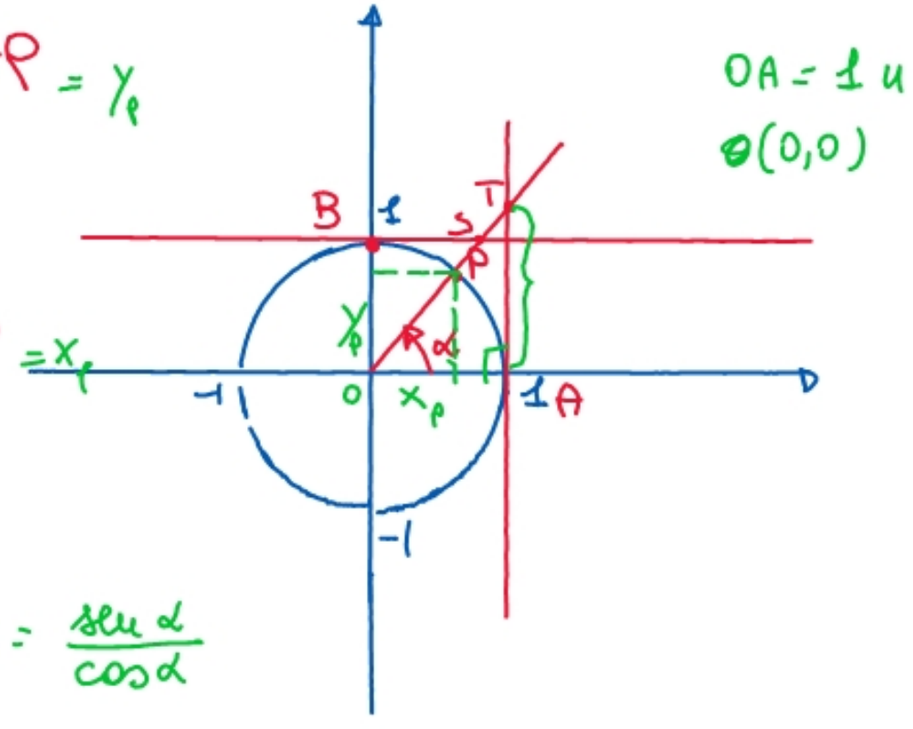
TRIGONOMETRIA

1. operatori goniometrici
2. funzioni goniometriche : rappresentazioni grafiche e trasformazioni
3. equazioni e disequazioni goniometriche

1. operatori goniometrici: $\alpha \in [0, 360^\circ] = [0, 2\pi]$

$\sin \alpha$ = ordinata di $P = y_P$

$\cos \alpha$ = ascissa di $P = x_P$



$$\operatorname{tg} \alpha = TA = \frac{y_P}{x_P} = \frac{\sin \alpha}{\cos \alpha}$$

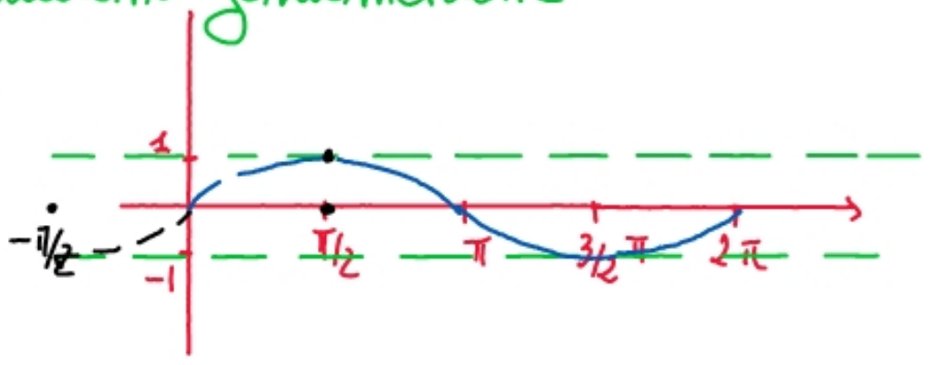
$$\operatorname{ctg} \alpha = SB = \frac{x_P}{y_P} = \frac{\cos \alpha}{\sin \alpha} = \frac{1}{\operatorname{tg} \alpha}$$

