# SVILUPPI CON I POLINOMI DI TAYLOR

**es 1**

$$f(x) = \log(3x+1) \qquad x_0 = 1$$

polinomio di Taylor di ordine 2 e centro $x_0 = 1$ :

$$T_1^2(f) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2$$

$$f(x_0) = \log(3+1) = \log 4 \qquad x_0 = 1$$

$$f'(x) = \frac{1}{3x+1} \cdot 3 = \frac{3}{3x+1} \qquad f'(x_0=1) = \frac{3}{4}$$

$$f''(x) = 3 \cdot \frac{-3}{(3x+1)^2} = -\frac{9}{(3x+1)^2} \qquad f''(x_0=1) = -\frac{9}{16}$$

$$T_1^2(f)(x) = \log 4 + \frac{3}{4}(x-1) - \overbrace{\frac{9}{16} \cdot \frac{1}{2!}}^{-9/32}(x-1)^2$$

$$T_1^2(f)(x) = \log 4 + \frac{3}{4}(x-1) - \frac{9}{32}(x-1)^2$$

$$f(x) = T_1^2(f)(x) + o(x-1)^2 =$$

nell'intorno di $x_0 = 1$

$$= \log 4 + \frac{3}{4}(x-1) - \frac{9}{32}(x-1)^2 + o(x-1)^2 =$$

$\approx o(x^2)$

$$= \log 4 + \frac{3}{4}x - \frac{3}{4} - \frac{9}{32}(x^2 + 1 - 2x) + o(x-1)^2 =$$

$$= \left(\log 4 - \frac{9}{32} - \frac{3}{4}\right) + \left(\frac{3}{4} - \frac{9}{16}\right)x - \frac{9}{32}x^2 + o(x-1)^2 =$$

$$= \left(\log 4 - \frac{33}{32}\right) + \frac{3}{16}x - \frac{9}{32}x^2 + o(x-1)^2$$

$o(x^2)$

# SVILUPPI DI TAYLOR E DI MACLAURIN

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2}x^2 + \frac{a(a-1)(a-2)}{6}x^3 + \dots + \binom{a}{n}x^n + o(x^n)$$

$n!$

$|x| < 1$

$a$ reale

$$\operatorname{sen} x = x - \frac{x^3}{6} + \frac{x^5}{120} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$x \in \mathbb{R}$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

$x \in \mathbb{R}$

$$\operatorname{tg} x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots + \frac{(-1)^{n-1}2^{2n}(2^{2n}-1)}{(2n)!}x^{2n-1} + o(x^{2n})$$

$$\operatorname{ctg} x = \frac{1}{x} - \frac{1}{3}x - \frac{1}{45}x^3 + \dots + \frac{(-1)^n 2^{2n}}{(2n)!}x^{2n-1} + o(x^{2n})$$

$$\operatorname{arcsen} x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \dots + \frac{(2n-1)!!}{(2n)!!\,(2n+1)}x^{2n+1} + o(x^{2n+2})$$

$|x| < 1$

$$\operatorname{arctg} x = x - \frac{x^3}{3} + \frac{x^5}{5} \cdots \quad + (-1)^n \frac{x^{2n+1}}{(2n+1)} + o\left(x^{2n+2}\right)$$

$$|x| \leq 1$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + o(x^n)$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1} \frac{x^n}{n} + o(x^n)$$

$$\operatorname{senh} x = x + \frac{x^3}{6} + \frac{x^5}{120} + \cdots \quad + \frac{x^{2n+1}}{(2n+1)!} + o\left(x^{2n+1}\right)$$

$$\cosh x = 1 + \frac{x^2}{2} + \frac{x^4}{24} + \cdots \quad + \frac{x^{2n}}{(2n)!} + o\left(x^{2n}\right)$$

Sviluppare al 2° ordine per $x_0 = 0$ le seguenti funzioni:

1) $y = \operatorname{sen}^2 x = \Big(\underbrace{x - \frac{x^3}{6} + o(x^4)}_{(\operatorname{sen} x)}\Big)^2 = x^2 + \frac{x^6}{6} + \underbrace{(o(x^4))^2}_{o(x^8)} - \frac{x^4}{3} - \underbrace{\frac{x^3}{3}\, o(x^4)}_{o(x^7)} + \underbrace{2x\, o(x^4)}_{o(x^5)} =$

$= x^2 - \frac{x^4}{3} + o(x^5)$

2) $y = \cos^2 x = 1 - x^2 + \frac{x^4}{3} + o(x^5)$

3) $y = \cos(2x) = 1 - 2x^2 + \frac{2}{3}x^4 + o(x^5)$

4) $y = \frac{1}{1+e^x} = (1+e^x)^{-1} =$ oppure $= \frac{1}{2} \cdot \frac{1}{1-t}$ (*) con $-t = \frac{1+e^x}{2} - 1$

