CONTINUITA' - DISCONTINUITA'

es 1

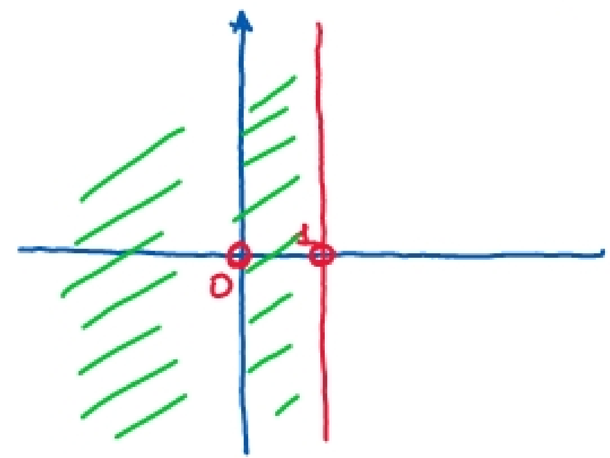
$$f(x)=\sqrt{x^2(x-1)}$$

- dominio
- continuità

dominio $\quad x^2(x-1)\geq 0 \qquad x-1\geq 0$

$x=0 \qquad (x^2>0 \quad \forall x\neq 0)$

$x=0$ pt. isolato, ivi è continua



$x\geq 1$ continua, perché composizione di funzioni continue es 2 $\quad f(x) = x^2 + \sqrt{-\left|\sin\frac{\pi}{x}\right|}$

- dominio
- continuità.

dominio $\quad \underbrace{-\left|\sin\frac{\pi}{x}\right|}_{\geq 0} \geq 0$

$\sin\frac{\pi}{x} = 0$

$\frac{\pi}{x} = k\pi \quad \forall k \in \mathbb{Z}$

$x = \frac{\pi}{k\pi} = \frac{1}{k} \quad \forall k \in \mathbb{Z} \setminus \{0\}$

$f\left(\frac{1}{k}\right) = \frac{1}{k^2} \quad \forall k \in \mathbb{Z} \setminus \{0\}$ pt. isolati

$\Downarrow$

la funzione è continua

es 3 $\qquad f(x) = \dfrac{x^2+x-6}{x^2-x-2}$

- dominio
- continuità

dominio $\qquad x^2-x-2 \neq 0 \qquad (x-2)(x+1) \neq 0 \qquad x \neq -1$

$x \neq 2$

$\forall x \in \mathbb{R} \setminus \{-1, 2\}$

$x = 2 \qquad \lim_{x \to 2^\pm} f(x) = \lim_{x \to 2^\pm} \dfrac{x^2+x-6}{x^2-x-2} =$

$= \lim_{x \to 2^\pm} \dfrac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}(x+1)} = \dfrac{5}{3}$

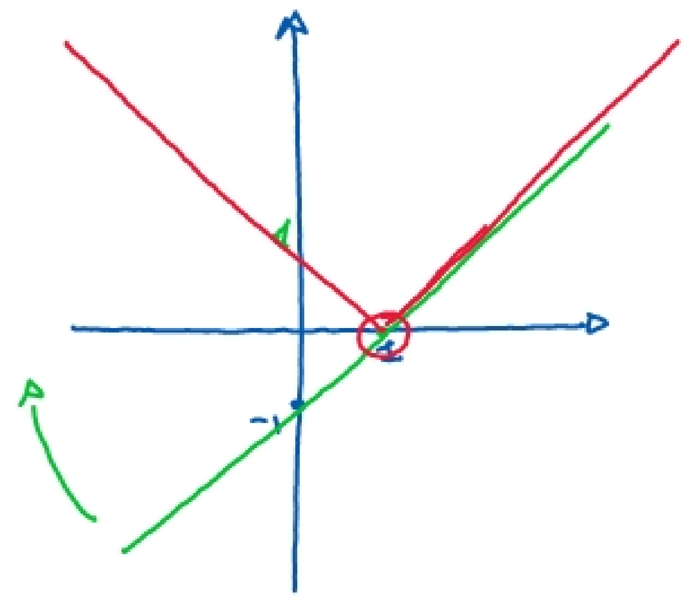
$x = -1 \qquad \lim_{x \to -1^\pm} f(x) = \lim_{x \to -1^\pm} \dfrac{\cancel{(x-2)}\overset{(+2)}{(x+3)}}{\cancel{(x-2)}(x+1)} = \pm\infty$