$$\sec \alpha = \frac{1}{\cos \alpha}$$

$$\operatorname{cosec} \alpha = \frac{1}{\sin \alpha}$$

2. funzioni goniometriche

$y = \operatorname{sen} x$



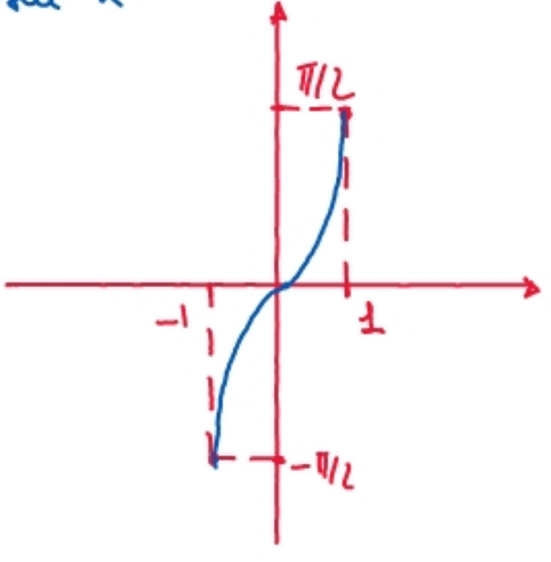
periodica: 2π

$\forall x \in \mathbb{R}$ (ampolo)

$-1 \le y \le 1$

$-1 \le \operatorname{sen} x \le 1$

$y = \operatorname{arcsen} x$

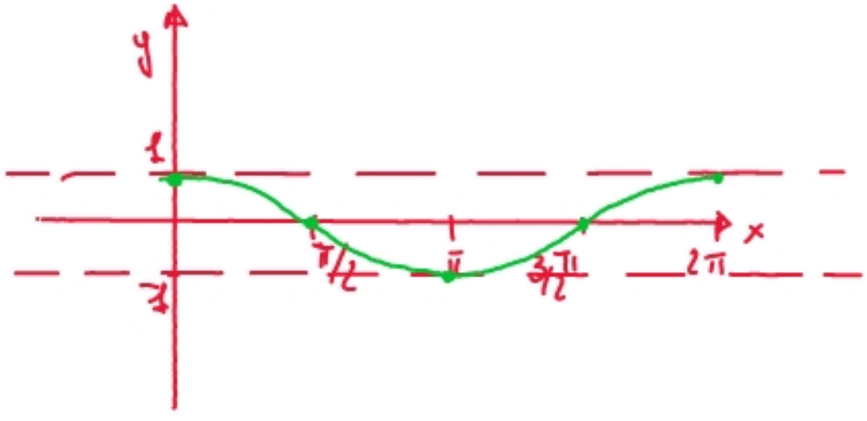


si inverte $y = \operatorname{sen} x$ nel tratto $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y = \operatorname{arcsen} x$

- dominio: $[-1, 1]$
- codominio: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- crescente

$y = \cos x$



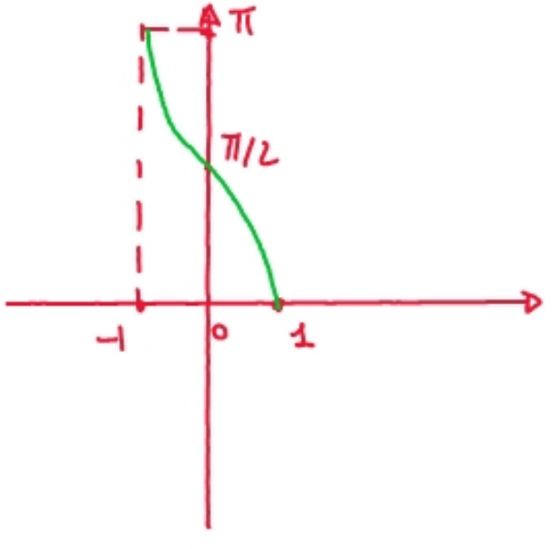
periodicità 2π

$\forall x \in \mathbb{R}$ (dominio)

