OPERATORI IPERBOLICI

1. definizione degli operatori
2. funzioni dirette e inverse
3. algebra degli operatori iperbolici

1. definizione degli operatori iperbolici

$$\operatorname{senh} x = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\coth x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

$$\operatorname{sech} x = \frac{2}{e^{x} + e^{-x}}$$

$$\operatorname{cosech} x = \frac{2}{e^{x} - e^{-x}}$$

operatori inversi

$$\text{arcsenh}\, x = \log\left(x + \sqrt{x^2+1}\right) \qquad \text{arccoth}\, x = \frac{1}{2} \log \frac{x+1}{x-1}$$

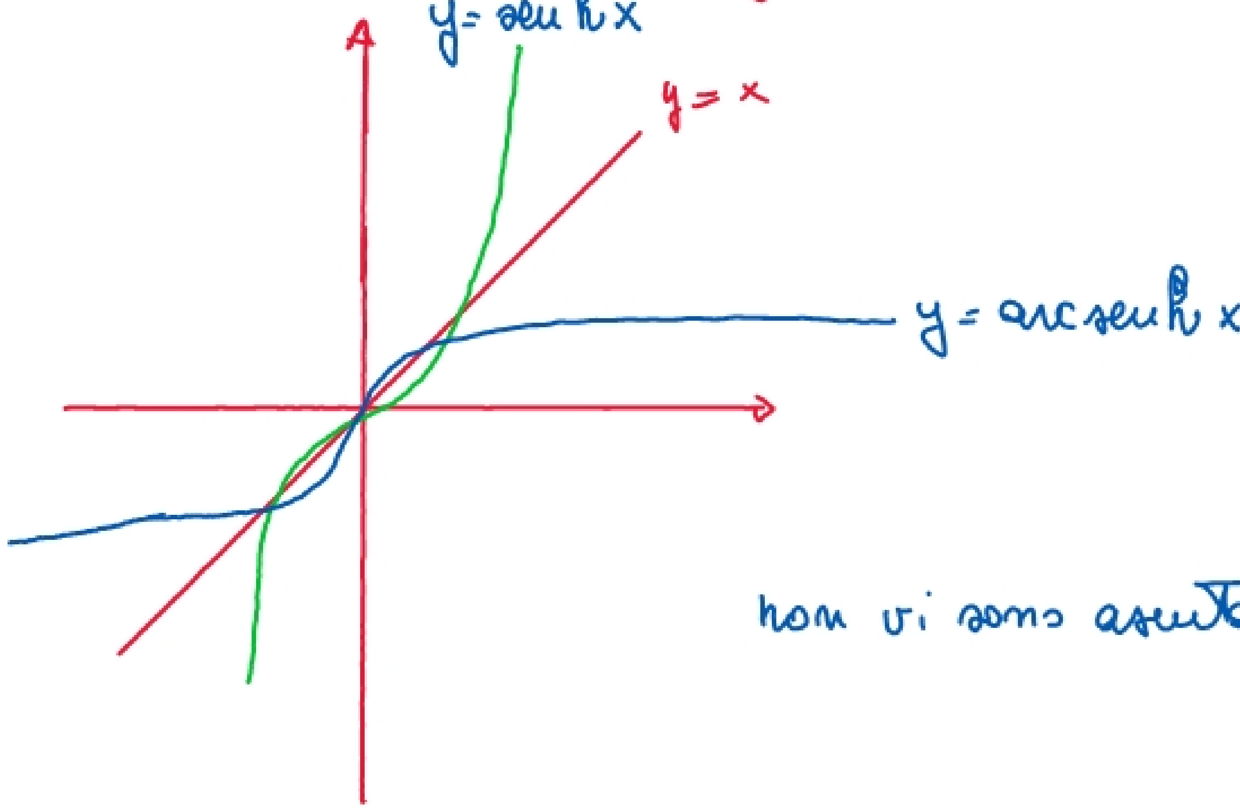
$$\text{arccosh}\, x = \log\left(x \pm \sqrt{x^2-1}\right) \qquad \text{arcsech}\, x = \log \frac{1 \pm \sqrt{1-x^2}}{x}$$

$$\text{arctanh}\, x = \frac{1}{2} \log \frac{1+x}{1-x} \qquad \text{arccosech}\, x = \log \frac{1 \pm \sqrt{1+x^2}}{x}$$