$$f(x) = \begin{cases} \dfrac{x^2+x-6}{x^2-x-2} & \text{per } x \neq 2,\ x \neq -1 \\ 5/3 & \text{per } x = 2 \end{cases}$$

es 4 $f(x) = |x-1| =$

dominio
continuità

$$= \begin{cases} x-1 & x \geq 1 \\ -x+1 & x < 1 \end{cases}$$



$f_+(1) = 0$

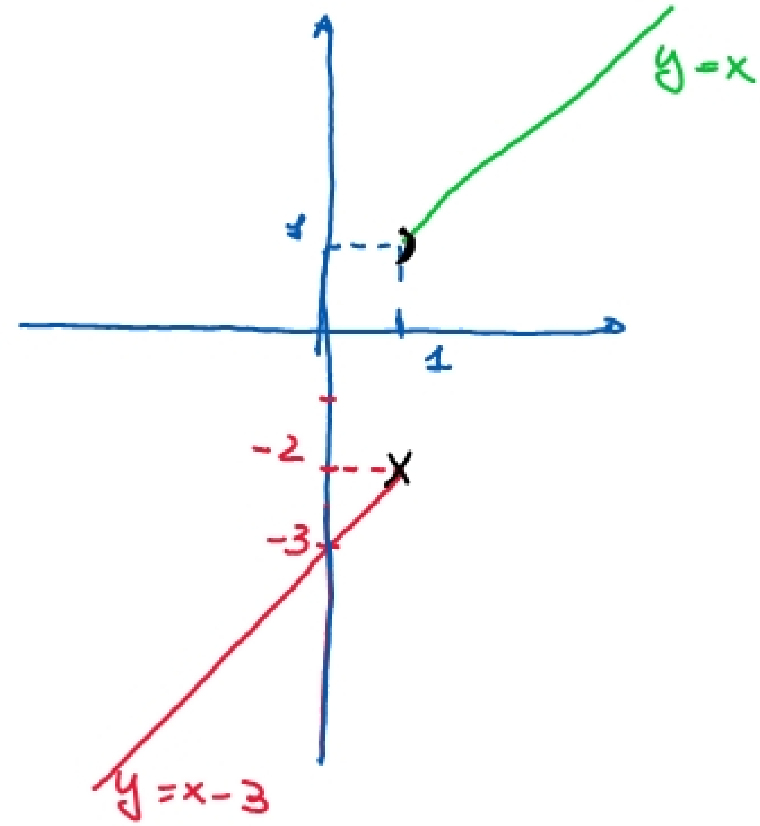
$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (-x+1) = 0$$

la funzione è continua in $x = 1$

es5

$$f(x) = \begin{cases} x & \text{per } x \geq 1 \\ x-3 & \text{per } x < 1 \end{cases}$$

dominio è $\mathbb{R}$



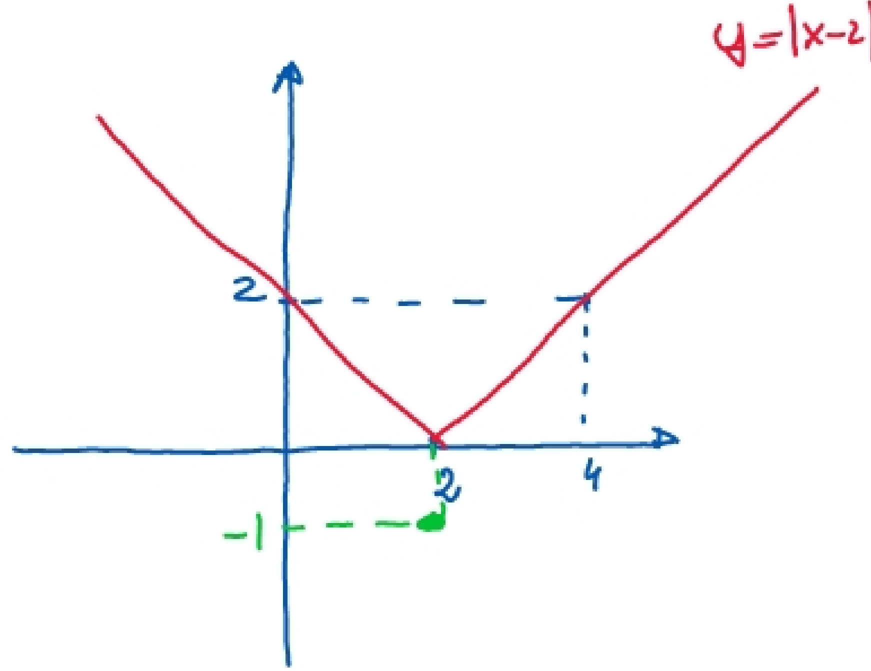
$f_+(1) = 1 \quad \left(\lim_{x \to 1^+} f(x) = 1 \right)$