5) $y = e^{\operatorname{sen} x} = 1 + \frac{\operatorname{sen}^2 x}{2} + o(x^2) = 1 + \frac{1}{2}\left[x^2 - \frac{x^4}{3} + o(x^5)\right] + o(x^2) = 1 + \frac{1}{2}x^2 + o(x^2)$

6) $y = \log(\cos x) = \log\Big(1 \underbrace{- \frac{x^2}{2} + o(x^3)}_{t}\Big) = -\frac{x^2}{2} + o(x^3)$ - - - -

(*) $= \frac{1}{2} \frac{1}{1 - \frac{1-e^x}{2}} = \frac{1}{2} \cdot \frac{1}{1 - \frac{1+x+o(x)}{2}} = \frac{1}{2} \frac{1}{1-\left(-\frac{x}{2} + o(x)\right)} = \frac{1}{2}\left[-\frac{x}{2} + o(x)\right]$

$e^x = 1 + x + o(x)$

## CALCOLO DEI LIMITI:

es 1

$$\lim_{x \to 0} \frac{e^x - 1}{\sqrt{e^x - x - 1}} =$$

(sviluppo al 1° ordine)

$$= \lim_{x \to 0} \frac{\cancel{1} + x + o(x) - \cancel{1}}{\sqrt{\cancel{1} + \cancel{x} + \frac{x^2}{2} - \cancel{x} - \cancel{1} + o(x^2)}} =$$

$$e^x = 1 + x + o(x)$$

$$e^x = 1 + x + \frac{x^2}{2} + o(x^2)$$

2° ordine

$$= \lim_{x \to 0} \frac{x + o(x)}{\sqrt{\frac{x^2}{2} + o(x^2)}}$$

$$\lim_{x \to 0^+} \frac{x + o(x)}{\sqrt{\frac{x^2}{2} + o(x^2)}} = + \frac{1}{1/\sqrt{2}} = +\sqrt{2}$$

$$\lim_{x \to 0^-} \frac{x + o(x)}{\sqrt{\frac{x^2}{2} + o(x^2)}} = -\sqrt{2}$$

es 2

$$\lim_{x\to 0^+}\left(\frac{4x}{Tg(4x)}\right)^{\frac{1}{x\,\mathrm{sen}\,3x}} = \lim_{x\to 0^+} e^{\log\left(\frac{4x}{Tg(4x)}\right)^{\frac{1}{x\,\mathrm{sen}\,3x}}} = e^{-16/9}$$

passando sull'esponente il calcolo del limite

$$\lim_{x\to 0^+} \frac{1}{x\,\mathrm{sen}\,3x}\cdot\log\frac{4x}{Tg\,4x} = \lim_{x\to 0^+}\frac{\log\frac{4x}{Tg\,4x}}{x\,\mathrm{sen}\,3x} =$$

$$= \lim_{x\to 0^+}\frac{\log\left(\frac{tg\,4x}{4x}\right)^{-1}}{x\,\mathrm{sen}\,3x} = \lim_{x\to 0^+} -\frac{\log\left(\frac{Tg\,4x}{4x}\right)}{x\,\mathrm{sen}\,3x} =$$

$$= \lim_{x\to 0^+} -\frac{\log\left(1+\frac{16}{3}x^2+o(x^2)\right)}{x(3x+o(x))} = \lim_{x\to 0^+} -\frac{\frac{16}{3}x^2+o(x^2)}{3x^2+o(x^2)} =$$

$$= -\frac{16}{9}$$

sviluppi

$$\log(1+x) = x + o(x)$$

$$\operatorname{Tg} x = x + \frac{1}{3} x^3 + o(x^3)$$

$$\frac{\operatorname{Tg} 4x}{4x} = \frac{4x}{4x} + \frac{1}{3} \frac{(4x)^3}{4x} + \frac{o(x^3)}{4x} =$$

$$= 1 + \left[\frac{1}{3} (4x)^2 + o(x)^2\right]$$

$$1 + \quad x$$

$$\operatorname{sen} x = x + o(x)$$

$$\operatorname{sen} 3x = 3x + o(3x) = 3x + o(x)$$

T.E.

