{4^n}+1\right)}{5^n\left(\frac{\ln n}{5^n}+1\right)} \sim \frac{4^n}{5^n} \quad \forall n > \bar{n} \right.$$

$\displaystyle\sum_{n=1}^{+\infty} \left(\frac{4}{5}\right)^n$ è convergente $\left(q = \frac{4}{5} < 1\right)$

$\Rightarrow \displaystyle\sum a_n = \sum_{n=1}^{+\infty} \frac{5n+4^n}{\ln n+5^n}$ converge per confronto asintotico!

es 9 $\displaystyle\sum_{n=1}^{+\infty} \frac{\cos n}{n^3}$ STP $-1 < \cos n < 1$

$[0,1]$

$$\left|\frac{\cos n}{n^3}\right| \leq \frac{|\cos n|}{n^3} \leq \frac{1}{n^3}$$

per il criterio del confronto: $\sum \frac{1}{n^3}$ converge, $\alpha = 3 > 1$,

quindi $\sum \frac{\cos n}{n^3}$ converge (conv. ass.)

es 10 $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2+1}{n^\alpha}$ $\alpha \in \mathbb{R}$ STP.

$$a_n = \frac{n^2+1}{n^\alpha} \simeq \frac{n^2}{n^\alpha} = \frac{1}{n^{\alpha-2}} \qquad \sum \frac{1}{n^{\alpha-2}} \qquad \alpha - 2 > 1 \Downarrow \alpha > 3$$

$\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n^{\alpha-2}}$ converge se $\alpha > 3$ $\Rightarrow$ $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2+1}{n^\alpha}$ converge per confronto.

es 11 $\displaystyle\sum_{n=2}^{+\infty} \frac{(n+1)^k}{n(n-1)}$

$$a_n = \frac{(n+1)^k}{n(n-1)} = \frac{(n+1)^k}{n^2-n} \simeq \frac{n^k}{n^2} = \frac{1}{n^{2-k}} \qquad 2-k>1 \Rightarrow k<1$$

$\displaystyle\sum \frac{1}{n^{2-k}}$ converge se $k<1 \Rightarrow$ per confronto $\displaystyle\sum_{n=2}^{+\infty} \frac{(n+1)^k}{n(n-1)}$ conv.

es 12 Sia $a_n = \begin{cases} \frac{1}{n} & \text{se } n \le 100 \\ \frac{1}{n^2} & \text{se } n \ge 101 \end{cases} \Rightarrow \displaystyle\sum_{n=1}^{+\infty} a_n = ?$

$$= \sum_{n=1}^{+\infty} a_n = \underbrace{\sum_{n=1}^{100} \frac{1}{n}}_{\text{somma finita}} + \underbrace{\sum_{n=101}^{+\infty} \frac{1}{n^2}}_{\text{convergente}} \Rightarrow \text{convergente}$$

(serie armonica generalizzata $\alpha = 2 > 1$)

es 13 $\quad \displaystyle\sum_{n=1}^{+\infty} \frac{e^n}{e^{2n}+2n}$ $\quad$ T.P.

$$\lim_{n\to+\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to+\infty} \frac{e^{n+1}}{e^{2(n+1)}+2(n+1)} \cdot \frac{e^{2n}+2n}{e^n} =$$

$$= \lim_{n\to+\infty} \frac{\cancel{e^n}\cdot e}{e^{2n}\cdot e^2+2(n+1)} \cdot \frac{e^{2n}+2n}{\cancel{e^n}} =$$

$$= \lim_{n\to+\infty} \frac{e}{\cancel{e^{2n}}\left(e^2+\underbrace{\frac{2(n+1)}{e^{2n}}}_{\to 0}\right)} \cdot \frac{\cancel{e^{2n}}\left(1+\underbrace{\frac{2n}{e^{2n}}}_{\to 0}\right)}{1} =$$

$$= \frac{e}{e^2} = \frac{1}{e} < 1$$

convergenza col criterio del rapporto.