I NUMERI COMPLESSI

$z \in \mathbb{C}$:

forma algebrica $= a + ib$

forma trigonometrica $= \rho(\cos\vartheta + i \sin\vartheta)$

forma esponenziale $= \rho\, e^{i\vartheta}$

1) $\rho = \sqrt{a^2 + b^2}$

$$\vartheta = \begin{cases} \cos\vartheta = \dfrac{a}{\sqrt{a^2+b^2}} \\ \sin\vartheta = \dfrac{b}{\sqrt{a^2+b^2}} \end{cases} \qquad \vartheta \in [0, \pi]$$

$\vartheta = \operatorname{arctg} \dfrac{b}{a}$

2) $\begin{cases} a = \rho \cos\vartheta \\ b = \rho \sin\vartheta \end{cases}$

$z = a + ib \qquad \bar{z} = a - ib$

verificare le seguenti uguaglianze:

$z + \bar{z} = 2a$

$z - \bar{z} = 2bi$

$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$

$\overline{\left(\dfrac{1}{z}\right)} = \dfrac{1}{\bar{z}}$

$|z| = \sqrt{a^2 + b^2}$

$|z| \geq 0$

$|\mathrm{Re}\, z| \leq |z|$

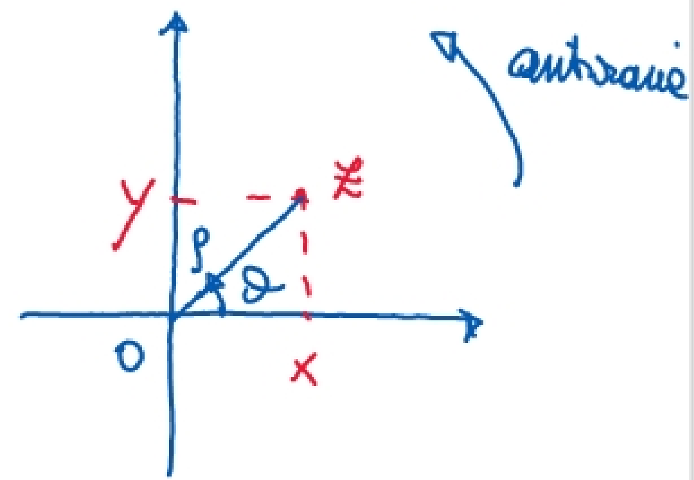
$|\mathrm{Im}\, z| \leq |z|$

$|z| \leq |\mathrm{Re}\, z| + |\mathrm{Im}\, z|$

$|z_1 + z_2| \leq |z_1| + |z_2|$

$|z_1 + z_2| \geq \big||z_1| + |z_2|\big|$

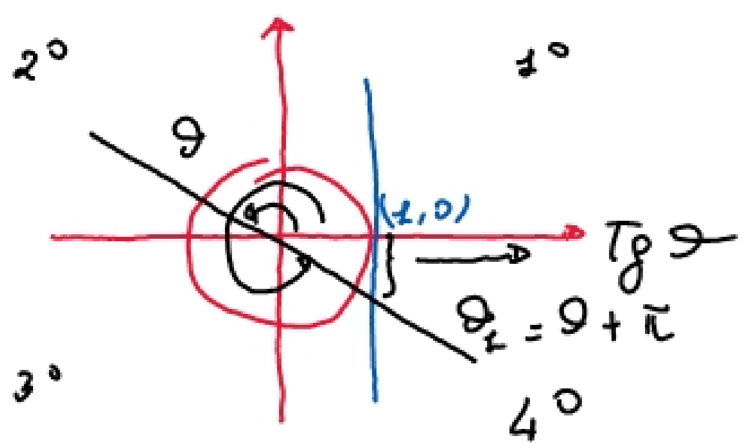
$$\rho = \sqrt{x^2+y^2} = |z| \qquad z = x + iy \qquad z(x, y)$$



$$\vartheta = \begin{cases} \cos\vartheta = \dfrac{x}{\rho} = \dfrac{x}{\sqrt{x^2+y^2}} \\ \sin\vartheta = \dfrac{y}{\rho} = \dfrac{y}{\sqrt{x^2+y^2}} \end{cases}$$

$$\frac{\sin\vartheta}{\cos\vartheta} = \frac{y}{\cancel{\sqrt{x^2+y^2}}} \cdot \frac{\cancel{\sqrt{x^2+y^2}}}{x} = y/x = \mathrm{Tg}\,\vartheta$$

$$\vartheta = \mathrm{arctg}\,\frac{y}{x}$$



1. Verificare se non essere equivalenti:

$z_1 = i - 1$ $\qquad\qquad$ $z_2 = 1 - i$

1. rappresentare z_i nel piano di Gauss

2. individuare le componenti ρ e ϑ corrispondenti

3. verificare : $tg\, \vartheta_1 = tg\, \vartheta_2$?