$$\lim_{x\to 1^+}\left\{e^{\frac{1}{\log x}}-e^{\frac{1}{x-1}}\right\}=$$

$$=\lim_{x\to 1^+}\left[e^{\left(\frac{1}{\log x}-\frac{1}{x-1}\right)}-1\right]e^{\frac{1}{x-1}}\ (-) =$$

$$=\lim_{x\to 1^+}\frac{e^{\frac{(x-1)-\log x}{(x-1)\log x}}-1}{e^{\frac{1}{1-x}}}=$$

$$=\lim_{t\to 0^+}\frac{e^{\frac{t-\log(t+1)}{t\cdot\log(t+1)}}-1}{e^{-\frac{1}{t}}}=$$

$$\log(1+t)=t-\frac{t^2}{2}+o(t^2)$$

$$\frac{1}{x-1}=-\frac{1}{1-x}$$

$$e^{\frac{1}{x-1}}=e^{-\frac{1}{1-x}}=\frac{1}{e^{\frac{1}{1-x}}}$$

$$x-1=t$$

$$\downarrow 1^+ \quad \downarrow 0^+$$

$$x=t+1$$

$$= \lim_{t\to 0^+} \frac{e^{\frac{t - t + t^2/2 + o(t^2)}{t\,(t + o(t))}} - 1}{e^{-\frac{1}{t}}} =$$

$\leftarrow 2^\circ$ ordine

$\nwarrow 1^\circ$ ordine

$$= \lim_{t\to 0^+} \frac{e^{\frac{t^2/2 + o(t^2)}{t^2 + o(t^2)}} - 1}{e^{-1/t}} \sim$$

$\to \frac{1}{2}$

$$\sim \underbrace{\left(e^{1/2} - 1\right)}_{>0} \lim_{t\to 0^+} \frac{1}{e^{-1/t}} = +\infty$$

$e^{-1/t} \to e^{-\infty} \to 0 > 0$

es: $\lim_{x\to 0^+} \dfrac{\sqrt{(\operatorname{arctg} x)^{x+\operatorname{sen}^2 x}} \cdot \log \frac{\operatorname{sen} x}{x}}{\log \sqrt{\cosh x}} =$

$$\lim_{x\to 0^+} (\operatorname{arctg} x)^{x+\operatorname{sen}^2 x} = \lim_{x\to 0^+} e^{\log (\operatorname{arctg} x)^{x+\operatorname{sen}^2 x}} =$$

$$= \lim_{x\to 0^+} e^{(x+\operatorname{sen}^2 x)\cdot \log(\operatorname{arctg} x)} = e^0 = 1$$

$$\operatorname{sen} x = x + o(x)$$

$$\operatorname{sen}^2 x = \underset{[x+o(x)]^2}{x^2 + 2x\, o(x) + (o(x))^2} = x^2 + o(x^2) + o(x^2) = x^2 + o(x^2)$$

$$\log(\operatorname{arctg} x) = \log(1 - 1 + \operatorname{arctg} x) = \log(1 + (\operatorname{arctg} x - 1))$$

$\log(1+t) = t + o(t)$ $\qquad = (\operatorname{arctg} x - 1) + o(x) =$

$$= \lim_{x \to 0^+} \frac{\log \frac{\operatorname{sen} x}{x}}{\log \sqrt{\cosh x}} = \lim_{x \to 0^+} \frac{\log \frac{\operatorname{sen} x}{x}}{\log (\cosh x)^{1/2}} =$$

$$= 2 \lim_{x \to 0^+} \frac{\log \frac{\operatorname{sen} x}{x}}{\log (\cosh x)} = 2 \lim_{x \to 0^+} \frac{-\frac{1}{6} x^2 + o(x^2)}{\frac{x^2}{2} + o(x^2)} =$$

$$= 2 \cdot \left(-\frac{1}{6}\right) \cdot 2 = -\frac{4}{6} = -2/3$$

$$\log\left(\frac{\operatorname{sen} x}{x}\right) = \log\left(\frac{x - \frac{x^3}{6} + o(x^3)}{x}\right) = \log\Big(\underbrace{1 - \frac{x^2}{6} + o(x^2)}_{1 + t}\Big)$$

$$\log(\cosh x) = \log\Big(\underbrace{1 + \frac{x^2}{2} + o(x^2)}_{1 + t}\Big)$$

$$\log(\operatorname{arctg} x) = \log(x + o(x))$$

$$x + \operatorname{sen}^2 x = x + x^2 + o(x^2)$$

$$\lim_{x \to 0} e^{(x + \operatorname{sen}^2 x) \cdot \log(\overbrace{x + o(x)}^{\to -\infty})}$$

$$= \lim_{x \to 0} e^{(x + \overbrace{x^2 + o(x^2)}^{\operatorname{sen}^2 x} \to 0) \cdot \log(\overbrace{x + o(x)}^{\to -\infty})} \overset{(x)}{=} e^0 = 1$$