$$\lim_{x \to 1^-} (x-3) = -2$$

per $x = 1$ $f(x)$ ha un salto

es 6

$$f(x) = \begin{cases} |x-2| & x \neq 2 \\ -1 & x = 2 \end{cases}$$



$$\lim_{x \to 2^+} (x-2) = 0$$

$$\lim_{x \to 2^-} (-(x-2)) = 0$$

$f(2) = -1 \quad \Rightarrow \quad x = 2$ discontinuità es 7 $\quad f(x) = \dfrac{x^2+2x-3}{|x-1|} = \begin{cases} \dfrac{x^2+2x-3}{x-1} & x>1 \\ \dfrac{x^2+2x-3}{-(x-1)} & x<1 \end{cases}$ $\quad x=1$ pt. ecc.

dominio di f: $|x-1| \neq 0 \Rightarrow x \neq 1 \Rightarrow \forall x \in \mathbb{R} \setminus \{1\}$

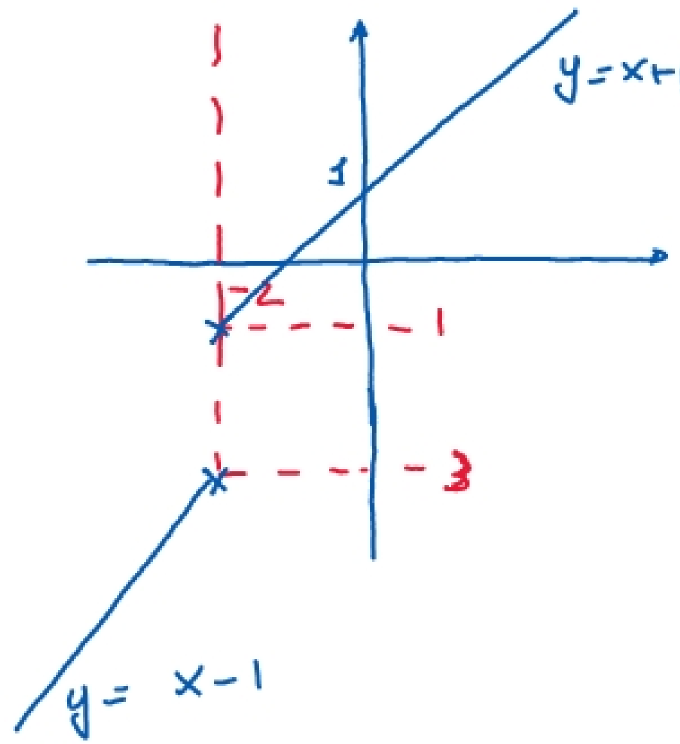
$$\lim_{x\to1^+} \frac{x^2+2x-3}{x-1} = \frac{0}{0} \overset{F.I.}{=} \lim_{x\to1^+} \frac{\cancel{(x-1)}(x+3)}{\cancel{x-1}} = 4$$

$$\lim_{x\to1^-} \frac{x^2+2x-3}{-(x-1)} = \frac{0}{0} \overset{F.I.}{=} \lim_{x\to1^-} \frac{\cancel{(x-1)}(x+3)}{-(\cancel{x-1})} = -4$$

La funzione per $x=1$ non può essere resa continua, perchè presenta un salto.

es 8 $f(x) = x + \frac{x+2}{|x+2|} = \begin{cases} x + \frac{x+2}{x+2} = x+1 & x > -2 \\ x + \frac{x+2}{-(x+2)} = x-1 & x < -2 \end{cases}$ $x = 2$? pt. occ.