Schema conclusivo

$\operatorname{tg}\vartheta = \frac{y}{x} \;\cancel{\Rightarrow}\; \vartheta = \operatorname{arctg} y/x$ (per il passaggio in coordinate polari) (o viceversa)

1. se $x>0$ l'implicazione è vera [1°, 4° quad]

2. se $x<0$ si deve aggiungere π [2°, 3° quad]

$\forall z \neq 0$

$$\arg z = \begin{cases} \operatorname{arctg} y/x \;(+2k\pi) & \text{se } x>0 \\ \pi + \operatorname{arctg} y/x \;(+2k\pi) & \text{se } x<0 \\ \frac{\pi}{2} \;(+2k\pi) & \text{se } x=0 \quad y>0 \\ \frac{3}{2}\pi \;(+2k\pi) & \text{se } x=0 \quad y<0 \end{cases}$$

Potenza di ordine n di z : (formule di De Moivre)

$$z^n = \rho^n (\cos n\vartheta + i \sin n\vartheta) = \rho^n e^{in\vartheta} \qquad [(a+ib)^n]$$

Radice n-esima di z :

A) $z = \rho(\cos\vartheta + i \sin\vartheta) = \rho e^{i\vartheta}$

$$\sqrt[n]{z} = \sqrt[n]{\rho} \cdot \left(\cos \frac{\vartheta + 2k\pi}{n} + i \sin \frac{\vartheta + 2k\pi}{n}\right)$$

$k \in \mathbb{N}_0$ e $k = 0, 1, \dots, n-1$

B) $z = \rho e^{i\vartheta}$

$$\sqrt[n]{z} = \sqrt[n]{\rho} \cdot e^{i\frac{\vartheta + 2k\pi}{n}}$$ con $k \in \mathbb{N}_0$, $k = 0, 1, \dots, n-1$

L'unità immaginaria

$$\boxed{i = \sqrt{-1}}$$

$$i^0 = 1$$
$$i^1 = i$$
$$i^2 = -1$$
$$i^3 = -i$$

R

$$i^4 = 1$$
$$i^5 = i$$
$$i^6 = -1$$
$$i^7 = -i$$

Calcolare

$$i^{23} = i^{20} \cdot i^3 = i^3 = -i$$

20 è divisibile per 4 $\quad i^{20} = i^0 = 1$

$$i^{231} = i^{228}\, i^3 = i^0 \cdot i^3 = i^3 \cdot 1 = -i$$

$231 : 4 = 57$ resto 3

$$i^{738} = i^{736} \cdot i^2 = i^0 \cdot i^2 = 1 \cdot (-1) = -1$$

$738 : 4 = 184$ resto $= 2$

Data l'equazione $z^n = k$, rappresentarle nel piano nelle sue soluzioni

1) algebricamente: $z = \sqrt[n]{k} \Rightarrow z_i$ passaggio in coord. polari (ρ, θ)