$$\log(x + o(x)) = \log(1 - 1 + x + o(x)) =$$

$$= \log(1 + \underbrace{(x + o(x) - 1)}) = \underbrace{x + o(x) - 1}_{*}$$

es $\lim_{x\to 0} \dfrac{e^{x\sqrt{x}} - \operatorname{sen}^3\sqrt{x} - \cos x^{5/4}}{x^2\sqrt{x}} =$

grado del denominatore $2+\frac{1}{2} = 5/2 = 2,5$

$e^{x\sqrt{x}}$: $\quad e^t = 1 + t + \dfrac{t^2}{2!} + o(t^2)$ $\qquad x\sqrt{x} = x \cdot x^{\frac{1}{2}} = x^{3/2}$

$e^{x\sqrt{x}} = 1 + x\sqrt{x} + \dfrac{(x\sqrt{x})^2}{2} + o\left((x\sqrt{x})^2\right) =$ $\qquad t = x\sqrt{x}$

$= 1 + x\sqrt{x} + \dfrac{x^3}{2} + o(x^3)$ $\qquad (x\sqrt{x})^2 = x^2 \cdot x = x^3$

$\operatorname{sen}^3\sqrt{x}$: $\quad \operatorname{sen} t = t - \dfrac{t^3}{3!} + o(t^4)$

$\operatorname{sen}^3\sqrt{x} = \left(t - \dfrac{t^3}{3!} + o(t^4)\right)^3 = \left(\sqrt{x} - \dfrac{(\sqrt{x})^3}{6} + o(\sqrt{x})^4\right)^3 =$

$= \left(\sqrt{x} - \dfrac{x\sqrt{x}}{6} + o(x^2)\right)^3 = \Big[\underbrace{\sqrt{x}}_{a} - \underbrace{\left(\dfrac{x\sqrt{x}}{6} + o(x^2)\right)}_{b}\Big]^3 =$ $\quad (a-b)^3$

$= x\sqrt{x} - 3(\sqrt{x})^2\left(\dfrac{x\sqrt{x}}{6} + o(x^2)\right) + 3\sqrt{x}\left(\dfrac{x\sqrt{x}}{6} + o(x^2)\right)^2 - \left(\dfrac{x\sqrt{x}}{6} + o(x^2)\right)^3 =$

$$= x\sqrt{x} - \frac{3x^2\sqrt{x}}{6} + o(x^3) = x\sqrt{x} - \frac{1}{2}x^2\sqrt{x} + o(x^3)$$

$\cos x^{5/4}$:

$$\cos t = 1 - \frac{t^2}{2!} + o(t^3)$$

$$\cos x^{5/4} = 1 - \frac{1}{2}\left(x^{5/4}\right)^2 + o\left(\left(x^{5/4}\right)^3\right) =$$

$$= 1 - \frac{1}{2}x^{5/2} + o\left(x^{15/4}\right) =$$

$$= 1 - \frac{1}{2}x^2\sqrt{x} + o\left(x^{15/4}\right) \qquad \frac{15}{4} > 3$$

$$= \lim_{x \to 0} \frac{\overbrace{1 + x\sqrt{x} + \frac{x^3}{2} + o(x^3)}^{e^t} - \overbrace{\left(x\sqrt{x} - \frac{1}{2}x^2\sqrt{x} + o(x^3)\right)}^{\operatorname{sen}^3 t} - \overbrace{\left(1 - \frac{1}{2}x^2\sqrt{x} + o\left(x^{15/4}\right)\right)}^{\cos t}}{x^2\sqrt{x}} =$$

$$= \lim_{x \to 0} \frac{\cancel{1} + \cancel{x\sqrt{x}} + \frac{x^3}{2} + \boxed{o(x^3)} - \cancel{x\sqrt{x}} + \frac{1}{2}x^2\sqrt{x} - \boxed{o(x^3)} - \cancel{1} + \frac{1}{2}x^2\sqrt{x} - \cancel{o\left(x^{15/4}\right)}}{x^2\sqrt{x}} =$$

$$= \lim_{x \to 0} \frac{\frac{x^3}{2} + \overbrace{x^2\sqrt{x}}^{} + o(x^3)}{\underbrace{x^2\sqrt{x}}_{}} = 1$$

$$= \lim_{x \to 0} \left[ \underbrace{\frac{x^2\sqrt{x}}{x^2\sqrt{x}}}_{\to 1} + \frac{1}{2} \underbrace{\frac{x^3}{x^2\sqrt{x}}}_{\frac{1}{2}\sqrt{x} \to 0} + \underbrace{o\left(\frac{x^3}{x^2\sqrt{x}}\right)}_{o(x^{1/2}) \to 0} \right] = 1$$

$$= \ell$$

es

$$\lim_{x\to 0} \frac{e^{\sin^3 x}-1}{\log(1-x^3)} = \lim_{x\to 0} \frac{\cancel{1}+x^3+o(x^3)\cancel{-1}}{-x^3+o(x^3)} = -1$$