$-1 \leq y \leq 1$

$-1 \leq \cos x \leq 1$

$y = \arccos x$

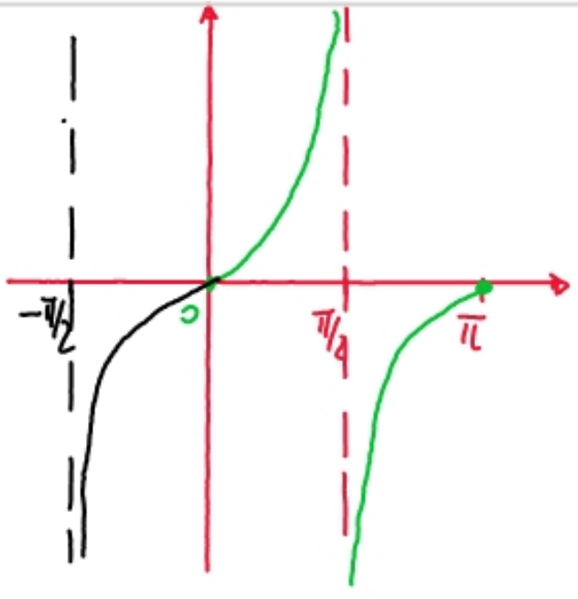


si inverte la proiezione di $y = \cos x$ nell'intervallo $[0, \pi]$

$y = \arccos x$

- dominio $[-1, 1]$
- codominio $[0, \pi]$
- decrescente

$y = tg\,x$



$tg\,x = \frac{sen\,x}{cos\,x}$

$cos\,x \neq 0 \qquad x \neq \frac{\pi}{2} + k\pi$

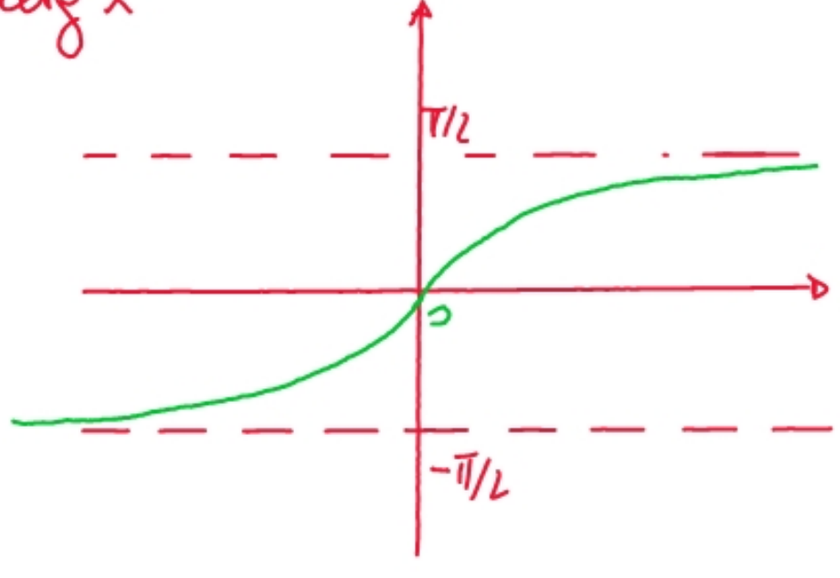
periodica : π

dominio $\quad x \neq \frac{\pi}{2} + k\pi \qquad k \in \mathbb{Z}$

strettamente crescente

asintoto verticale $\quad x = \frac{\pi}{2} + k\pi$