2) rappresentazione grafica: poligono di n-lati

es: $z^4 = 1$ $\qquad z^4 - 1 = 0$

(si può scomporre in
$(z^2-1)\ (z^2+1) = 0$
$(z-1)(z+1)\ (z-i)(z+i) = 0$

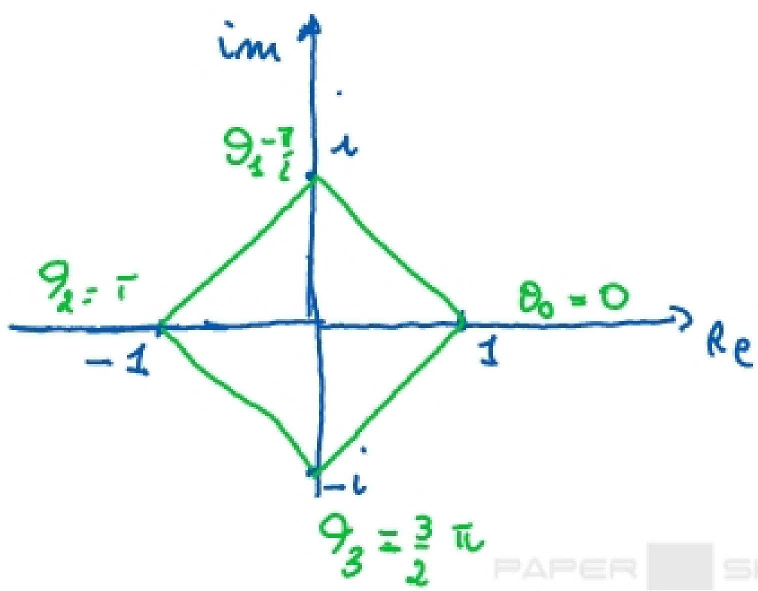
$z = 1$

$z = -1$

$z = i$

$z = -i$

Graficamente



Calcolare le soluzioni delle seguenti equazioni, visualizzando nel piano di Gauss:

$z^2 = 2$	$z^3 = 9$	$z^4 = +16$
$z^2 = -2$	$z^3 = -9$	$z^4 = -16$
$z^2 = +2i$	$z^3 = +9i$	$z^4 = +16i$
$z^2 = -2i$	$z^3 = -9i$	$z^4 = -16i$

In oltre:

$z^5 = 1$	$z^5 = i$	$z^5 = 1 + i$

(Tema d'esame 03.09.09)

• Radici complesse : $z^4 - i\,|i-1|^2 z = 0$

$$z\left[z^3 - i\,|i-1|^2\right] = 0 \quad \rightarrow \quad z = 0 \rightarrow (0,0)$$

origine

numero reale

0

$$z^3 = i\,|i-1|^2 = i \cdot \left(\sqrt{1+1}\right)^2 = i \cdot \left(\sqrt{2}\right)^2 =$$

$$z^3 = 2i$$

$$z = \sqrt[3]{2i} = \sqrt[3]{2} \cdot \sqrt[3]{i} \qquad i = \left(1 \;\; \tfrac{\pi}{2}\right)$$

$$i = e^{\frac{\pi}{2} i}$$

$$z_1 = \sqrt[3]{2}\; e^{\pi/2/3\, i} = \sqrt[3]{2}\, e^{\frac{\pi}{6} i}$$

$$z_2 = \sqrt[3]{2}\; e^{\left(\frac{\pi}{2} + 2\pi\right)\cdot\frac{1}{3} i} = \sqrt[3]{2}\; e^{\frac{5}{6}\pi i}$$

$$z_3 = \sqrt[3]{2}\; e^{\left(\frac{\pi}{2} + 4\pi\right)\frac{1}{3} i} = \sqrt[3]{2}\; e^{\frac{9}{6}\pi i} = \sqrt[3]{2}\; e^{\frac{3}{2}\pi i}$$

scritti in notazione esponenziale

* determinare il luogo del piano t.c. (Tema d'esame 06-09-10)

$$2(z+\bar{z}) - 3\,\mathrm{Im}(z) = z^2 - 3|z|^2$$

sostituzione $z = x+iy$ $\quad Re\, z = x$ $\quad x,y \in \mathbb{R}$

$Im\, z = y$

$$2(\underbrace{x+\cancel{iy}}_{z} + \underbrace{x-\cancel{iy}}_{\bar{z}}) - 3y = (x+iy)^2 - 3(\cancel{\sqrt{x^2+y^2}})^{\cancel{2}}$$

$$4x - 3y = x^2 - y^2 + 2xyi - 3x^2 - 3y^2$$

$$\underbrace{2x^2 + 4y^2 + 4x - 3y}_{Reale} \; \underbrace{-2xy}_{Im}\, i = 0$$

$Re\, T = 0$

$Im\, T = 0$

$$\begin{cases} 2x^2 + 4y^2 + 4x - 3y = 0 \\ -2xy = 0 \longrightarrow \text{se } x=0 \text{ o } y=0 \end{cases}$$

$$\begin{cases} x = 0 \\ 4y^2 - 3y = 0 \end{cases} \qquad \begin{cases} y = 0 \\ 2x^2 + 4x = 0 \end{cases}$$

$$\left[\, 2x^2 + 4y^2 + 4x - 3y = 0 \,\right]$$

$$\begin{cases} x = 0 \\ y(4y - 3) = 0 \end{cases} \qquad \begin{cases} y = 0 \\ 2x\,(x + 2) = 0 \end{cases}$$