$\sin x = x + o(x)$

$e^t = 1+t+o(t)$

$\log(1+t) = t + o(t)$

$t = -x^3$

$\log(1-x^3) = -x^3 + o(x^3)$

$$\Rightarrow e^{\sin^3 x} = 1+\sin^3 x + o(\sin^3 x) =$$

$$= 1 + (x+o(x))^3 + o\left[(x+o(x))^3\right]$$

$$e^{\sin^3 x} = e^{\overbrace{(x+o(x))^3}^{t}} =$$

$$= 1 + (x+o(x))^3 + \underbrace{o\left[(x+o(x))^3\right]}_{o(x^3)}$$

$$(x+o(x))^3 = x^3 + (o(x))^3 + 3x^2 o(x) + 3x(o(x))^2 =$$

$$= x^3 + \underbrace{o(x^3) + 3\,o(x^3) + 3\,o(x^3)}_{o(x^3)}$$

es $\lim_{x \to 0} \dfrac{1-\cos 2x}{\log(1-x)+\log(1+x)} =$

$$\cos 2x = 1 - \frac{4x^2}{2} + o(x^2) = 1 - 2x^2 + \overset{o(x^3)}{o(x^2)}$$

$$\log(1+x) + \log(1-x) = \log\big((1+x)(1-x)\big) = \log(1-x^2)$$

$$\log(1-x^2) = -x^2 + \underbrace{\frac{x^4}{2} + o(x^4)}_{\text{Trascurare guardando al numeratore}}$$

$t = -x^2$

$$= \lim_{x \to 0} \frac{1-(1-2x^2+o(x^2))}{-x^2+o(x^2)} = \lim_{x \to 0} \frac{\cancel{1}-\cancel{1}+2x^2+o(x^2)}{-x^2+o(x^2)} = -2$$

es $\lim\limits_{x\to 0^+} \dfrac{4\left(\sqrt{\cos x^2}-\sqrt{\cos x}\right)}{3\left(\cos x^3-\cos x\right)} =$

$$= \lim_{x\to 0^+} \frac{4\left(\sqrt{\cos x^2}-\sqrt{\cos x}\right)\left(\sqrt{\cos x^2}+\sqrt{\cos x}\right)}{3\left(\cos x^3-\cos x\right)\left(\sqrt{\cos x^2}+\sqrt{\cos x}\right)} =$$

$$= \lim_{x\to 0^+} \frac{4\left(\cos x^2-\cos x\right)}{3\left(\underbrace{\sqrt{\cos x^2}+\sqrt{\cos x}}_{1\,+\,1}\right)\left(\cos x^3-\cos x\right)} = \frac{2}{3}\underbrace{\lim_{x\to 0^+} \frac{\cos x^2-\cos x}{\cos x^3-\cos x}}_{1} = 2/3$$

es $\displaystyle \lim_{x\to 0} \frac{2(\cos x-1)\cdot\cos x + x^2}{x^4 \underbrace{\cos x}}$ $\displaystyle = \left(\lim_{x\to 0}\frac{1}{\cos x}\right)\cdot \lim_{x\to 0}\frac{2(\cos x-1)\cdot \cos x + x^2}{x^4} =$

$= 7/12$

ordine → prendo denominatore è $(x^4)$

$$= \lim_{x\to 0}\underset{\downarrow \atop 1}{\frac{1}{\cos x}} \cdot \lim_{x\to 0}\frac{1}{x^4}\left(2\underbrace{\left(-\frac{x^2}{2}+\frac{x^4}{4!}+o(x^5)\right)}_{\cos x - 1}\cdot\underbrace{\left(1-\frac{x^2}{2}+o(x^3)\right)}_{\cos x}+x^2\right) =$$

$$= \lim_{x\to 0}\frac{1}{x^4}\left[\cancel{-x^2}+\frac{x^4}{12}+\frac{x^4}{2}\cancel{+x^2}+o(x^5)\right] =$$

$$= \frac{1}{12}+\frac{1}{2} = 7/12$$

N.B.: $2\left(-\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)\right)\cdot\left(1-\frac{x^2}{2}+o(x^3)\right) =$

$$= 2\left(-\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)+\frac{x^4}{4}-\frac{x^6}{48}-o(x^7)+o(x^5)+o(x^7)+o(x^8)\right) =$$

$$= 2\left(-\frac{x^2}{2}+\frac{x^4}{24}+\frac{x^4}{4}+o(x^5)\right.$$

es $$\lim_{x\to 0^+} \frac{\log\frac{1+x}{e} + e^{-(\sin x)}}{\sinh x \cdot \log(1+x^2)} = 1/3$$