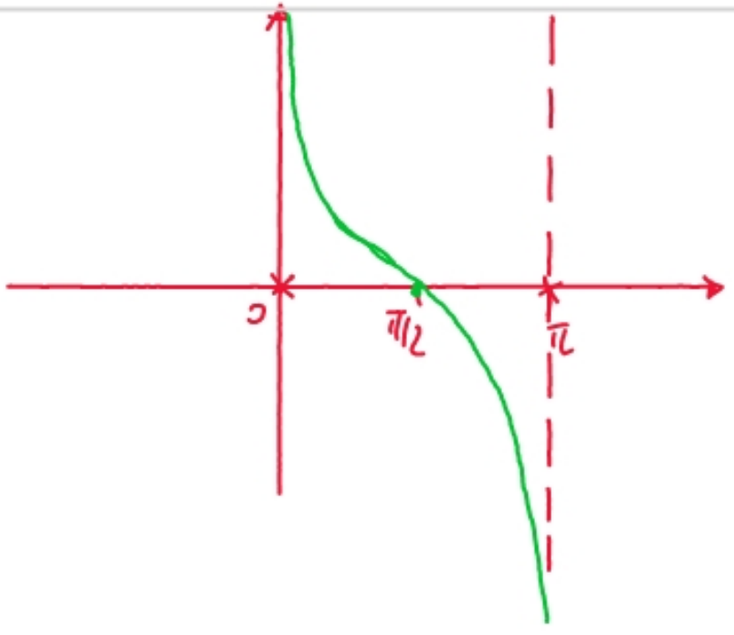
$y = arctg\,x$



$y = arctg\,x$

- dominio $\quad \forall x \in \mathbb{R}$
- codominio $\quad \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$
- asintoti orizzontali $y = -\frac{\pi}{2}$, $y = \frac{\pi}{2}$
- crescente

$y = \operatorname{ctg} x$

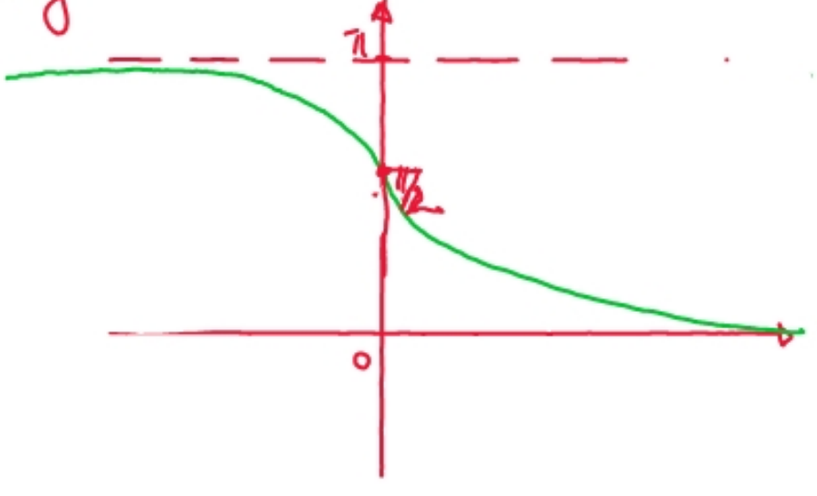


$\operatorname{ctg} x = \frac{\cos x}{\operatorname{sen} x}$

$\operatorname{sen} x \neq 0 \qquad x \neq k\pi \qquad k \in \mathbb{Z}$

dominio $x \neq k\pi \quad k \in \mathbb{Z}$
codominio $\mathbb{R}$
strettamente decrescente
periodica: π