$y = 0$ $\quad$ $y = 3/4$ $\qquad$ $x = 0$ $\quad$ $x = -2$

(x,y): $\quad (0,0) \quad (0, 3/4) \qquad (0,0) \quad (-2, 0)$

$z = x + iy$ $\qquad z_1 = 0 \qquad z_2 = \frac{3}{4} i \qquad z_3 = -2$

2 sol. reali

1 sol. immaginaria

(Tema d' esame 10-09-07)

- Calcolare il valore del seguente numero complesso:

$$w = \frac{2\,(\sqrt{3}-i)^4}{(7+7i)^{10}} = \frac{2\,(\sqrt{3}-i)^4}{7^{10}\,(1+i)^{10}} \qquad (*)$$

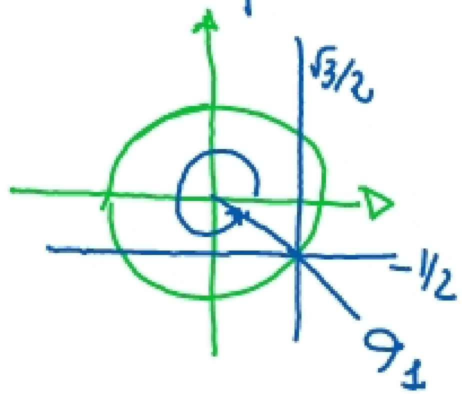
[esponenti

$$\frac{11}{\cancel{6}_3}\cdot\cancel{4}^2 - \frac{\cancel{10}^5}{\cancel{4}_2} = \frac{22}{3}-\frac{5}{2} = \frac{29}{6}$$
]

$$\sqrt{3}-i = 2\,e^{\frac{11}{6}\pi i} \qquad\qquad 1+i = \sqrt{2}\cdot e^{\frac{\pi}{4} i}$$

$$\rho_1 = \sqrt{(\sqrt{3})^2+(-1)^2} = \sqrt{4} = 2 \quad (\rho>0) \qquad\qquad \rho_2 = \sqrt{1+1} = \sqrt{2}$$

$$\vartheta_1 = \begin{cases} \cos\vartheta_1 = \frac{\sqrt{3}}{2} \\ \operatorname{sen}\vartheta_1 = -\frac{1}{2} \end{cases} \qquad\qquad \vartheta_2 = \begin{cases} \cos\vartheta_2 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \operatorname{sen}\vartheta_2 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{cases}$$



$$\vartheta_1 = \frac{11}{6}\pi \qquad\qquad \vartheta_2 = \frac{\pi}{4}$$

$$(*) = \frac{2\,\left(2\,e^{\frac{11}{6}\pi i}\right)^4}{7^{10}\,\left(\sqrt{2}\,e^{\frac{\pi}{4} i}\right)^{10}} = \frac{2\cdot\cancel{2^4}\;e^{\frac{11}{6}\cdot 4\pi i}}{7^{10}\,\underbrace{(\sqrt{2})^{10}}_{2^{10/2}=\cancel{2^5}}\;e^{\frac{\pi}{4}\cdot 10 i}} = \frac{1}{7^{10}}\cdot e^{\frac{29}{6}\pi i} = \frac{1}{7^{10}}\,e^{\frac{5}{6}\pi i} =$$

$$= \frac{1}{7^{10}}\left(\cos\frac{5}{6}\pi + i\operatorname{sen}\frac{5}{6}\pi\right)$$

Determinare i numeri complessi $(x+iy)$ soddisfacenti la condizione $(x+iy)^2 = 3+4i$

$$(x+iy)^2 = \overbrace{x^2-y^2}^{**} + \overbrace{2xy}^{*}\,i \qquad ** \ \mathrm{Re} = x^2-y^2 \qquad * \ \mathrm{Im} = 2xy$$

$$(iy)^2 = i^2 y^2 = -1\cdot y^2$$

$$(iy)^2 = i^2\cdot y^2 = -y^2$$

$x, y \in \mathbb{R}$

si uguagliano i termini: $x^2-y^2+2xyi = 3+4i$ (identità polinomiale)

$$\begin{array}{l} \mathrm{Re} = \\ \mathrm{Im} = \end{array}\begin{cases} x^2-y^2 = 3 \\ 2xy = 4 \end{cases} \qquad \begin{cases} x^2-y^2 = 3 \\ xy = 2 \end{cases} \qquad \begin{cases} \dfrac{4}{y^2} - y^2 = 3 \quad (*) \\ x = \dfrac{2}{y} \end{cases}$$