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^x - \frac{1}{e^x}}{2} = \frac{e^{2x}-1}{2e^x}$$

$$= \lim_{x\to 0^+} \frac{\log(1+x) - \log e^{=1} + e^{-\sin x}}{\frac{e^{2x}-1}{x\,2e^x}\cdot x \cdot \frac{\log(1+x^2)}{x^2}\cdot x^2} \approx$$

$$\lim_{t\to 0} \frac{e^t-1}{t} = 1 \qquad \lim_{x\to 0} e^x = e^0 = 1 \qquad \lim_{t\to 0} \frac{\log(1+t)}{t} = 1$$

$$\approx \lim_{x\to 0^+} \frac{\log(1+x) - 1 + e^{-\sin x}}{x^3} =$$

$$= \lim_{x\to 0^+} \frac{\overbrace{x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)}^{\log} - 1 + \overbrace{1 - \sin x + \frac{\sin^2 x}{2} - \frac{\sin^3 x}{6} + o(x^3)}^{e^t}}{x^3}$$

$$= \lim_{x\to 0^+} \frac{\cancel{x} - \cancel{\frac{x^2}{2}} + \frac{x^3}{3} - \cancel{1} + \cancel{1} - \cancel{x} + \cancel{\frac{x^2}{2}} + o(x^3) -}{x^3}$$

$$\sin x = \left(x - \frac{x^3}{6}\right)$$

$$1 - \operatorname{sen} x + \frac{\operatorname{sen}^2 x}{2} - \frac{\operatorname{sen}^3 x}{6} + o(x^3) =$$

$$\operatorname{sen} x = \left( x - \frac{x^3}{6} + o(x^3) \right)$$

$$1 - \left( x - \frac{x^3}{6} + o(x^3) \right) + \frac{\left( x - \frac{x^3}{6} + o(x^3) \right)^2}{2} - \frac{\left( x - \frac{x^3}{6} + o(x^3) \right)^3}{6} + o(x^3) =$$

ciò che è superiore a $o(x^3)$

lo trascuro

$$= 1 - x + \cancel{\frac{x^3}{6}} + o(x^3) + \frac{x^2}{2} - \cancel{\frac{x^3}{6}} = \text{- - - - - - - -} =$$

es $\lim_{x \to 0^+} \dfrac{x e^x \operatorname{sen} x - \operatorname{sen}(x^2) - \operatorname{tg} x^3}{\operatorname{sen}^2 x \cdot \log(1+2x^2)} = \dfrac{1}{6}$ (4° grado) $o(x^4)$

$$N: \; x\underbrace{\left(1+x+\frac{x^2}{2}+o(x^2)\right)}_{e^x}\underbrace{\left(x-\frac{x^3}{6}+o(x^3)\right)}_{\operatorname{sen} x} - \underbrace{x^2+o(x^5)}_{\operatorname{sen}(x^2)} \underbrace{- x^3+o(x^3)}_{\operatorname{tg} x^3} =$$

$$= x\left(x-\frac{x^3}{6}+x^2+\frac{x^3}{2}+o(x^3)\right) - x^2 - x^3 + o(x^3) =$$

$$= x^2 + \frac{1}{4}x^4 + x^3 - x^2 - x^3 + o(x^4) = \frac{1}{3}x^4 + o(x^4)$$

$$D: \; \underbrace{\left(x^2+o(x^2)\right)}_{\operatorname{sen}^2 x} \cdot \underbrace{\left(2x^2+o(x^2)\right)}_{\log(1+2x^2)} = 2x^4 + o(x^4)$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{3}x^4 + o(x^4)}{2x^4 + o(x^4)} = 1/6$$

es

$$\lim_{x\to 0^+} \frac{tg(\operatorname{sen}(\operatorname{sen} x)) - \operatorname{sen}(\operatorname{sen} x)}{Tg(\operatorname{sen}(tg x)) - \sin(\sin(Tg x))} = \lim_{x\to 0^+} \frac{\frac{1}{3} x^3 + o(x^3)}{\frac{1}{2} x^3 + o(x^3)} = 2/3$$