N.B per $x>0 \rightarrow$ segno $+$ per $x<0 \rightarrow$ segno $-$

2. funzioni dirette e inverse: grafici

$y = \operatorname{senh} x$ $\qquad\qquad$ $y = \operatorname{arcsenh} x$



non vi sono asintoti

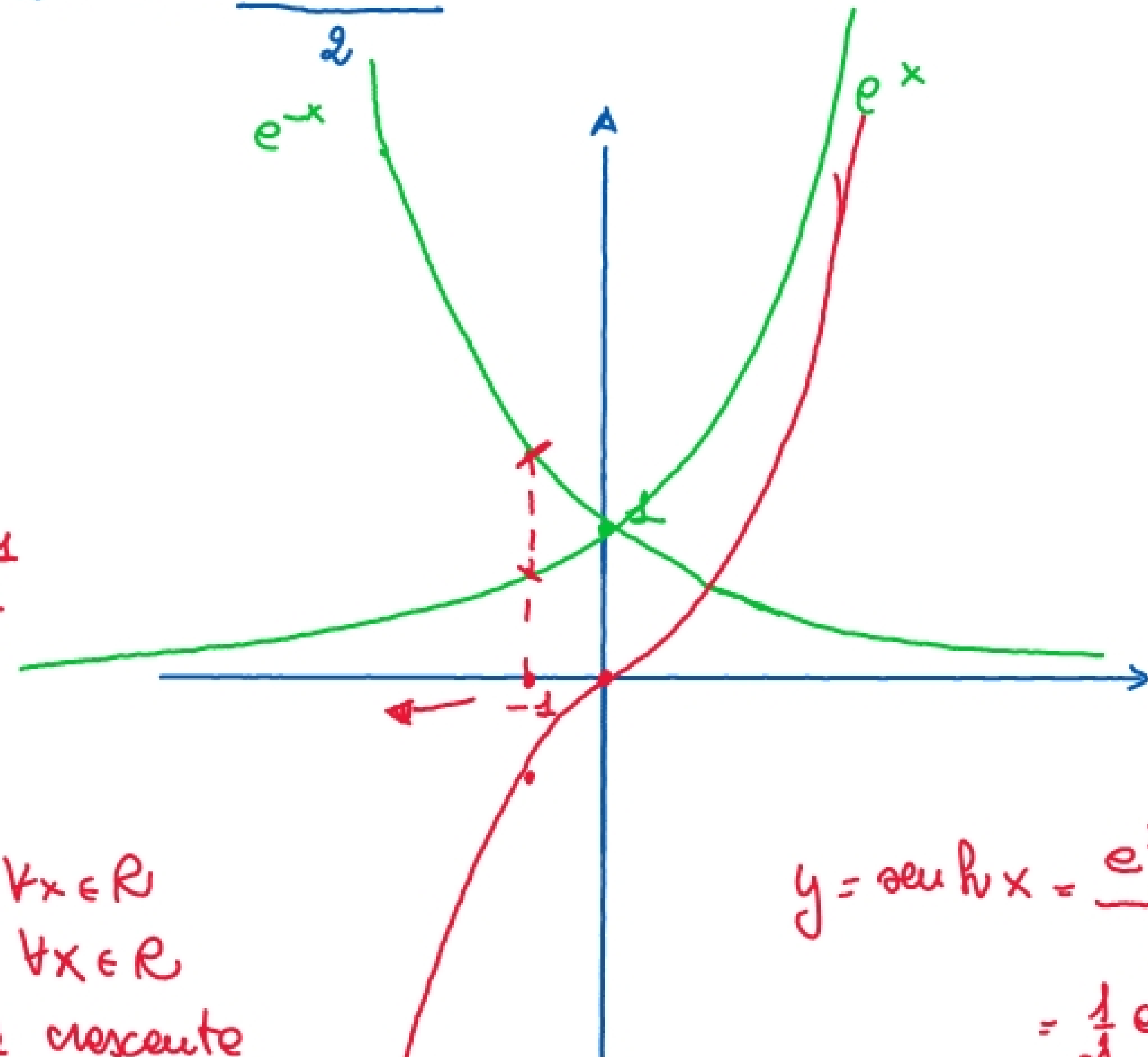
$$y = \operatorname{senh} x = \frac{e^x - e^{-x}}{2}$$