$y \neq 0$

$(*)$ $4 - y^4 - 3y^2 = 0$

$$y^4+3y^2-4 = 0$$
$$(y^2+4)(y^2-1) = 0$$
$y^2 = -4$ no sol in $\mathbb{R}$
$$y^2 = 1$$
$$y = \pm 1$$

$$\begin{cases} y_1 = 1 \\ x_1 = 2 \end{cases} \qquad \begin{cases} y_2 = -1 \\ x_2 = -2 \end{cases}$$

$$z_1 = 2+i \qquad z_2 = -2-i$$

Risolvere la seguente equazione a coefficienti complessi, in $\mathbb{C}$

$$\underbrace{3i}\, x^2 - \underbrace{(4+5i)}\, x + \underbrace{(3+i)} = 0 \qquad \text{eq. di 2° grado}$$

$$x_{1,2} = \frac{(4+5i) \pm \sqrt{(4+5i)^2 - 4 \cdot 3i\,(3+i)}}{6i} = \frac{(4+5i) \pm \sqrt{16+40i-25-36i+12}}{6i} =$$

$$= \frac{(4+5i) \pm \sqrt{3+4i}}{6i} \implies$$

$$\Delta = 3+4i = 4 - \overset{(+i)^2}{1} + 4i = \overset{2^2}{4} + i^2 + 4i = \underline{(2+i)^2}$$

$$x_1 = \frac{4+5i + \overbrace{2+i}^{\sqrt{\Delta}}}{6i} = \frac{6+6i}{6i} = \frac{1+i}{i} \cdot \frac{i}{i} = \frac{i + \overset{-1}{i^2}}{\underset{-1}{i^2}} = +1-i$$

$$x_2 = \frac{4+5i \overbrace{-2-i}^{-\sqrt{\Delta}}}{6i} = \frac{2+4i}{6i} \cdot \frac{i}{i} = \frac{1+2i}{3i} \cdot \frac{i}{i} = \frac{i + 2\overset{-1}{i^2}}{3\underset{-1}{i^2}} =$$

$$= -\frac{1}{3}i + \frac{2}{3}$$

Calcolare $\left[\dfrac{2(i-1)}{1+i}\right]^4$ (Tema d'esame 21.03.05)

ALGEBRICO

$$z = \frac{2(i-1)}{1+i} \cdot \frac{(1-i)}{(1-i)} = \frac{\cancel{2}\,\overbrace{(i-1)(1-i)}^{-(1-i)^2}}{\underbrace{1-\underbrace{i^2}_{-1}}_{\cancel{2}}} = i - \cancel{1} + \cancel{1} + i = 2i$$

Denominatore $(1+i)(1-i) = 1 - i^2 = 1+1 = 2$

$$z = 2i$$

$$z^4 = (2i)^4 = 2^4 \cdot \underbrace{i^4}_{1} = 2^4 = 16$$

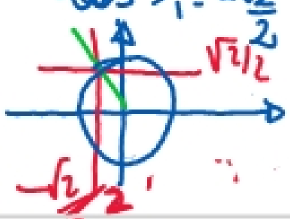
(rappresenta un numero reale)

N.B. provare a trasformarlo in notazione esponenziale.