$$\sin x = x + o(x) \qquad \left( x - \frac{x^3}{3!} + o(x^3) \right)$$

$$Tg\, x = x + o(x) \qquad \left( x + \frac{x^3}{3} + o(x^3) \right)$$

$$N: \operatorname{sen}(\operatorname{sen} x) + \frac{[\operatorname{sen}(\operatorname{sen} x)]^3}{3} + o\left((\operatorname{sen}(\operatorname{sen} x))^3\right) - \operatorname{sen}(\operatorname{sen} x) =$$

$$= \frac{[\operatorname{sen}(\operatorname{sen} x)]^3}{3} + o\left((\operatorname{sen}(\operatorname{sen} x))^3\right) = \frac{1}{3} \operatorname{sen}^3(x+o(x)) + o\left(\operatorname{sen}^3(x+o(x))\right) =$$

$$= \frac{1}{3} [x+o(x)]^3 + o\left((x+o(x))^3\right) = \frac{1}{3} x^3 + o(x^3)$$

$$D = \underbrace{\cancel{\operatorname{sen}(\operatorname{tg}(x))} + \frac{\operatorname{sen}^3(\operatorname{tg}(x))}{\underset{x}{3}} + o(x^3)}_{\text{sviluppo } \operatorname{Tg}(t)} - \underbrace{\cancel{\operatorname{sen}(\operatorname{tg}(x))} + \frac{\operatorname{sen}^3(\operatorname{tg}(x))}{\underset{3!\,x}{6}} + o(x^3)}_{\text{sviluppo } (-\operatorname{sen}(t))} =$$

$$\operatorname{tg} x = x + o(x)$$

$$= \frac{1}{2} \operatorname{sen}^3(\operatorname{tg}(x)) + o(x^3) =$$

$$= \frac{1}{2} \operatorname{sen}^3(x + o(x)) + o(x^3) = \frac{1}{2} (x + o(x))^3 + o(x^3) =$$

$$= \frac{1}{2} x^3 + o(x^3)$$

es: $\lim_{x\to 0^+}\left(\frac{3\,\mathrm{Tg}(2x)}{2\,\mathrm{sen}(3x)}\right)^{\frac{1}{2x^2}} = e^{17/12}$ $\quad\left(e^{\log}\right)$

esaminando l'esponente:

$$= \lim_{x\to 0^+}\left(\frac{1}{2x^2}\log\frac{3\,\mathrm{Tg}(2x)}{2\,\mathrm{sen}(3x)}\right) = \lim_{x\to 0^+}\left(\frac{1}{2x^2}\log\frac{3\,\mathrm{Tg}\,2x}{\phantom{x}} - \log\frac{2\,\mathrm{sen}\,3x}{6x}\right) =$$

$$\frac{3\,\mathrm{Tg}(2x)}{2\,\mathrm{sen}(3x)} = \frac{3\,\mathrm{Tg}\,2x}{6x}\cdot\frac{6x}{2\,\mathrm{sen}(3x)} = \left(\frac{3\,\mathrm{Tg}\,2x}{6x}\right) : \left(\frac{2\,\mathrm{sen}\,3x}{6x}\right)$$

$$\log\frac{3\,\mathrm{Tg}\,2x}{6x} = \log\frac{3\left(2x + \frac{8}{3}x^3 + o(x^3)\right)}{6x}$$

$$\log\frac{2\,\mathrm{sen}\,3x}{6x} = \log\frac{2\left(3x - \frac{9}{2}x^3 + o(x^3)\right)}{6x}$$

$$= \lim_{x\to 0^+}\left(\frac{1}{2x^2}\left(\frac{4}{3}x^2 + o(x^2) + \frac{3}{2}x^2 + o(x^2)\right)\right) = \lim_{x\to 0^+}\left(\frac{1}{2x^2}\left(\frac{4}{3}x^2 + \frac{3}{2}x^2 + o(x^2)\right)\right) =$$

$$= \frac{1}{2}\cdot\left(\frac{4}{3} + \frac{3}{2}\right) = 17/12$$

es

$$\lim_{x \to 0} \frac{x^3 + 2(-1 + \cos x)\,\mathrm{sen}\,x}{x^4 \cdot \mathrm{senh}\left(\frac{x}{2}\right)} =$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} + o(x^5)$$

$$\mathrm{sen}\,x = x - \frac{x^3}{3!} \overset{\text{stop}}{/\!/} \frac{x^5}{5!} + o(x^6)$$

$$\mathrm{senh}\,\frac{x}{2} = \frac{x}{2} + o(x^2)$$

$$= \lim_{x \to 0} \frac{x^3 + 2\left(1 - 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right)\left(x - \frac{x^3}{6} + o(x^4)\right)}{x^4 \cdot \left(\frac{x}{2} + o(x^2)\right)} =$$