$y = e^x$

$y = e^{-x} = \left(\frac{1}{e}\right)^x$

$x = -1$

$$y = \frac{e^{-1} - e^{1}}{2}$$



dominio $\forall x \in \mathbb{R}$

codominio $\forall x \in \mathbb{R}$

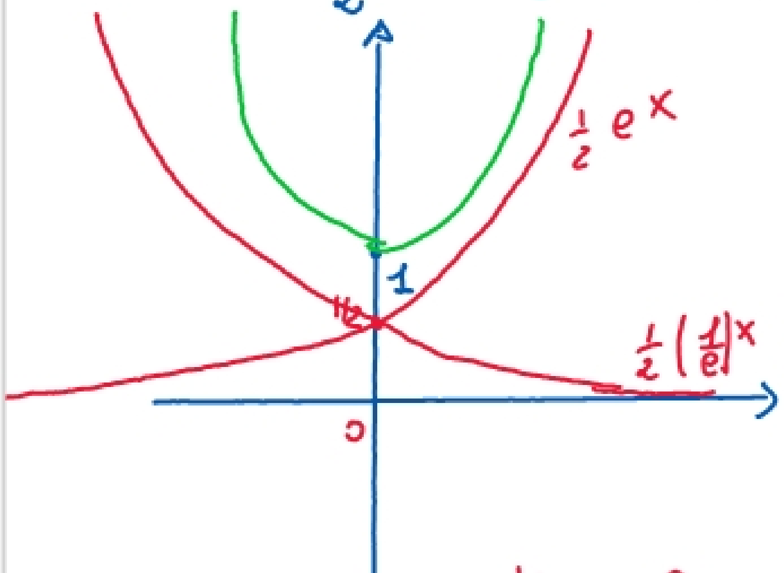
strettamente crescente

$f(-x) = -f(x)$ dispari (simmetrica rispetto all'origine)

$$y = \operatorname{senh} x = \frac{e^x - e^{-x}}{2} = \frac{1}{2} e^x - \frac{1}{2}\left(\frac{1}{e}\right)^x$$

$y = \cosh x =$

$$= \frac{e^x + e^{-x}}{2} = \frac{1}{2}e^x + \frac{1}{2}\left(\frac{1}{e}\right)^x$$