$z_1 = i - 1 = \sqrt{2}\, e^{\frac{3}{4}\pi i}$

$\rho_1 = \sqrt{2}$

$\theta_1 = \begin{cases} \operatorname{sen} \vartheta_1 = +\frac{1}{\sqrt{2}} \\ \cos \vartheta_1 = -\frac{\sqrt{2}}{2} \end{cases}$

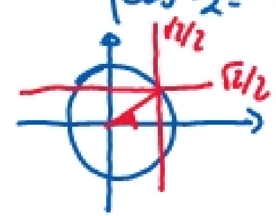


$\vartheta_1 = 3/4\,\pi$

$z_2 = 1 + i = \sqrt{2}\, e^{\frac{\pi}{4} i}$

$\rho_2 = \sqrt{2}$

$\vartheta_2 = \begin{cases} \operatorname{sen} \vartheta_2 = \sqrt{2}/2 \\ \cos \vartheta_2 = \sqrt{2}/2 \end{cases}$



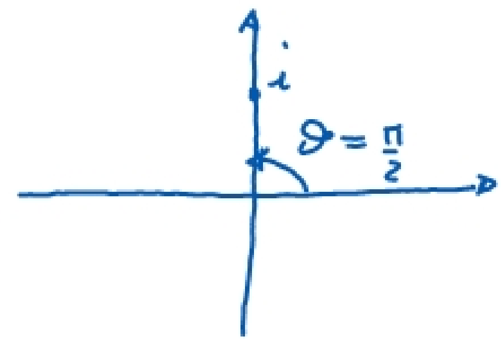
$$\left[\frac{2(i-1)}{1+i}\right]^4 = \left[2 \cdot \frac{e^{3/4\pi i} \cdot \cancel{\sqrt{2}}}{e^{\pi/4\, i}\, \cancel{\sqrt{2}}}\right]^4 = \left[2\, e^{\frac{2}{4}\pi}\right]^4 = \left[2\, e^{\frac{1}{2}\pi}\right]^4 = 2^4 \cdot (+i)^4 = 16$$

(Tema d'esame 9-12-04)

Calcolare le radici terze di $z = 10(1+i)^2$

$$z = 10(1+i)^2 = 10\,(1-1+2i) = 10\,(2i) = 20i =$$

$$= 20\, e^{\frac{\pi}{2} i}$$



$$\sqrt[3]{z} = \sqrt[3]{20\, e^{\frac{\pi}{2} i}} = \sqrt[3]{20}\ \sqrt[3]{e^{\frac{\pi}{2} i}}$$

$$w = e^{\frac{\pi}{2} i} \qquad \rho = 1 \qquad \vartheta = \frac{\pi}{2}$$

$n = 3$

$$k_0 = 0 \qquad w_0 = \left(\cos \frac{\frac{\pi}{2} + 2\cdot 0\pi}{3} + i \sin \frac{\pi/2}{3}\right) = \cos\frac{\pi}{6} + i \sin\frac{\pi}{6}$$

$$k_1 = 1 \qquad w_1 = \cos \frac{\frac{\pi}{2} + 2\cdot 1\cdot\pi}{3} + i \sin \frac{5}{6}\pi = \cos \frac{5}{6}\pi + i \sin 5/6\,\pi$$

$$k_2 = 2 \qquad w_2 = \cos \frac{\frac{\pi}{2} + 2\cdot 2\cdot\pi}{3} + i \sin \frac{3}{2}\pi = \cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi$$

$$z_0 = \sqrt[3]{20}\,\underbrace{\left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right)}_{w_0} \qquad z_1 = \sqrt[3]{20}\,\underbrace{\left(-\frac{\sqrt{3}}{2} + \frac{1}{2} i\right)}_{w_2} \qquad z_3 = \sqrt[3]{20}\,(-i)$$

(Tema d'esame 04/07/08)

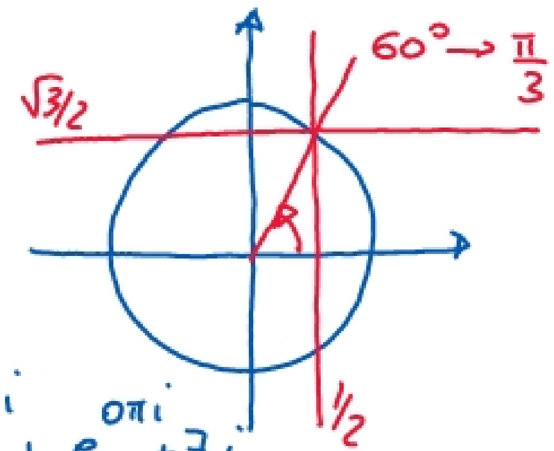
Dato il numero complesso $z=\frac{1}{2}+i\,\frac{\sqrt{3}}{2}$, calcolare il valore della seguente espressione:

$$\operatorname{Re}\left(z^{40}+z^{36}+7i\right)\cdot\left[\operatorname{Im}\left(z^{39}-z^{36}\right)+3\right] = (*)$$