dominio di f : $|x+2| \neq 0 \Rightarrow x \neq -2 \Rightarrow \forall x \in R \setminus \{-2\}$



$\lim_{x \to 2^-} (x-1) = -3$

$\lim_{x \to -2^+} (x+1) = -1$

la funzione non può essere

estesa per continuità in $x = -2$, dato che presenta un salto

es 9

$$f(x) = \begin{cases} x+1 & x \leq 1 \\ 3 - a x^2 & x > 1 \end{cases}$$

per quali valori di a è continua? ($a \in \mathbb{R}$)

$$\lim_{x \to 1^-} f(x) = f(1) = 2 \qquad \left[\lim_{x \to 1^-} (x+1) = 2 \right]$$

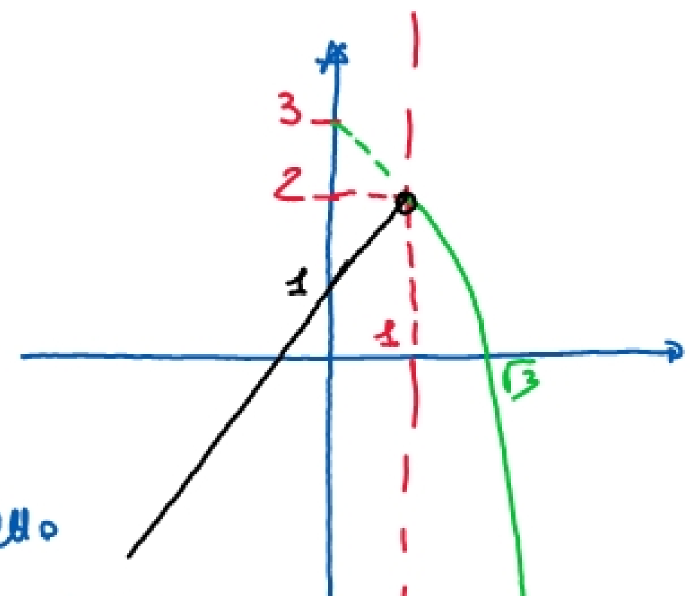
$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (3 - a x^2) = 3 - a$$

si ha la continuità se

$$3 - a = 2 \implies a = 1$$

La funzione diviene:

$$f(x) = \begin{cases} x+1 & x \leq 1 & \text{retta} \\ 3 - x^2 & x > 1 & \text{parabola} \end{cases}$$



es 10

$$f(x) = \begin{cases} \log_3(1+x) & -1 < x \le 0 \\ a\sin x + b\cos x & 0 < x < \pi/2 \\ x & x \ge \pi/2 \end{cases}$$

$a, b \in \mathbb{R}$, continuità di f ?

continuità

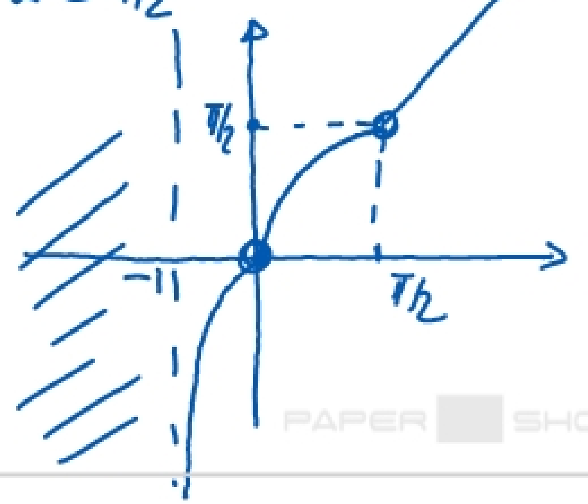
$x = 0$

$$\left.\begin{aligned} & f_-(0) = \log_3(1+0) = 0 \\ & \lim_{x\to 0^+} (a\sin x + b\cos x) = b \end{aligned}\right] \Rightarrow b = 0$$

$x = \pi/2$

$$\left.\begin{aligned} & \lim_{x\to \frac{\pi}{2}^-} (a\sin x + b\cos x) = a \\ & f_+\left(\tfrac{\pi}{2}\right) = \pi/2 \end{aligned}\right] \Rightarrow a = \pi/2$$

$$\Rightarrow f(x) = \begin{cases} \log_3(1+x) & x \in (-1, 0] \\ \pi/2 \sin x & x \in (0, \pi/2) \\ x & x \in [\pi/2, +\infty) \end{cases}$$



<u>es11</u>

$$f(x) = \begin{cases} 3^{-x} + a & x \leq 0 \\ c\,\dfrac{\operatorname{sen} x}{x+b} & 0 < x < \pi \\ 0 & x \geq \pi \end{cases}$$

determinare $a, b, c \in \mathbb{R}$: $f(x)$ sia continua.