dominio $\forall x \in \mathbb{R}$

codominio $[1, +\infty[$

decrescente $]-\infty, 0[$

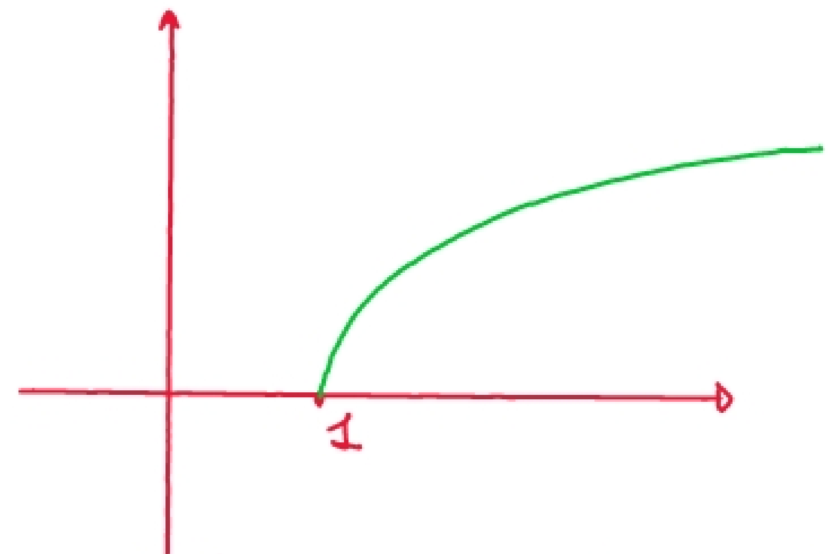
crescente $[0, +\infty[$

$f(-x) = f(x)$ pari

sempre positiva

minimo $x = 0$ $P(0, 1)$

$y = \operatorname{arccosh} x$

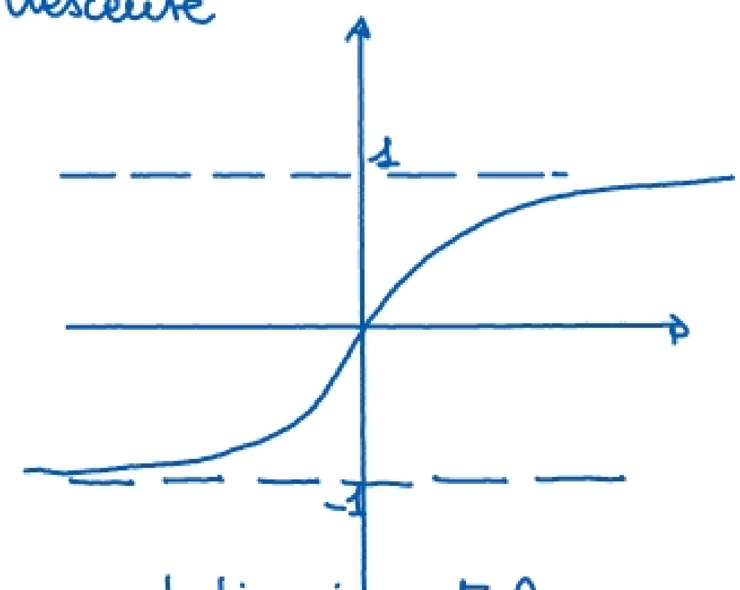


si inverte il tratto crescente $[0, +\infty[$

$$y = \tanh x =$$

$$= \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

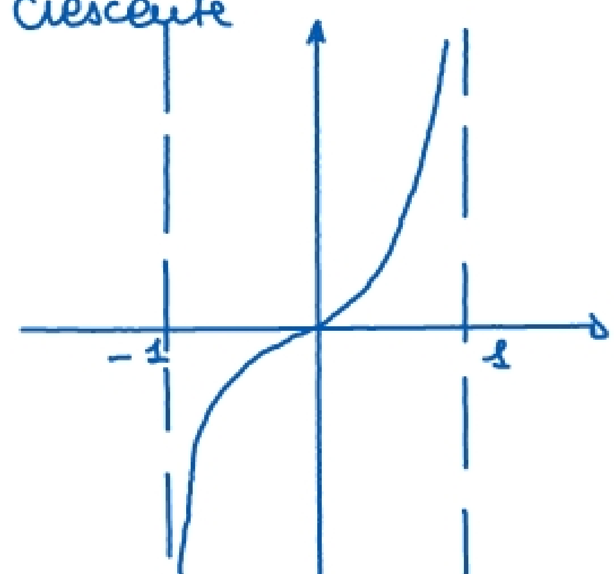
funzione dispari
definita in $\mathbb{R}$
crescente



2 asintoti orizzontali
$y = 1 \qquad y = -1$

$$y = \operatorname{arctanh} x$$

funzione dispari
dominio $]-1, 1[$
crescente



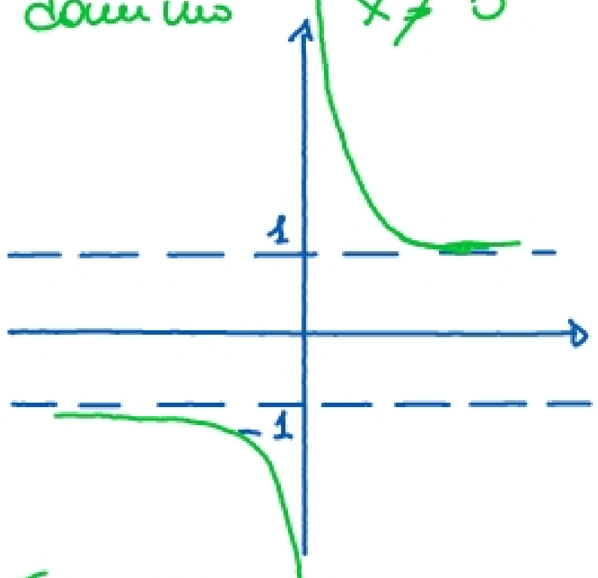
2 asintoti verticali
$x = -1 \qquad x = 1$