$$z=\frac{1}{2}+\frac{\sqrt{3}}{2}i \qquad a=\frac{1}{2} \qquad b=\frac{\sqrt{3}}{2}$$

$$\rho=\sqrt{\frac{1}{4}+\frac{3}{4}}=1$$

$$\vartheta=\begin{cases}\sin\vartheta=\frac{\sqrt{3}}{2}\\ \cos\vartheta=\frac{1}{2}\end{cases}$$



$$z=1\,e^{\frac{\pi}{3}i}=e^{\frac{\pi}{3}i}$$

$$z^{40}+z^{36}+7i = e^{\frac{40}{3}\pi i}+e^{\frac{36}{3}\pi i}+7i = e^{\frac{4}{3}\pi i}+e^{0\pi i}+7i$$

$$z^{39}-z^{36} = e^{\frac{39}{3}\pi i}-e^{\frac{36}{3}\pi i} = e^{\pi i}-e^{0i}$$

$$(*) = \operatorname{Re}\Big(\underbrace{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}_{e^{\frac{4}{3}\pi i}}\ \underbrace{+1}_{e^{0\pi i}}+7i\Big)\cdot\Big[\operatorname{Im}\big(\underbrace{-1}_{e^{\pi i}}\ \underbrace{-1}_{e^{0i}}\big)+3\Big] = \frac{1}{2}(0+3) = \frac{3}{2}$$

Calcolare le radici in $\mathbb{C}$ della seguente equazione:

$$z^2 - z - 5 - 5i = 0$$

$$z^2 - z - (5+5i) = 0$$

$$z_{1,2} = \frac{1 \pm \sqrt{1 + 4(5+5i)}}{2} = \frac{1 \pm \sqrt{\overset{21}{1+20}\overset{+20i}{+20i}}}{2} =$$

$$= \frac{1 \pm \sqrt{25 - 4 + 20i}}{2} = \frac{1 \pm \sqrt{(5+2i)^2}}{2} =$$

$$z_1 = \frac{1 + (5+2i)}{2} = \frac{6+2i}{2} = 3 + i$$

$$z_2 = \frac{1 - (5+2i)}{2} = \frac{-4-2i}{2} = -2 - i$$

Esaminare se esistono valori reali della x, per i quali il numero $\dfrac{x+2+ix}{x+i}$ sia immaginario puro.

$x \in \mathbb{R}$

$$\frac{x+2+ix}{x+i} \cdot \frac{(x-i)}{(x-i)} = \frac{x^2+2x+x^2 i - xi - 2i + x}{x^2+1} =$$

[somma · differenze → elimino la i ($\neq 0$) al denominatore

$$(z) = \underbrace{\frac{x^2+3x}{x^2+1}}_{\text{parte reale}} + \underbrace{\frac{x^2-x-2}{x^2+1}}_{\text{parte immaginaria}} \cdot i$$

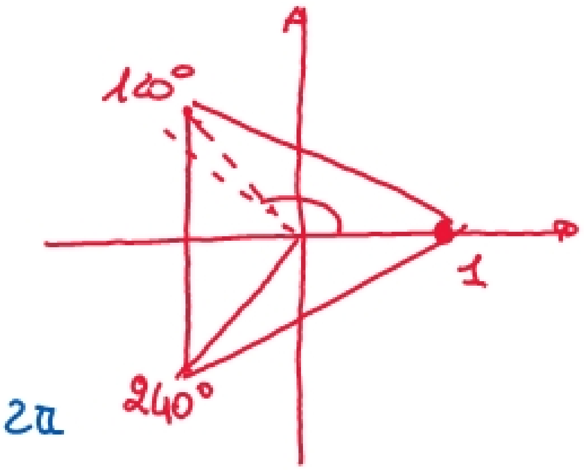
immaginario puro : $Re(z) = 0$ $\qquad \dfrac{x^2+3x}{x^2+1} = 0$

$x^2+3x=0 \qquad x_1 = 0$

$x_2 = -3$

[N.B. $x^2+1 \neq 0 \ \forall x \in \mathbb{R} \Rightarrow$ non devo porre condizioni sul denominatore, prima di procedere a semplificazione]