$x = 0$ $\quad f_-(0) = 3^0 + a = a + 1$

$$\lim_{x \to 0^+} \left(c\,\frac{\operatorname{sen} x}{x+b} \right) =$$

- se $b = 0$ $\quad \displaystyle\lim_{x \to 0^+} \left(c\,\overbrace{\frac{\operatorname{sen} x}{x}}^{\to 1} \right) = c$
- se $b \neq 0$ $\quad \displaystyle\lim_{x \to 0^+} \left(c\,\frac{\overset{\to 0}{\operatorname{sen} x}}{\underset{\to b}{x+b}} \right) = 0 \quad \forall c \in \mathbb{R}$

$x = \pi$ $\quad f_+(\pi) = 0$

$$\lim_{x \to \pi^-} \left(c\,\frac{\operatorname{sen} x}{x+b} \right) =$$

- se $b = -\pi$ $\quad \displaystyle\lim_{x \to \pi} \left(c\,\overbrace{\frac{\operatorname{sen} x}{x - \pi}}^{\to 1} \right)^{*} = -c$
- se $b \neq -\pi$ $\quad \displaystyle\lim_{x \to \pi^-} \left(c\,\frac{\overset{\to 0}{\operatorname{sen} x}}{\underset{\pi + b}{x+b}} \right) = 0$

dominio $x \neq -b$: se $b = -\pi$

$$(*) \quad \lim_{x \to \pi} \left(c\,\frac{\operatorname{sen} x}{x - \pi} \right) = \lim_{x \to \pi} c \left(\frac{\operatorname{sen}(\pi - x)}{-(\pi - x)} \right) = -c$$

se (per la continuità)

$(x=0)$

$b=0 \qquad c=a+1$

$b\neq 0 \qquad 1+a=0 \qquad a=-1 \qquad \forall c\in\mathbb{R}$

$b=-\pi \qquad -c=0 \qquad c=0$

$b\neq-\pi \qquad 0=0 \qquad \forall c\in\mathbb{R}$ (continuità per $x=\pi$)

es 12

$$f(x) = \begin{cases} \dfrac{\log x^2 - 1}{\log x^2 + 1} & \text{se } x \neq 0 \ , \ x \neq \pm e^{-1/2} \\ 1 & \text{se } x = 0 \ , \ x = \pm e^{-1/2} \end{cases}$$

è continua?

dominio $\begin{cases} x \neq 0 & (\text{Arg. del log.}) \\ \log x^2 + 1 \neq 0 & \log x^2 \neq -1 \quad x^2 = e^{-1} \quad x \neq \pm\sqrt{e^{-1}} = \pm e^{-1/2} \end{cases}$

$-1 \log e = \log e^{-1}, \ >0$

$$\lim_{x \to 0^\pm} \frac{\log x^2 - 1}{\log x^2 + 1} = \lim_{x \to 0^\pm} \frac{\cancel{\log x^2}\left(1 - \frac{1}{\log x^2}\right)}{\cancel{\log x^2}\left(1 + \frac{1}{\log x^2}\right)} = 1$$

$\to 0$

$$\lim_{x \to 0^\pm} \log x^2 = -\infty$$

$x = 0 \quad f(x)$ continua

$$\lim_{x \to \pm e^{-\frac{1}{2}} \pm} \frac{\log x^2 - 1}{\log x^2 + 1} = \frac{-1-1}{-1+1} = \frac{-2}{0} = \infty$$

$x = \pm e^{-1/2}$ pt di infinito

$$x^2 = \left(\pm e^{-\frac{1}{2}}\right)^2 = \pm e^{-1} \qquad \log x^2 = \log(e^{-1}) = -1 \log e = -1$$

T.E. $f(x) = \begin{cases} -1 & x = 1, 2 \\ \log|x-2| + 3\,\dfrac{e^{x-1}-1}{x-1} & x \neq 1,\ x \neq 2 \end{cases}$ è continua?