$y = \operatorname{arcctg} x$

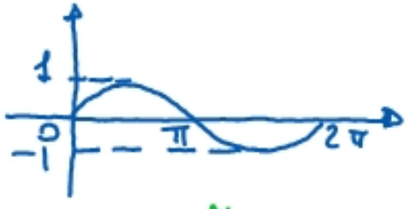


si inverte nel tratto $[0, \pi]$

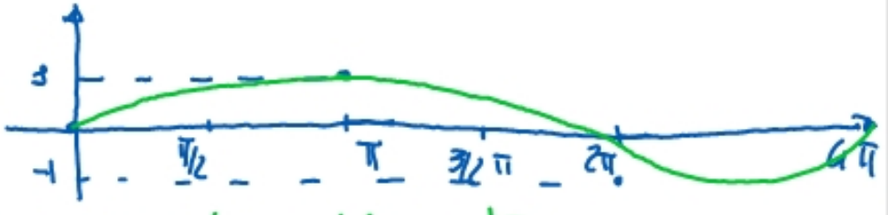
$y = \operatorname{arcctg} x$
dominio $\mathbb{R}$
codominio $]0, \pi[$
asintoti orizzontali $y = 0$ $y = \pi$
strettamente decrescente

3. grafici e algebra per le risoluzioni di equazioni e disequazioni

$y = \sin x$



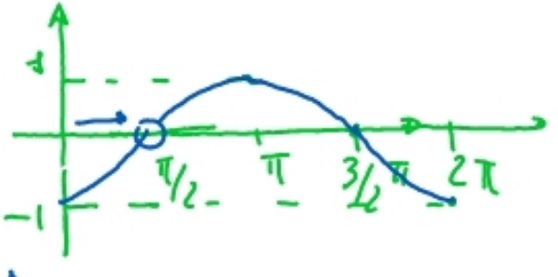
$y = \sin\left(\frac{x}{2}\right)$ se divido l'argomento di un valore $k > 1$, il periodo si dilata : : 2 → periodo raddoppiato



$y = \sin(2x)$ periodo si dimezza ($k > 1$)

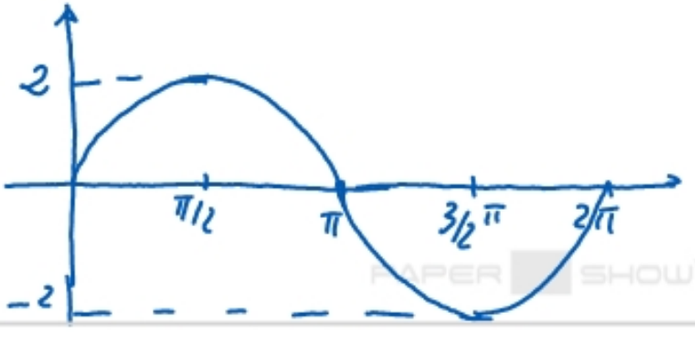
si contrae

$y = \sin\left(x - \frac{\pi}{2}\right)$ una traslazione dell'asse delle ordinate verso destra di $\pi/2$



$y = 2 \sin x$ $\quad -1 \le \sin x \le 1$

$-2 \le 2 \sin x \le 2$



$y = \sin x + 1$ $\quad -1 \le \sin x \le 1$

$0 \le \sin x + 1 \le 2$

Equazione goniometrica

$\sin^2 x - \sin x = 0$

- eq. goniometrica di 2° grado
- un unico operatore goniometrico

1° Tappa: trovare il valore dell'operatore goniometrico

$$\sin x (\sin x - 1) = 0 \quad \Rightarrow \quad \sin x = 0 \quad (1)$$

($\sin x = t$ cambio di variabile $t' - t = 0$)

$$\sin x = 1 \quad (2)$$

2° Tappa: calcolare il valore degli angoli, con la loro periodicità

(1) $x_1 = 0 + 2k\pi$ $\quad k \in \mathbb{Z}$ $\quad x = k\pi$

$x_2 = \pi + 2k\pi$

(2) $x_3 = \frac{\pi}{2} + 2k\pi$

$\sec(45° + x) + 3\sec(45° + x) = 2$

$4\sec(45° + x) = 2$

Argumento $= 45° + x$

$\sec(45° + x) = \frac{2}{4} = \frac{1}{2}$

1) $45° + x = 30° + K360°$

esplicitando: $x = 30° - 45° + K360° = -15° + K360°$

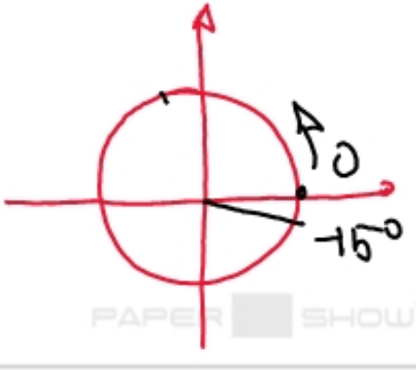
$= 360° - 15° + K360° = \underline{345° + K360°}$