Calcolare $\sqrt[3]{1}$



$$1 = 1\,(\cos 0 + i \operatorname{sen} \vartheta)$$

$a = 1$ $\qquad \rho = 1$

$b = 0$

$$\vartheta \begin{cases} \cos\vartheta = 1 \\ \operatorname{sen}\vartheta = 0 \end{cases} \Rightarrow \vartheta = 0 \text{ oppure } 2\pi$$

$$z = 1 \cdot \underbrace{e^{0\pi i}}_{1} \qquad \vartheta = 0$$

$n = 3$

$\sqrt[3]{1}:$

$$z_0 = 1 \qquad k_0 = 0 \quad \vartheta_0 = \frac{0 + 2\cdot 0\pi}{3} = 0$$

$$z_1 = 1 \cdot \left(\cos \frac{2}{3}\pi + i \operatorname{sen} \frac{2}{3}\pi\right) = -\frac{1}{2} + i\,\frac{\sqrt{3}}{2} \qquad k_1 = 1 \quad \vartheta_1 = \frac{0 + 2\pi}{3} = \frac{2}{3}\pi$$

$$z_2 = 1 \cdot \left(\cos \frac{4}{3}\pi + i \sin \frac{4}{3}\pi\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2} i \qquad k_2 = 2 \quad \vartheta_2 = \frac{0 + 4\pi}{3} = \frac{4}{3}\pi$$

$k = 0, 1, 2 \quad (3-1)$

Trovare modulo e argomento del seguente numero complesso:

$z = -1 + \sqrt{3}\, i$

$a = -1$

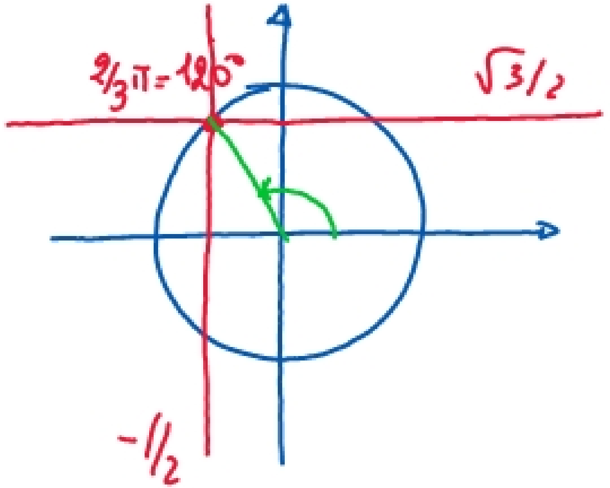
$b = \sqrt{3}$

$\rho = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2 \qquad (\rho \geq 0)$

$\vartheta: \quad \cos\vartheta = \frac{a}{\rho} = -\frac{1}{2}$

$\operatorname{sen}\vartheta = \frac{b}{\rho} = \sqrt{3}/2$

$\Rightarrow \vartheta = \frac{2}{3}\pi$



$z = -1 + \sqrt{3}\, i = 2 \cdot e^{\frac{2}{3}\pi i}$

$\rightarrow$ continua: $\sqrt[4]{z} = ?$

$\sqrt[5]{z} = ?$

Calcolare la seguente potenza:

$$\left[2\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)\right]^4=\left[2\cdot e^{\frac{\pi}{12}i}\right]^4_{(*)} \qquad \rho=2 \quad \vartheta=\pi/12$$

$$=2^4\left(\cos 4\cdot\frac{\pi}{12}+i\sin 4\frac{\pi}{12}\right)=$$

(la potenza di un numero complesso in notazione trigonometrica:
- si calcola la potenza di ρ
- l'angolo si moltiplica per l'esponente della potenza in oggetto)

$$=16\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$$

$$=16\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=$$

$$=\frac{16}{2}\left(1+\sqrt{3}\,i\right)=8\left(1+\sqrt{3}\,i\right)$$

$$\left[\begin{matrix}\rho=16\\ \vartheta=\frac{\pi}{3}\end{matrix}\quad\Rightarrow\quad\begin{matrix}a=\rho\cos\vartheta=16\cdot\frac{1}{2}=8\\ b=\rho\sin\vartheta=16\cdot\frac{\sqrt{3}}{2}=8\sqrt{3}\end{matrix}\right]$$

$$(*)\quad\left(\frac{\pi}{12}i\right)\cdot 4=\frac{\pi}{3}i \longrightarrow \vartheta_1=\pi/3$$