$\underline{x = 2}$

$$\lim_{x \to 2^{\pm}} \left(\underbrace{\log|x-2|}_{\log 0 \to -\infty} + 3\,\frac{e^{x-1}-1}{x-1} \right) = -\infty$$

$$3\,\frac{e^{1}-1}{\cancel{2-1}\ 1}$$

$x = 2$ pt. di infinito

$\underline{x = 1}$

$$\lim_{x \to 1^{\pm}} \left(\log|x-2| + \overbrace{3\,\frac{e^{x-1}-1}{x-1}}^{3} \right) = 3$$

pt. di disc. eliminabile

$\log|-1| =$
$= \log 1 = 0$

$$\lim_{x \to 1} \frac{e^{x-1}-1}{x-1} = 1$$

T.E. $f(x) = \begin{cases} \dfrac{\cos x - 1}{x}\, e^{x-2} + \cos \dfrac{7}{|x-7|} & x \neq 0,\ x \neq 7 \\ 0 & x = 0,\ x = 7 \end{cases}$ continuité ?

$x = 0$ $\quad \lim_{x \to 0^\pm} \left(\dfrac{\cos x - 1}{x}\, e^{x-2} + \cos \dfrac{7}{|x-7|} \right) = 0 \cdot e^{-2} + \cos 1 = \cos 1 \neq 0$

$e^{0-2} = e^{-2}$

$\cos \dfrac{7}{|7|} = \cos 1$

$-\dfrac{(1 - \cos^2 x)}{x\,(1 + \cos^2 x)} = -\dfrac{\sin^2 x}{x\,(1 + \cos x)} = -x\, \dfrac{\sin^2 x}{x^2} \cdot \dfrac{1}{1 + \cos x}$

$\lim_{x \to 0^\pm} \left(-x\, \underbrace{\dfrac{\sin^2 x}{x^2}}_{\to 1} \cdot \underbrace{\dfrac{1}{1 + \cos x}}_{\to 1/2} \right) = 0$

$\underline{x=7}$

$$\lim_{x \to 7^{\pm}} \left(\frac{\cos x - 1}{x} e^{x-2} + \cos \frac{7}{|x-7|} \right) = \nexists$$

$$\frac{\cos 7 - 1}{7} e^{5}$$

$$\cos \frac{7}{0}$$

$[-1, 1]$

T.E.

$$f(x) = \begin{cases} \dfrac{x+1}{|x+1|}\left(\dfrac{x+8}{x-7}\right)^{5/3} & x \neq -1,\ x \neq 7 \\ 1 & x = -1,\ x = 7 \end{cases}$$

continua?

$$f(x) = \begin{cases} -\left(\frac{x+8}{x-7}\right)^{5/3} & x < -1,\ x \neq 7 \\ \left(\frac{x+8}{x-7}\right)^{5/3} & x > -1,\ x \neq 7 \\ 1 & x = -1,\ x = 7 \end{cases}$$

$$\lim_{x \to -1^-} -\left(\frac{x+8}{x-7}\right)^{5/3} = -\left(\frac{7}{-8}\right)^{5/3} = \left(7/8\right)^{5/3} > 0$$

$$\lim_{x \to -1^+} \left(\frac{x+8}{x-7}\right)^{5/3} = \left(\frac{7}{-8}\right)^{5/3} = -\left(\frac{7}{8}\right)^{5/3} < 0$$

$x = -1$ salto

$$\lim_{x \to 7^\pm} \frac{x+1}{|x+1|}\left(\frac{x+8}{x-7}\right)^{5/3} = \pm\infty$$

(a seconda che sia $x \to 7^-$, $x \to 7^+$)

$x = 7$ punto di infinito.

T.E. $f(x) = \begin{cases} 2\left(x-\frac{\pi}{2}\right) e^{\frac{1}{x-\pi/2}} + 7\,\dfrac{\sin x}{|x-\pi|} & x \neq \frac{\pi}{2},\ x \neq \pi \\ 2 & x = \frac{\pi}{2},\ x = \pi \end{cases}$

continuità?