$$= \lim_{x \to 0} \frac{x^3 + 2\left(-\frac{x^3}{2} + \frac{x^5}{12} + o(x^6) + \overbrace{\frac{x^5}{24} - \frac{x^7}{24 \cdot 6} + o(x^8) - - -}^{\text{Trascurabile}}\right)}{\frac{x^5}{2} + o(x^6)} =$$

$$= \lim_{x \to 0} \frac{\cancel{x^3} - \cancel{x^3} + \frac{x^5}{6} + \frac{x^5}{12} + o(x^6)}{\frac{x^5}{2} + o(x^6)} = \frac{3}{12} \cdot 2 = \frac{1}{2}$$

es $\lim_{x \to 0} \dfrac{e^{\mathrm{sen}\, x} - \frac{\mathrm{sen}\, x}{x} - x}{\mathrm{sen}^2(3x)} =$

N: $e^{\mathrm{sen}\, x} = 1 + \mathrm{sen}\, x + \dfrac{\mathrm{sen}^2 x}{2} + o(\mathrm{sen}^2 x) =$

$$= 1 + \left(x - \frac{x^3}{6} + o(x^4)\right) + \frac{(x + o(x^2))^2}{2} + o\left((x + o(x^2))^2\right) =$$

$$= 1 + x + \frac{x^3}{6} + o(x^4) + \frac{x^2 + 2x\, o(x^2) + o(x^4)}{2} + o(x^2) =$$

• Trascurabile

$$= 1 + x + \frac{x^2}{2} + o(x^2)$$

D: $\mathrm{sen}^2(3x) = (3x + o(x^2))^2 = 9x^2 + 6x\, o(x^2) + o(x^4) = 9x^2 + o(x^3) \approx 9x^2 + o(x^2)$

$$\frac{\mathrm{sen}\, x}{x} = \frac{x - x^3/6 + o(x^4)}{x} = 1 - \frac{x^2}{6} + o(x^3)$$

$$= \lim_{x \to 0} \frac{2/3\, x^2 + o(x^2)}{9x^2 + o(x^2)} = \frac{2}{3} \cdot \frac{1}{9} = \frac{2}{27}$$

es $\lim_{x\to 0} \left( \frac{1}{\operatorname{sen} x} - \frac{x}{\log(1+x^2)} \right) \cdot \frac{\arcsin x}{1-\cos x} =$

$$= \lim_{x\to 0} \frac{\ln(1+x^2) - x \operatorname{sen} x}{\operatorname{sen} x \ \ln(1+x^2)} \cdot \frac{\arcsin x}{1-\cos x}$$

$$\ln(1+x^2) - x \operatorname{sen} x = x^2 - \frac{x^4}{2} + o(x^4) - x\left(x - \frac{x^3}{6} + o(x^4)\right) =$$

$$= x^2 - \frac{x^4}{2} + o(x^4) - x^2 + \frac{x^4}{6} + o(x^5) \qquad N_1$$

$$\operatorname{sen} x \cdot \ln(1+x^2) = \left(x - \frac{x^3}{6} + o(x^4)\right)\left(x^2 + o(x^2)\right) \qquad D_1$$

$$\arcsin x = x + \frac{x^3}{6} + o(x^4)$$

$$1 - \cos x = 1 - 1 + \frac{x^2}{2} - \frac{x^4}{24} + o(x^5)$$

$$N_1 = -\frac{1}{3}x^4 + o(x^4)$$

$$D_1 = x^3 - \frac{x^5}{6} + o(x^6) + o(x^3) - o(x^5) + o(x^6) = x^3 + o(x^3)$$

$$= \lim_{x \to 0} \frac{\left(-\frac{1}{3}x^4 + o(x^4)\right) \cdot \left(x + \frac{x^3}{6} + o(x^4)\right)}{\left(x^3 + o(x^3)\right)\left(\frac{x^2}{2} - \frac{x^4}{24} + o(x^5)\right)} =$$

$$= \lim_{x \to 0} \frac{-\frac{1}{3}x^5 + o(x^5) \overbrace{- \frac{x^7}{18} + o(x^7) + o(x^8) + o(x^8)}}{\frac{x^5}{2} + o(x^5) \underbrace{- \frac{x^7}{24} + o(x^7) + o(x^8) + o(x^8)}_{\text{Trascurabile}}} =$$

$$= \lim_{x \to 0} \frac{-\frac{1}{3}x^5 + o(x^5)}{\frac{1}{2}x^5 + o(x^5)} = -\frac{1}{3} \cdot 2 = -2/3$$