$[0, 360°]$

2) $45° + x = 150° + K360°$

$x = 150° - 45° + K360° =$

$= \underline{105° + K360°}$

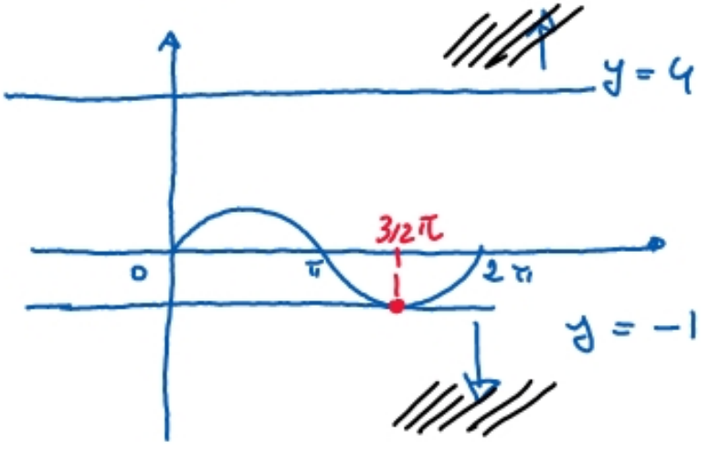


$\sin^2 x - 3\sin x - 4 \geq 0$

$(\sin x - 4)(\sin x + 1) \geq 0 \longrightarrow \begin{cases} \sin x - 4 = 0 & \sin x = 4 \\ \sin x + 1 = 0 & \sin x = -1 \end{cases}$

$\sin x = 4, -1$

DICE: $\sin x \leq -1$, $\sin x \geq 4$ (IMP.)



$[0, 2\pi]$

$x = \frac{3}{2}\pi$

Soluzione generale

$x = \frac{3}{2}\pi + 2k\pi$

$\sqrt{2\sin^2 x - 1} > 2$ procedimento per confronto

$-1 \le \sin x \le 1$

$(\ )^2$

$0 \le \sin^2 x \le 1$

$(\cdot 2)$

$0 \le 2\sin^2 x \le 2$

(-1)

$0 - 1 \le 2\sin^2 x - 1 \le 2 - 1 = 1$

$\sqrt{-1} \le \sqrt{2\sin^2 x - 1} \le \sqrt{1}$

↑ IMP.

$\Downarrow$

$0 \le \sqrt{2\sin^2 x - 1} \le 1$

$\sqrt{2\sin^2 x - 1} > 2$ IMPOSSIBILE

$\sqrt{2\sin^2 x - 1} > 2$

$$\begin{cases} 2\sin^2 x - 1 \geq 0 \\ 2\sin^2 x - 1 > 4 \end{cases}$$

$$\begin{cases} \frac{\pi}{4} \leq x \leq \frac{3}{4}\pi \cup \frac{5}{4}\pi \leq x \leq \frac{7}{4}\pi \\ \text{IMPOSSIBILE} \end{cases}$$

1° disequazione $\quad 2\sin^2 x - 1 \geq 0$

$(\sqrt{2}\sin x - 1)(\sqrt{2}\sin x + 1) \geq 0$

$\sin x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sin x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\Downarrow$

$\sin x \leq -\frac{\sqrt{2}}{2} \qquad \sin x \geq \frac{\sqrt{2}}{2}$

2° disequazione

$2\sin^2 x > 5$

$\sin^2 x > 5/2 = 2{,}5 > 1$

IMPOSSIBILE