$x = \pi/2$

$$\lim_{x \to \frac{\pi}{2}} \left\{ 2\left(x-\frac{\pi}{2}\right) e^{\frac{1}{x-\pi/2}} + \underbrace{7\,\frac{\overset{\nearrow 1}{\sin x}}{\underset{\downarrow}{|x-\pi|}}}_{7 \cdot \frac{2}{\pi}} \right\} = \begin{cases} \lim\limits_{x \to \pi/2^-} f(x) = 14/\pi \\ \lim\limits_{x \to \pi/2^+} f(x) = +\infty \end{cases}$$

$$|-\pi/2| = \pi/2$$

$x = \pi/2$ punto di infinito

$$\lim_{x \to \pi/2^-} \left(\underset{\downarrow 0}{2\left(x-\frac{\pi}{2}\right)} e^{\frac{1}{x-\pi/2}} \right) = 0$$

$$e^{\frac{1}{0^-}} \to e^{-\infty} \to 0$$

$$\lim_{x \to \pi/2^+} \left(\underset{\downarrow 0}{2\left(x-\frac{\pi}{2}\right)} \underset{\downarrow}{e^{\frac{1}{x-\pi/2}}} \right) = \text{F.I.} = \lim_{t \to +\infty} \frac{2e^t}{t} = +\infty$$

$$e^{\frac{1}{0^+}} \to e^{+\infty} \to +\infty$$

(ordine di infinito)

posto $t = \frac{1}{x-\pi/2} \to 0^+$, $t \to +\infty$

$x = \pi$

$$\lim_{x \to \pi} \left(2\left(x - \frac{\pi}{2}\right) e^{\frac{1}{x-\pi/2}} + 7 \frac{\operatorname{sen} x}{|x-\pi|} \right) =$$

$$= \lim_{t \to 0} \left\{ \frac{7 \operatorname{sen}(t+\pi)}{|t|} \right\} + \pi e^{\frac{2}{\pi}} = \lim_{t \to 0} \left(\frac{7(-\operatorname{sen} t)}{\pm t} \right) + \pi e^{\frac{2}{\pi}}.$$

$$\underbrace{x - \pi}_{\downarrow \pi} = \underbrace{t}_{\downarrow 0}$$

$$\Rightarrow \lim_{t \to 0^-} \left\{ 7 \underbrace{\frac{(-\operatorname{sen} t)}{-t}}_{\downarrow 1} \right\} + \pi e^{\frac{2}{\pi}} = 7 + \pi e^{2/\pi}$$

salto!

$$\lim_{t \to 0^+} \left\{ 7 \underbrace{\frac{(-\operatorname{sen} t)}{t}}_{-\downarrow 1} \right\} + \pi e^{2/\pi} = -7 + \pi e^{2/\pi}$$

T.E.

$$f(x) = \begin{cases} \sin(x-1)\ \dfrac{|x-1|}{e^{x-1}-1} \cdot \dfrac{1}{\sqrt{|x-8|}} & x \neq 1,\ x \neq 8 \\ 0 & x = 1, 8 \end{cases}$$

continuità ?

$x = 1$

$$\lim_{x \to 1^\pm} \left(\underbrace{\sin(x-1)}_{\to 0}\ \underbrace{\frac{|x-1|}{e^{(x-1)}-1}}_{\to \pm 1} \cdot \underbrace{\frac{1}{\sqrt{|x-8|}}}_{\frac{1}{\sqrt{|-7|}} = \frac{1}{\sqrt{7}}} \right) = 0$$

$\Rightarrow$ CONTINUA

$$\left(\frac{e^{(x-1)}-1}{|x-1|} \right)^{-1} = ?$$

$$\lim_{x \to 1^-} \frac{e^{(x-1)}-1}{-(x-1)} = -1$$

$$\lim_{x \to 1^+} \frac{e^{x-1}-1}{(x-1)} = 1$$

$x = 8$

$$\lim_{x \to 8^\pm} \left(\sin(x-1) \cdot \frac{|x-1|}{e^{(x-1)}-1} \cdot \frac{1}{\sqrt{|x-8|}} \right) = +\infty$$

disc. pr. di infinito

$$\underbrace{\sin 7}_{>0} \cdot \frac{7}{e^7-1} \cdot \frac{1}{\sqrt{0}}$$

$2\pi < 7 < 2\pi + \pi/2$