$y = \coth x =$

$$= \frac{\cosh x}{\operatorname{senh} x} = \operatorname{tgh}^{-1} x =$$

$$= \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$$

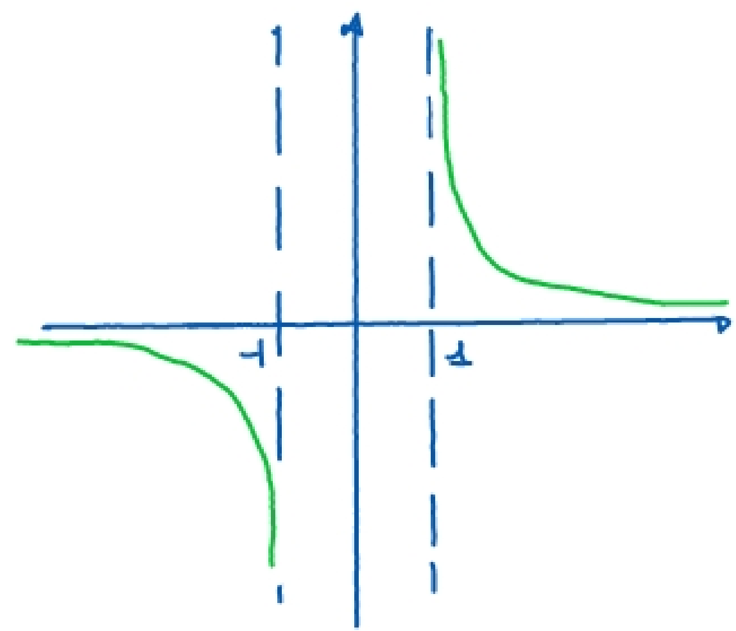
funzione dispari
dominio $x \neq 0$



Tratto decrescente,
1 asintoto verticale $x = 0$
2 asintoti orizzontali $y = 1$, $y = -1$

$y = \operatorname{arccoth} x$

funzione dispari
dominio $\mathbb{R} \setminus [-1, 1]$
decrescente



1 asintoto orizzontale $y = 0$
2 asintoti verticali $x = -1$, $x = 1$

RELAZIONI FONDAMENTALI TRA LE PRINCIPALI FUNZIONI IPERBOLICHE

$$\cosh^2 x - \operatorname{senh}^2 x = 1$$

$$\operatorname{Tanh} x = \frac{\operatorname{senh} x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\operatorname{senh} x} = \operatorname{Tanh} x^{-1}$$

FORMULE DI ADDIZIONE

$$\operatorname{senh}(x \pm y) = \operatorname{senh} x \cosh y \pm \cosh x \operatorname{senh} y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \operatorname{senh} x \operatorname{senh} y$$

FORMULE DI DUPLICAZIONE

$$\operatorname{senh} 2x = 2 \operatorname{senh} x \cosh x$$

$$\cosh 2x = \cosh^2 x + \operatorname{senh}^2 x = \begin{cases} 1 + 2 \sinh^2 x \\ 2 \cosh^2 x - 1 \end{cases}$$

(per sostituzione)

COME RICAVARE LA FUNZIONE INVERSA

$y = \sinh x = \dfrac{e^x - e^{-x}}{2} \longrightarrow y = \operatorname{arcsinh} x = \ln\left(x + \sqrt{x^2+1}\right)$

pongo: $x = \sinh y = \dfrac{e^y - e^{-y}}{2} = \dfrac{e^{2y} - 1}{2e^y}$

$e^{2y} - 2e^y x - 1 = 0$

$e^y = \dfrac{2x \pm \sqrt{4x^2+4}}{2} = x \pm \sqrt{x^2+1}$ (Risoluzione eq. 2° grado)

quindi:

$e^y = x + \sqrt{x^2+1}$

[N.B. si esclude la soluzione: $e^y = x - \sqrt{x^2+1}$ perché non è mai positiva]

$y = \log\left(x + \sqrt{x^2+1}\right)$